INFLUENCE DIAGRAM MODELS WITH CONTINUOUS VARIABLES

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INTRODUCTION

OUTLINE OF PRESENTATION

- THEORY
  - INFLUENCE DIAGRAMS
  - NORMAL INFLUENCE DIAGRAM
- APPLICATIONS
  - DISCRETE - TIME FILTERING
  - ΔV - ACCURACY ANALYSIS

INFLUENCE DIAGRAMS

DEFINITIONS

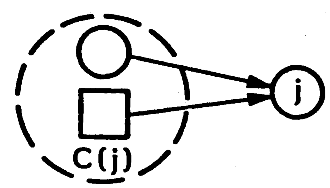
- ACYCLIC DIRECTED GRAPH WITH NODES $N = \{1, \ldots, n\}$
- VECTOR OF VARIABLES $X = (X_1, \ldots, X_n)^T = X_N$
- NODES

○ = CHANCE NODE IN $C \subseteq N$ (RANDOM VARIABLES)

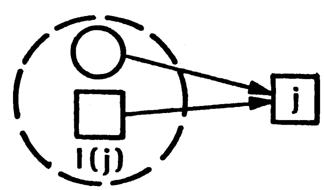
□ = DECISION NODE IN $D \subseteq N$ (DECISION VARIABLES)

◇ = VALUE NODE (CONDITIONAL EXPECTATION OF VALUE FUNCTION)

- ARCS



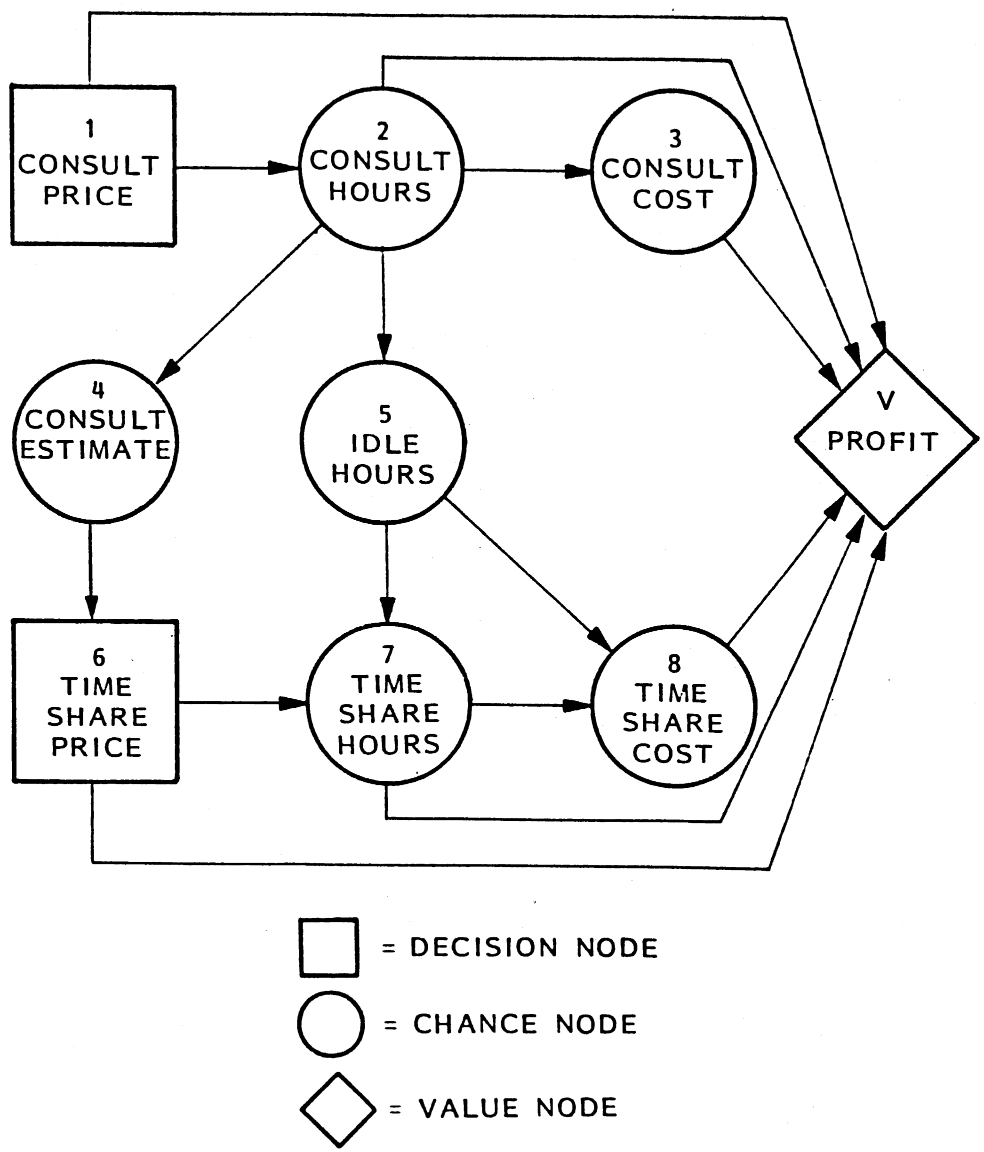
PROBABILISTIC CONDITIONING OF X_j ON $X_{C(j)}$



INFORMATION OBSERVABLE IS $X_{I(j)}$ WHEN SELECTING DECISION X_j

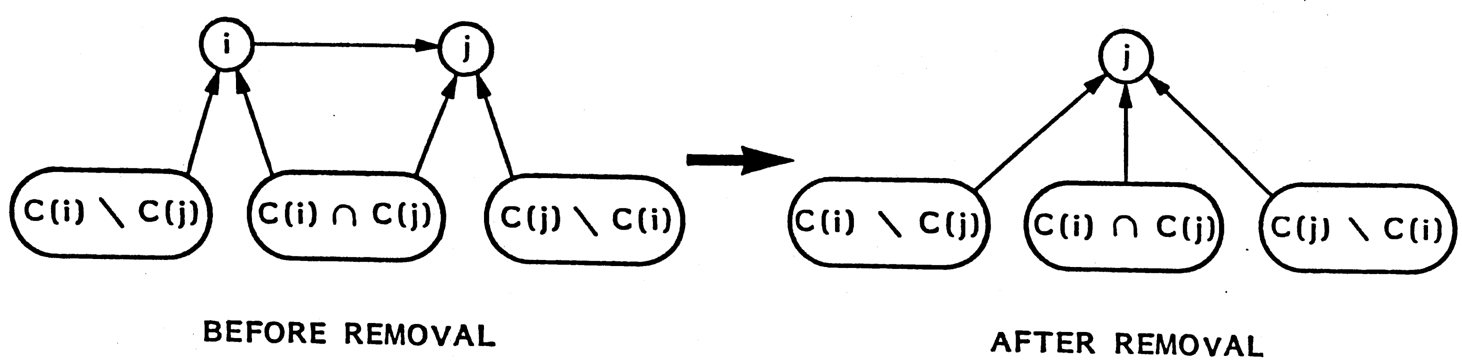
INFLUENCE DIAGRAMS

CONSULTANT'S PROBLEM



INFLUENCE DIAGRAMS

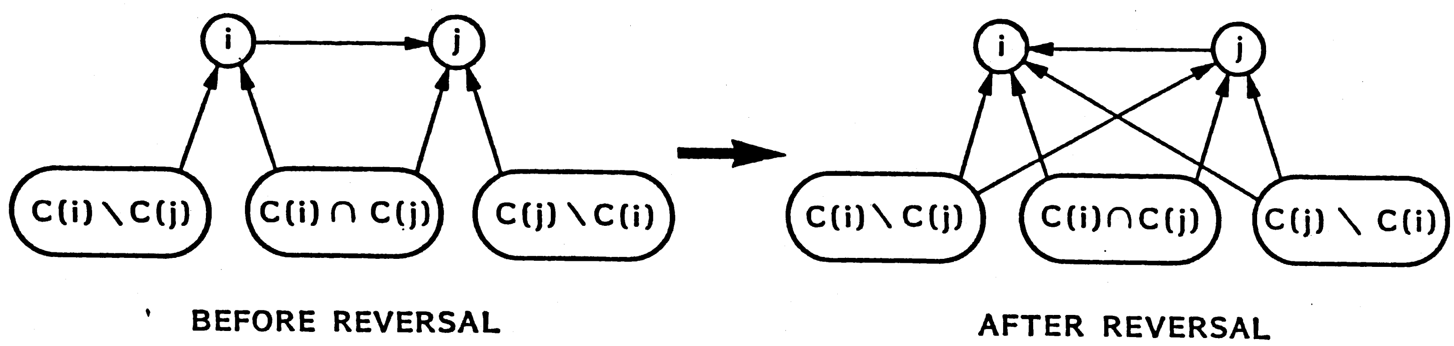
REMOVAL OF CHANCE NODE i INTO CHANCE NODE j



- "\" = SET SUBTRACTION
- INTEGRATION OF JOINT PROBABILITY WITH RESPECT TO R.V. X_i
- NODE i MUST HAVE SINGLE SUCCESSOR NODE j

INFLUENCE DIAGRAMS

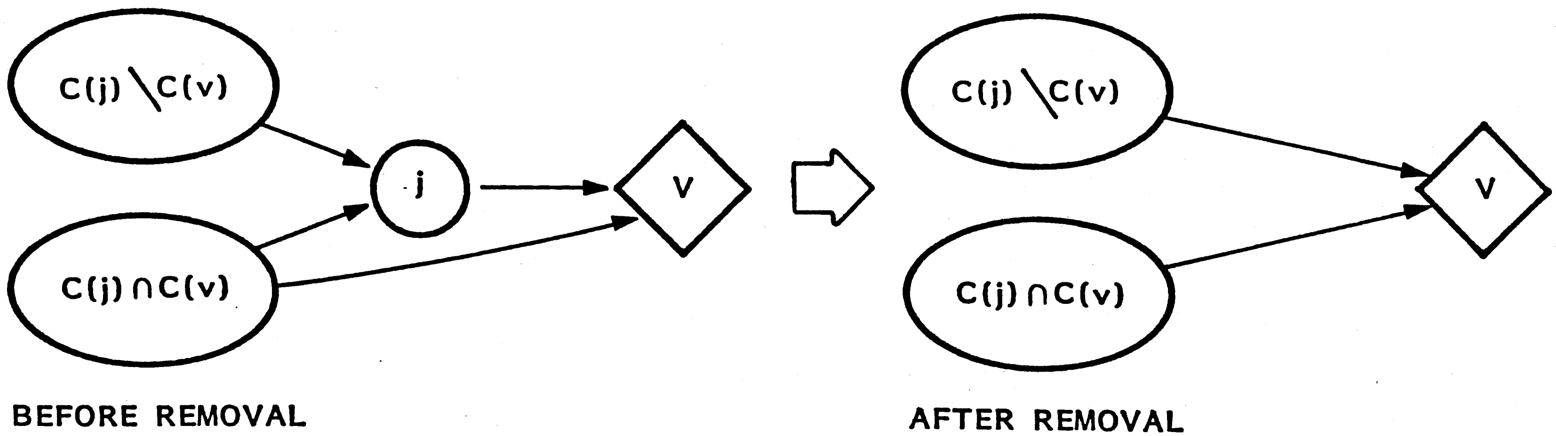
REVERSAL OF ARC FROM CHANCE NODE i TO CHANCE NODE j



- INFLUENCE DIAGRAM VERSION OF BAYES' RULE
- REVERSAL CANNOT CREATE CYCLE (NO OTHER PATH FROM i TO j)
- NODES i AND j HAVE SAME CONDITIONING SET, $C(i) \cup C(j)$, AFTER REVERSAL

INFLUENCE DIAGRAMS

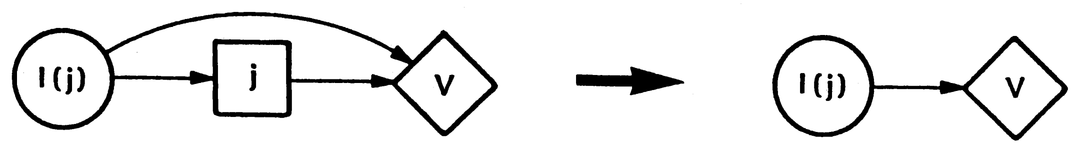
REMOVAL OF CHANCE NODE j INTO VALUE NODE v



- EXPECTATION OF VALUE FUNCTION WITH RESPECT TO X_j
- NODE j HAS SINGLE SUCCESSOR NODE V

INFLUENCE DIAGRAMS

REMOVAL OF DECISION NODE j INTO VALUE NODE V



BEFORE REMOVAL

AFTER REMOVAL

- MAXIMIZATION OF VALUE FUNCTION BY CHOICE BY X_j
- NODE j HAS SINGLE SUCCESSOR NODE V
- $C(V) \subseteq \{j\} \cup I(j)$
- ALL NODES HAVING AN EFFECT ON VALUE NODE MUST BE OBSERVABLE

MULTIVARIATE NORMAL DISTRIBUTION

DEFINITIONS

$N = \{1, \ldots, n\}$, A SET OF INTEGERS

FOR $j \in N$, X_j IS A SCALAR RANDOM VARIABLE

X_N IS MULTIVARIATE NORMAL

- $E[X_N] = \mu_N$
- $Cov(X_N) = E[X_N X_N^T] - E[X_N]\, E[X_N^T] = \Sigma_{NN}$

CONDITIONAL DISTRIBUTIONS

- $\left\{X_j \mid X_{C(j)}\right\}$ UNIVARIATE NORMAL
- $E\left[X_j \mid X_{C(j)} = x_{C(j)}\right] = \mu_j + \Sigma_{jC(j)} \left(\Sigma_{C(j)C(j)}\right)^{-1} (X_{C(j)} - \mu_{C(j)})$
- $VAR\left[X_j \mid X_{C(j)} = x_{C(j)}\right] = \Sigma_{jj} - \Sigma_{jC(j)} \left(\Sigma_{C(j)C(j)}\right)^{-1} \Sigma_{C(j)\,j}$

NORMAL INFLUENCE DIAGRAM

DEFINITIONS

- FOR CHANCE NODES $j \in C$

$$E\left[X_j \mid X_{C(j)} = x_{C(j)}\right] = \mu_j + \sum_{k \in C(j)} b_{kj}(x_k - \mu_k)$$

$$\mathrm{Var}\left[X_j \mid X_{C(j)} = x_{C(j)}\right] = v_j$$

$v_j = 0 \longleftrightarrow X_j$ DETERMINISTIC, LINEAR FUNCTION OF $X_{c(j)}$

- FOR DECISION NODES $j \in D$

μ_j = REFERENCE DECISION VALUE

$v_j = 0$

- VALUE NODE IS QUADRATIC

$$V(X_N) = 1/2\, X_N^T Q X_N + p^T X_N + r$$

NORMAL INFLUENCE DIAGRAM

ASSESSMENT

1. ASSESS AN INFLUENCE DIAGRAM

   DETERMINE VARIABLES X_N

   DETERMINE GRAPH (C, D, C(j) FOR $j \in C$, I(j) FOR $j \in D$)

2. ORDER THE VARIABLES X_N SO THAT
   $j \in C(k) \rightarrow j < k$
   $j \in I(k) \rightarrow j < k$
3. FOR j = 1 TO n

   IF ($j \in C$)

   $$\mu_j = E\left[X_j \mid X_{C(j)} = \mu_{C(j)}\right]$$
   $$v_j = VAR\left[X_j \mid X_{C(j)} = \mu_{C(j)}\right]$$

   FOR $i \in C(j)$

   $$b_{ij} = \frac{\partial E\left[X_j \mid X_{C(j)\setminus\{i\}} = \mu_{C(j)\setminus\{i\}}\ X_i = x_i\right]}{\partial x_i}$$

   END

   ELSE ($j \in D$)

   μ_j = REFERENCE DECISION VALUE

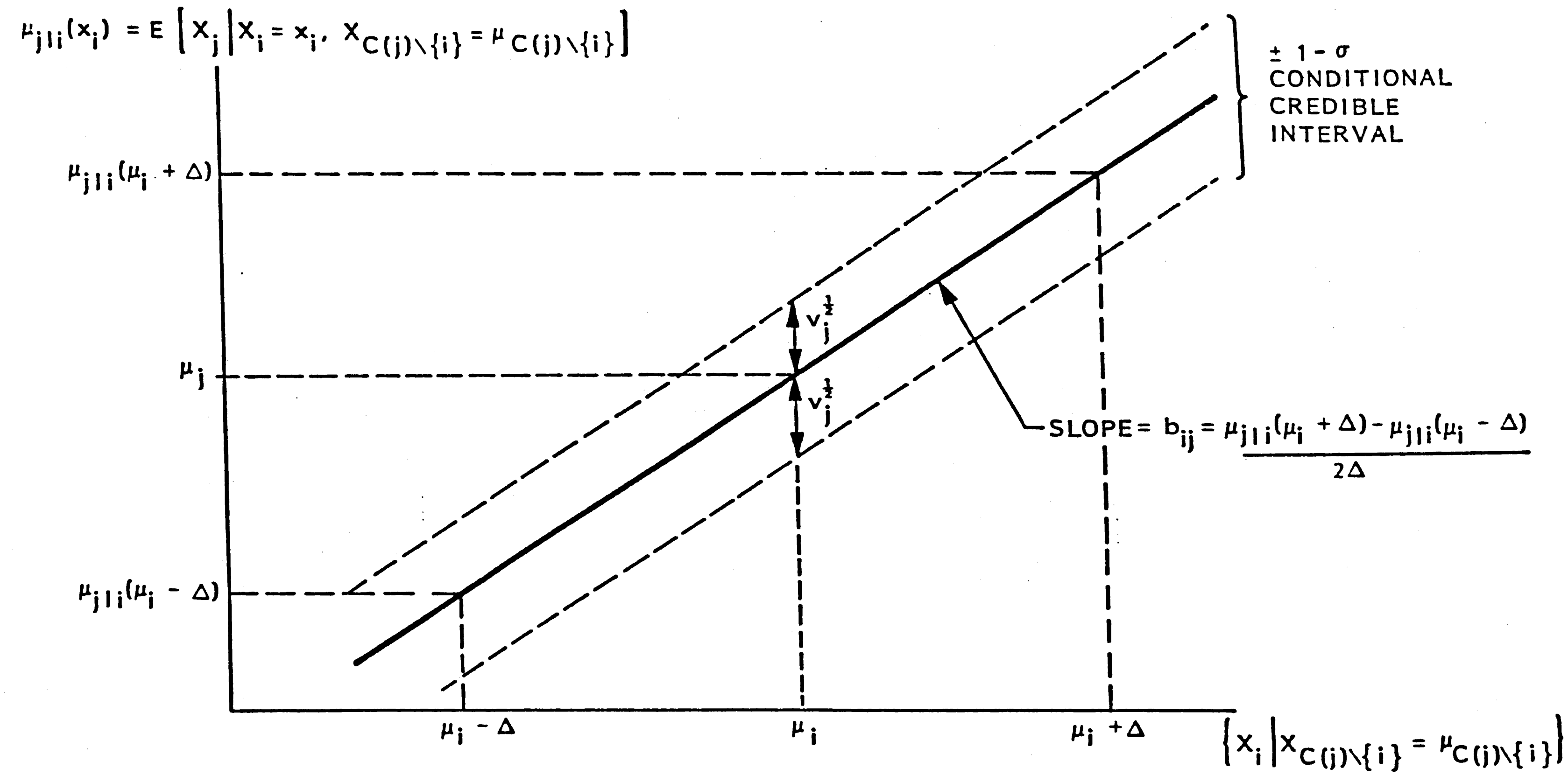
   v_j = 0

   ENDIF

   END

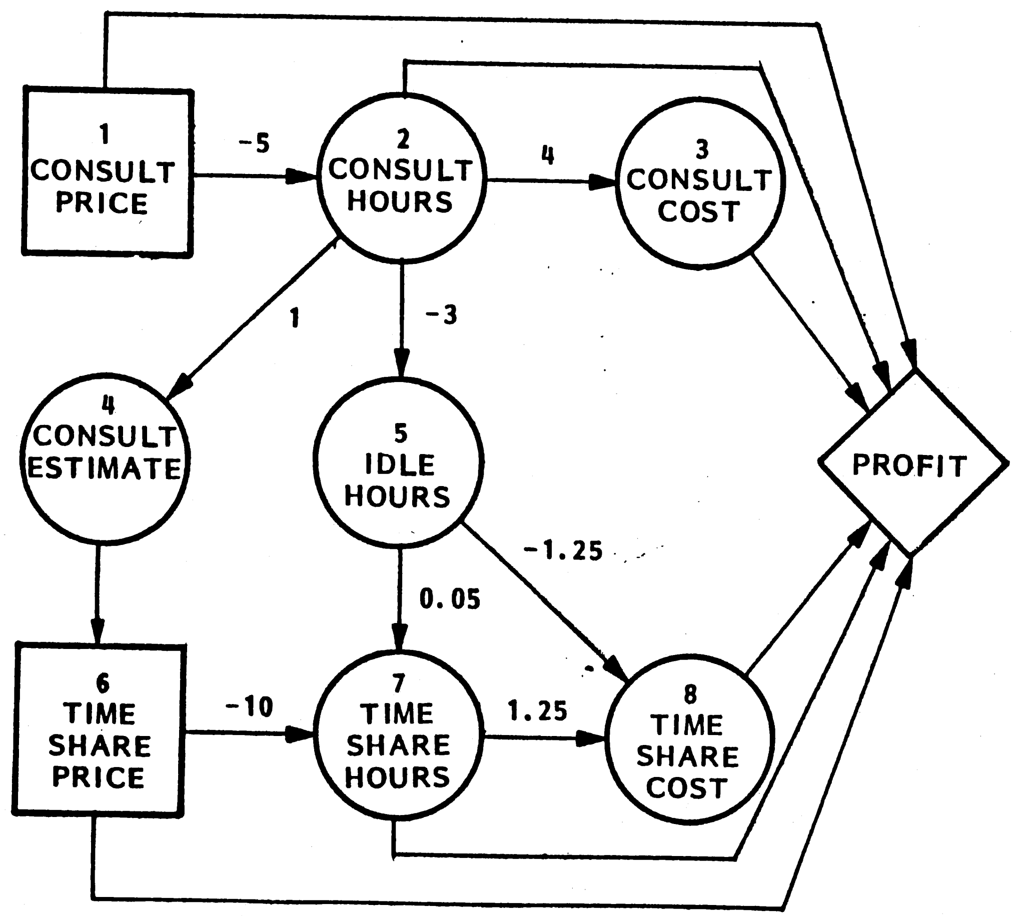
NORMAL INFLUENCE DIAGRAM

ASSESSMENT



NORMAL INFLUENCE DIAGRAM

CONSULTANT'S PROBLEM



$$v = (0\ \ 40{,}000\ \ 4{,}000{,}000\ \ 250{,}000\ \ 100\ \ 0\ \ 10{,}000\ \ 40{,}000)^T$$

$$\mu = (100\ \ 1{,}500\ \ 58{,}000\ \ 1{,}500\ \ 3{,}500\ \ 25\ \ 750\ \ 5{,}000)^T$$

$$\text{PROFIT} = X_1X_2 - X_3 + X_6X_7 - X_8$$

$$N = \{1,2,3,4,5,6,7,8\}$$

$$Q = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$p = (0\ 0\ -1\ 0\ 0\ 0\ 0\ -1)^T$$

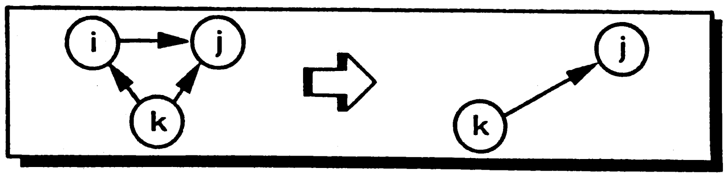
$$r = 0$$

□ = DECISION NODE

○ = CHANCE NODE

NORMAL INFLUENCE DIAGRAM

REMOVAL OF NODE i INTO NODE j



- EXPECTATION WITH RESPECT TO NODE i
- NODE i MUST HAVE SINGLE DIRECT SUCCESSOR NODE j
- ALGORITHM

$$C(j) \leftarrow C(i) \cup C(j) \setminus \{i\}$$

$$b_{kj} \leftarrow b_{kj} + b_{ki} b_{ij} \text{ FOR } k \in C(j)$$

$$v_j \leftarrow v_j + b_{ij}^2 v_i$$

$$N \leftarrow N \setminus \{i\}$$

- NOTE: v_j STAYS NON-NEGATIVE

NORMAL INFLUENCE DIAGRAM

REVERSAL OF ARC FROM NODE i TO NODE j

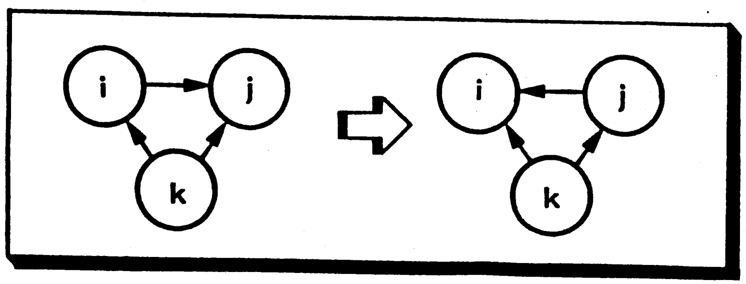
- INFLUENCE DIAGRAM VERSION OF BAYES' RULE
- $i \in C(j)$
- MUST HAVE NO OTHER DIRECTED PATH FROM i TO j (OTHERWISE ACYCLIC LOOP FORMED)
- ALGORITHM

$C(j) \leftarrow C(i) \cup C(j) \setminus \{i\}$

$b_{kj} \leftarrow b_{kj} + b_{ki}\, b_{ij}$ for $k \in C(j)$

$v_j^{old} \leftarrow v_j$

$v_j \leftarrow v_j^{old} + b_{ij}^2\, v_i$



NOTE: v_i AND v_j STAY NON-NEGATIVE

IF $v_j > o$

$C(i) \leftarrow C(j) \cup \{j\}$

$v_{RATIO} \leftarrow v_i / v_j$

$v_i \leftarrow v_j^{old}\ v_{RATIO}$

$b_{ji} \leftarrow b_{ij}\ v_{RATIO}$

$b_{ki} \leftarrow b_{ki} - b_{kj}\, b_{ji}$ for $k \in C(i)$

ELSE

$b_{ji} \leftarrow 0$

END

NORMAL INFLUENCE DIAGRAM

REMOVING CHANCE NODE INTO VALUE NODE

- d = SCALAR RANDOM VARIABLE WITH NO SUCCESSORS OBSERVABLE PRIOR TO REMAINING DECISIONS

- $s = N \setminus \{d\}$

- $Q = \begin{bmatrix} Q_{ss} & Q_{sd} \\ Q_{ds} & Q_{dd} \end{bmatrix}$

- ALGORITHM

$$Q_{ss} \leftarrow Q_{ss} + Q_{sd} B_{sd}^{T} + B_{sd} Q_{ds} + B_{sd} Q_{dd} B_{sd}^{T}$$

$$p_s \leftarrow p_s + B_{sd} p_d + (Q_{sd} + B_{sd} Q_{dd}) (\mu_d - B_{sd}^{T} \mu_s)$$

$$r \leftarrow r + \tfrac{1}{2} Q_{dd} v_d + \tfrac{1}{2} (\mu_d - B_{sd}^{T} \mu_s) Q_{dd} (\mu_d - B_{sd}^{T} \mu_s) + p_d (\mu_d - B_{sd}^{T} \mu_s)$$

$$N \leftarrow N \setminus \{d\}$$

NORMAL INFLUENCE DIAGRAM

REMOVING DECISION NODE INTO VALUE NODE

- X_d = CONTROLLABLE SCALAR VARIABLE SELECTED AFTER OBSERVING X_s WHEN $s = N \setminus \{d\}$
- Q_{dd} NEGATIVE SCALAR ELEMENT OF Q
- ALGORITHM

$$X_d^* (X_s) = - Q_{dd}^{-1} \; p_d - Q_{dd}^{-1} \; Q_{ds} \; X_s$$

$$Q_{ss} \leftarrow Q_{ss} - Q_{sd} \, Q_{dd}^{-1} \, Q_{ds}$$

$$p_s \leftarrow p_s - Q_{sd} \, Q_{dd}^{-1} \, p_d$$

$$r \leftarrow r - \tfrac{1}{2} \, p_d^T \, Q_{dd}^{-1} \, p_d$$

$$N \leftarrow N \setminus \{d\}$$

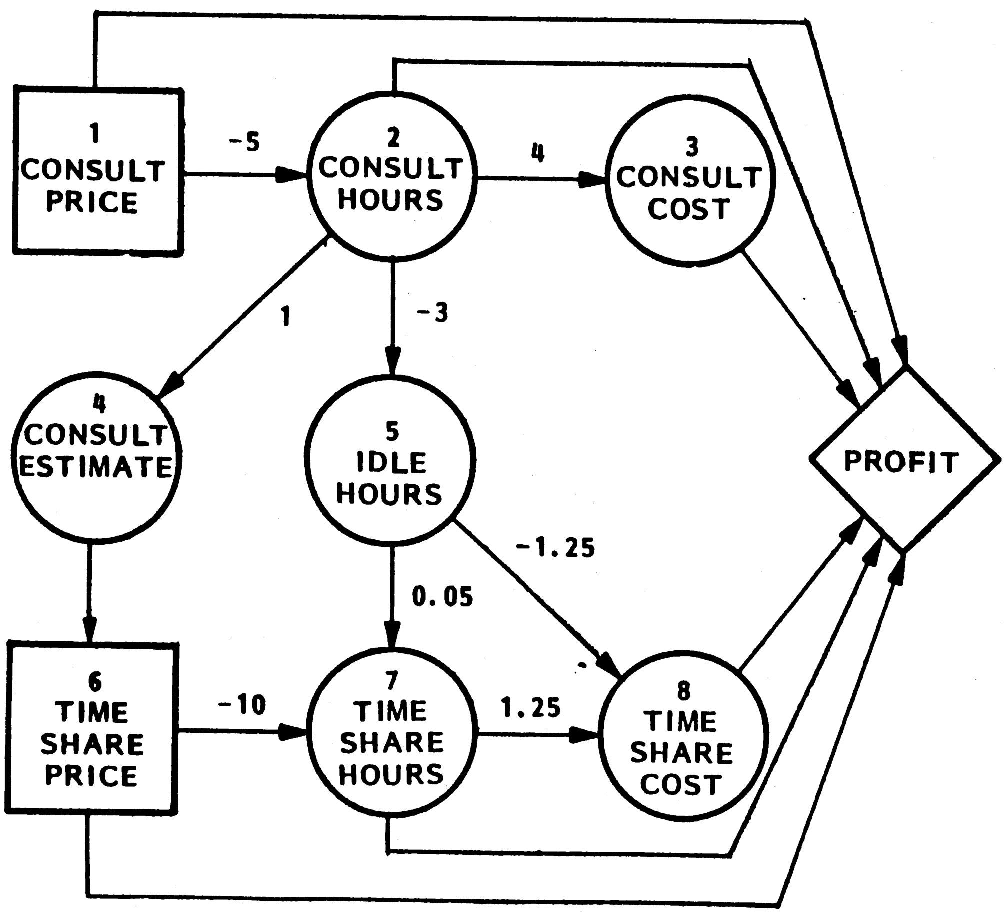
NORMAL INFLUENCE DIAGRAM

SOLUTION PROCEDURE FOR CONSULTANT'S PROBLEM

1. SUCCESSIVELY REMOVE CHANCE NODES x_8, x_7, x_5, AND x_3 INTO THE VALUE NODE
2. REVERSE THE INFLUENCE FROM x_2 to x_4
3. REMOVE CHANCE NODE x_2 INTO THE VALUE NODE
4. REMOVE DECISION NODE x_6 INTO THE VALUE NODE
5. REMOVE CHANCE NODE x_4 INTO THE VALUE NODE
6. REMOVE DECISION NODE x_1 INTO THE VALUE NODE

NORMAL INFLUENCE DIAGRAM

CONSULTANT'S PROBLEM



$$PROFIT = X_1X_2 - X_3 + X_6X_7 - X_8$$

$$N = \{1,2,3,4,5,6,7,8\}$$

$$Q = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$p = (0\ 0\ -1\ 0\ 0\ 0\ 0\ -1)^T$$

$$r = 0$$

$$v = (0\ 40{,}000\ 4{,}000{,}000\ 250{,}000\ 100\ 0\ 10{,}000\ 40{,}000)^T$$

$$\mu = (100\ 1{,}500\ 58{,}000\ 1{,}500\ 3{,}500\ 25\ 750\ 5{,}000)^T$$

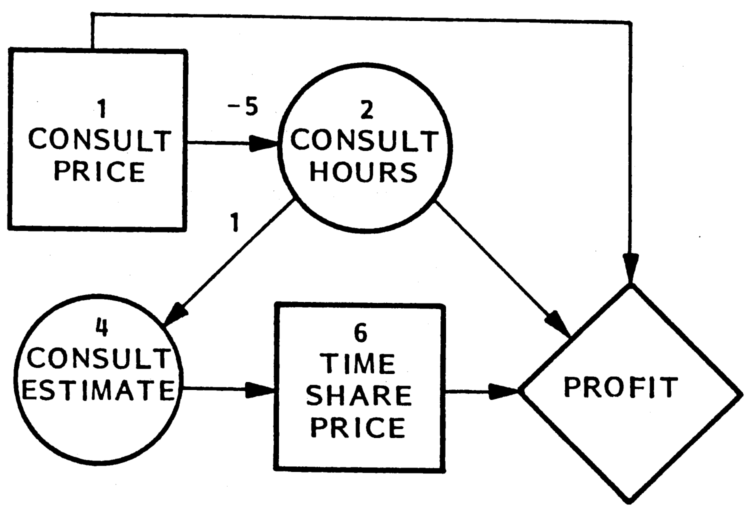
□ = DECISION NODE

○ = CHANCE NODE

NORMAL INFLUENCE DIAGRAM

REVERSAL STEP

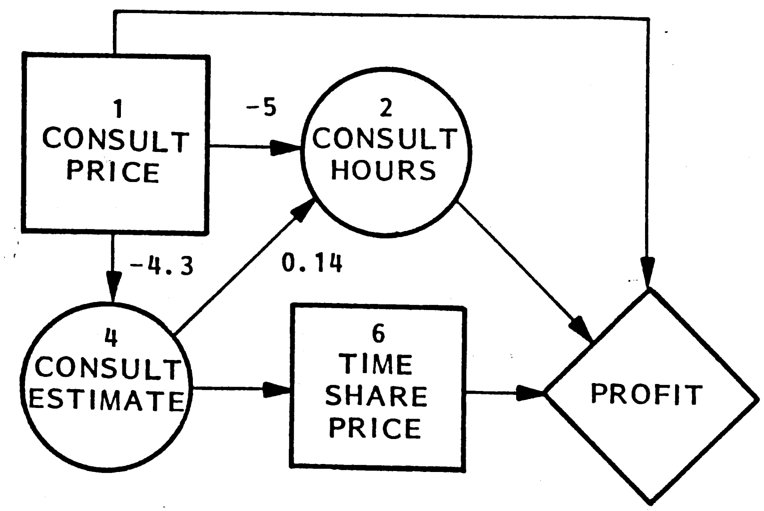
$N = \{1,2,4,6,\}$



$V = (0\ 40{,}000\ 250{,}000\ 0)^T$

BEFORE REVERSAL

$N = \{1,4,2,6\}$

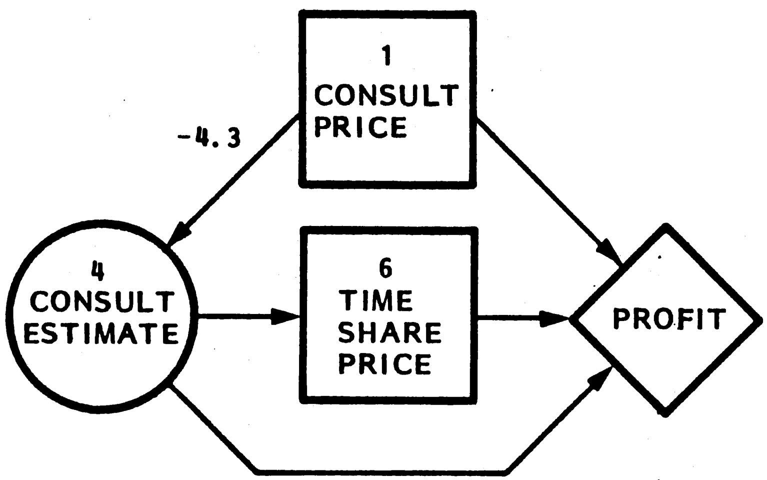


$v = (0\ 290{,}000\ 34{,}482.76\ 0)^T$

AFTER REVERSAL

DISCRETE - TIME FILTERING

MINIMAL REPRESENTATION OF CONSULTANT'S PROBLEM



$N = \{1,\ 4,\ 6\}$

$$Q = \begin{bmatrix} -8.6207 & 0.1379 & 0.6466 \\ 0.1379 & 0.0 & -0.0207 \\ 0.6466 & -0.0207 & -20.0 \end{bmatrix}$$

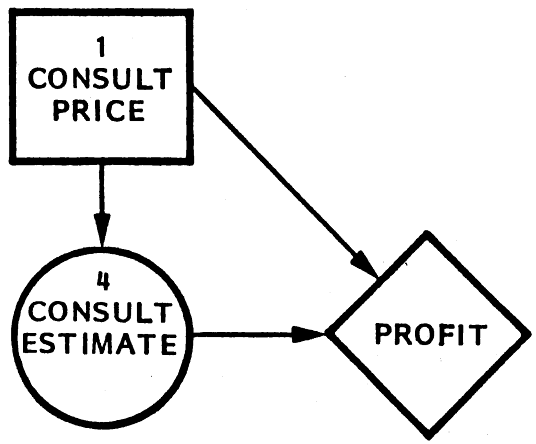
$p = (1756.73\ -1.043\ 978.9)^T$

$r = -65{,}007.54$

$\nu = (0\ 34{,}482.76\ 0)^T$

NORMAL INFLUENCE DIAGRAM

OPTIMAL SOLUTION OF THE CONSULTANT'S PROBLEM



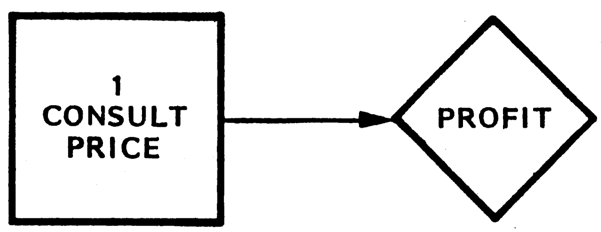
$N = \{1, 4\}$

$Q = \begin{bmatrix} -8.5998 & 0.1373 \\ -0.1373 & 2.1403 \end{bmatrix} \times 10^{-5}$

$p = (1988.38 \ \ -2.506)^T$

$r = -41,052.43$

$x_6^* = 48.944 + 0.3233\, x_1 - 1.0345 \times 10^{-3}\, x_4$



$N = \{1\}$

$Q = \begin{bmatrix} -9.9719 \end{bmatrix}$

$p = (2072.97)$

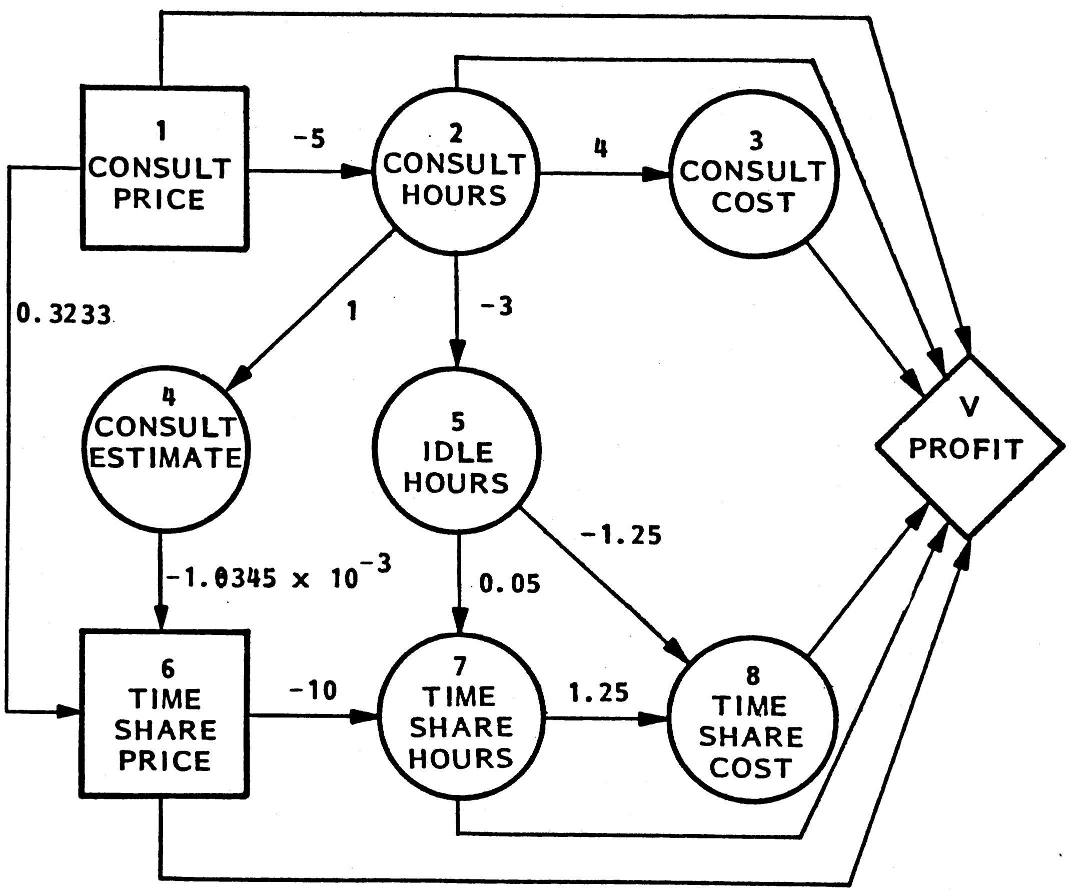
$r = -50,207.58$

$x_1^* = 207.88$

$r = 170,347.98$

NORMAL INFLUENCE DIAGRAM

CONSULTANT'S POLICY DIAGRAM



$$PROFIT = X_1X_2 - X_3 + X_6X_7 - X_8$$

$$N = \{1,2,3,4,5,6,7,8\}$$

$$Q = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$p = (0\ 0\ -1\ 0\ 0\ 0\ 0\ -1)^T$$

$$r = 0$$

$$v = (0\ 40{,}000\ 4{,}000{,}000\ 250{,}000\ 100\ 0\ 10{,}000\ 40{,}000)^T$$

$$\mu = (207.88\ 960.59\ 55842\ 960.59\ 5118.2\ 54.67\ 543.21\ 2707.48)^T$$

□ = POLICY NODE

○ = CHANCE NODE

DISCRETE - TIME FILTERING

MATHEMATICAL MODEL

DYNAMIC PROCESS: $x(k+1) = \Phi(k)x(k) + \Gamma(k)w(k)$ $\quad k = 0,\ldots,N.$

MEASUREMENT PROCESS: $z(k) = H(k)x(k) + v(k)$ $\quad k = 0,\ldots,N.$

PROBABILISTIC STRUCTURE:

$E[x(0)] = \mu_0.$

$Cov[x(0)] = P_0.$

$E[w(k)] = 0$ for $k = 0,\ldots,N.$

$Cov[w(j),w(k)] = \delta_{jk}Q_k$ for $j = 0,\ldots,N$ and $k = 0,\ldots,N.$

Q_k are diagonal for $k = 0,\ldots,N.$

$Cov[x(0),w(0)] = 0.$

$E[v(k)] = 0$ for $k = 0,\ldots,N.$

$Cov[v(j),v(k)] = \delta_{jk}R_k$ for $j = 0,\ldots,N$ and $k = 0,\ldots N.$

R_k are diagonal for $k = 0,\ldots,N.$

$Cov\ [w(j),v(k)] = 0$ for $j = 1,\ldots,N$ and $k = 0,\ldots,N.$

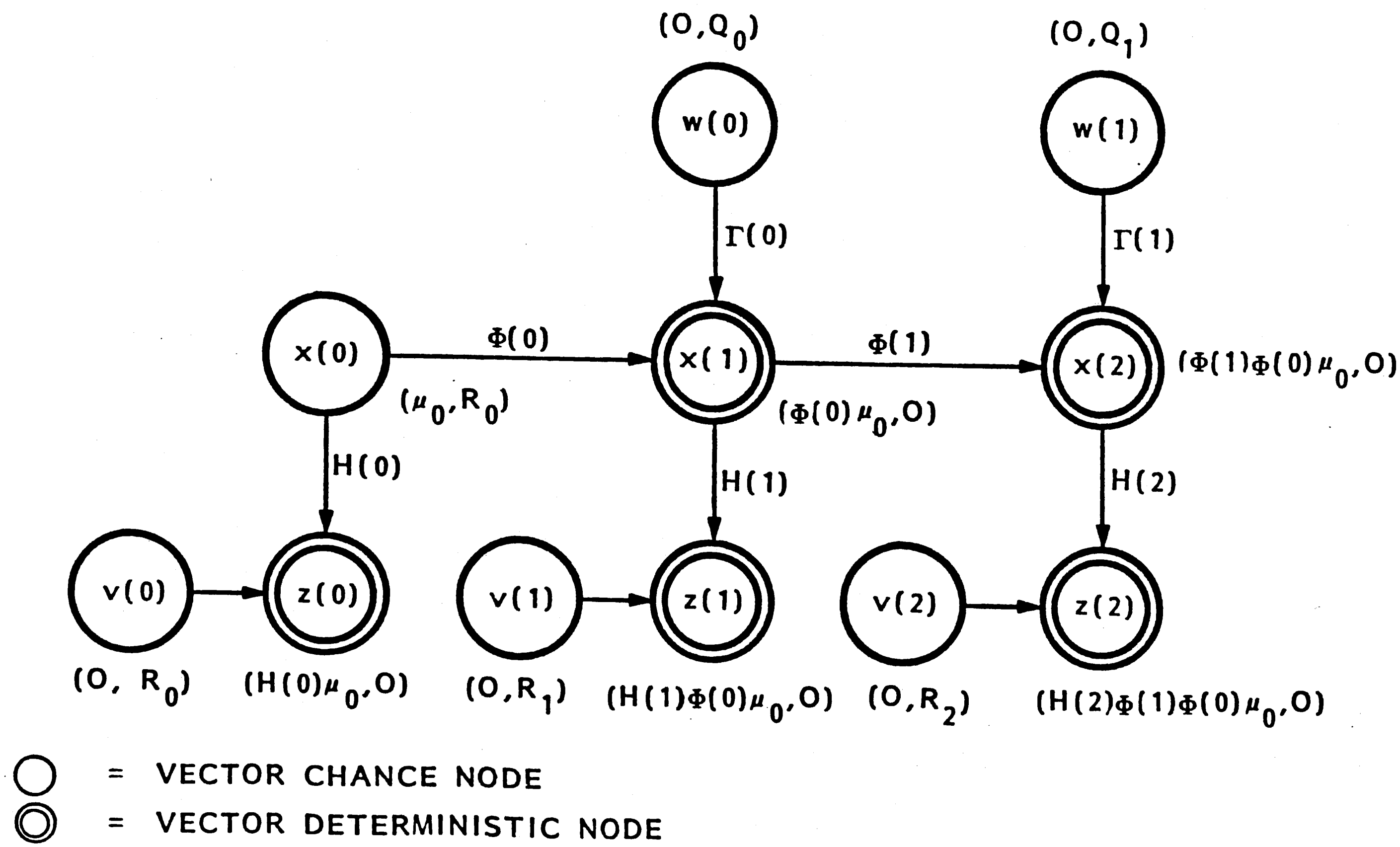
$Cov[x(0),v(k)] = 0$ for $0 = 1,\ldots,N.$

DIMENSIONS OF VECTORS: $x(k) \in R^n$, $w(k) \in R^r$, $z(k) \in R^P$, and $v(k) \in R^P$.

OBJECTIVE: FIND RECURSIVE ALGORITHM FOR GENERATING $P\{x(k+1) \mid z(0), \ldots, z(k)\}$ FROM $P\{x(k) \mid z(0), \ldots, z(k-1)\}$ FOR $k = 0, \ldots, N$

DISCRETE - TIME FILTERING

INFLUENCE DIAGRAM REPRESENTATION



DISCRETE - TIME FILTERING

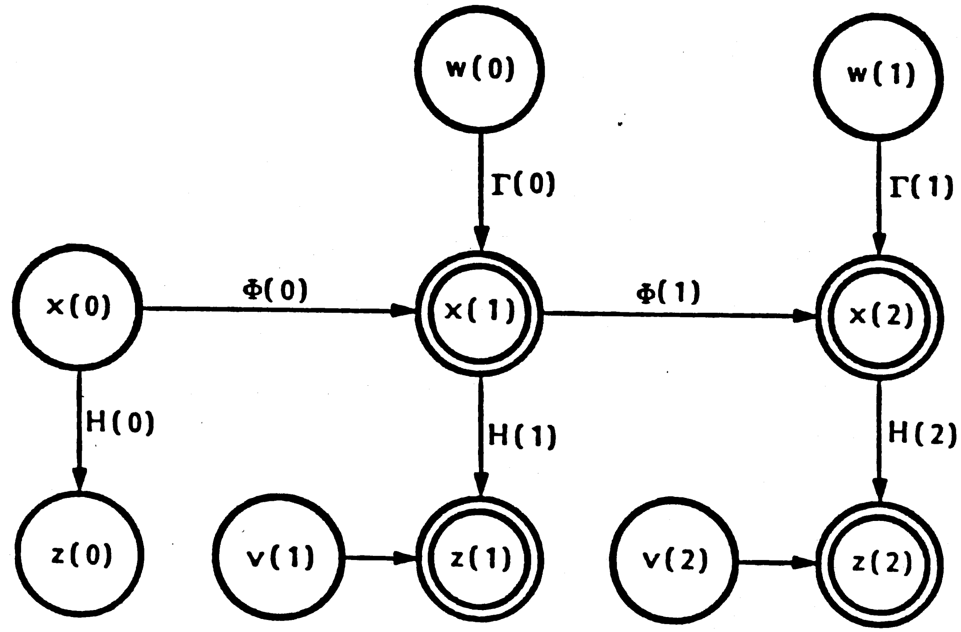
INCORPORATION OF MEASUREMENT $z(k)$

A. MEASUREMENT UPDATE
   1. REMOVE $v(k)$ INTO $z(k)$
   2. REVERSE ARCS FROM $x(k)$ TO $z(k)$
   3. INSTANTIATE $z(k)$ AND UPDATE $E[x(k)]$

B. TIME UPDATE
   1. REMOVE $x(k)$ INTO $x(k+1)$
   2. REMOVE $w(k)$ INTO $x(k+1)$

DISCRETE - TIME FILTERING

V(0) REMOVED INTO Z(0)

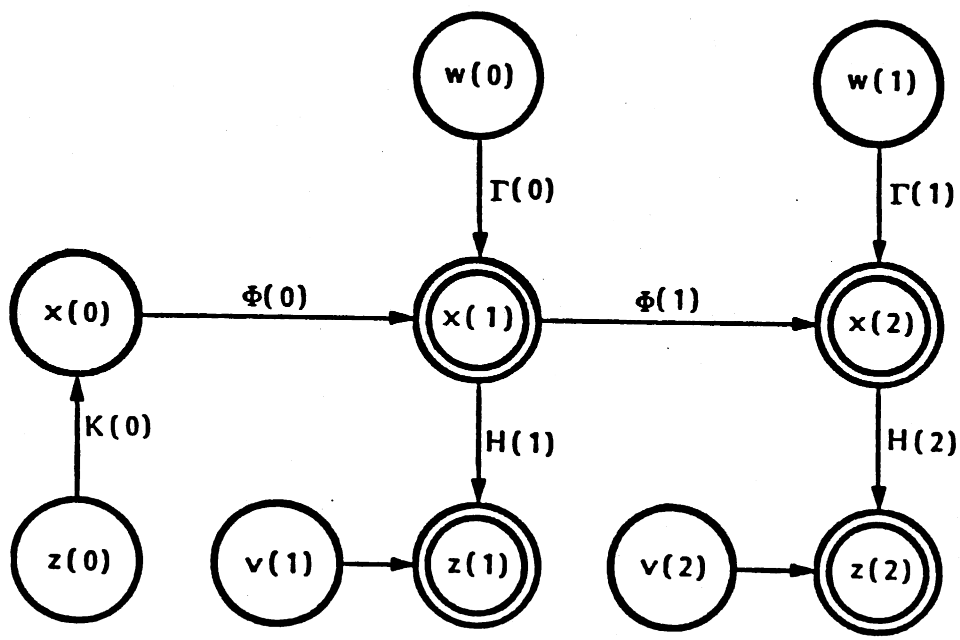


○ = VECTOR CHANCE NODE

◎ = VECTOR DETERMINISTIC NODE

DISCRETE - TIME FILTERING

X(0) AND Z(0) REVERSED

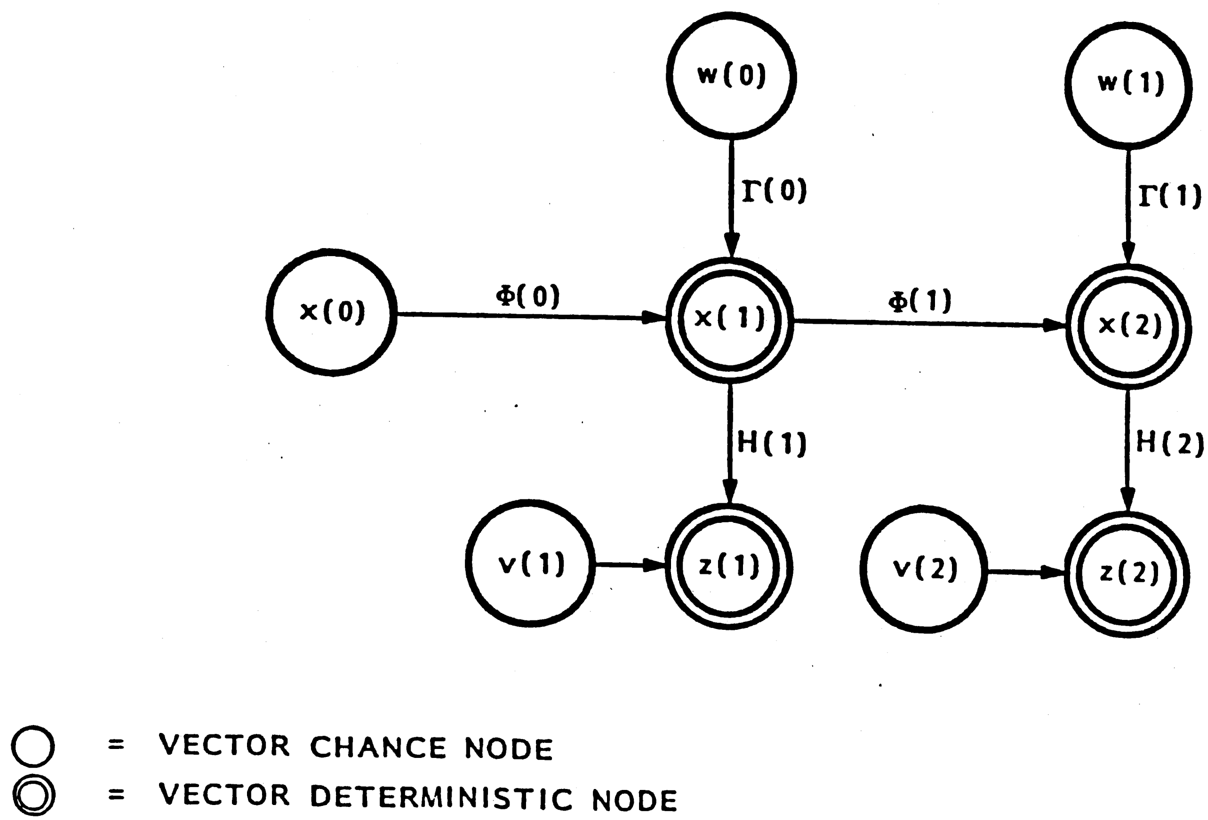


○ = VECTOR CHANCE NODE

◎ = VECTOR DETERMINISTIC NODE

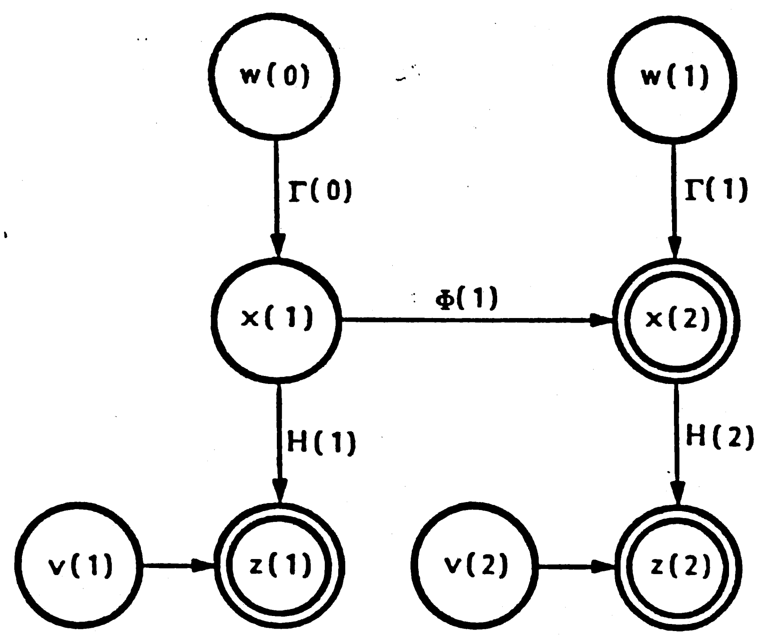
DISCRETE - TIME FILTERING

Ż(0) INSTANTIATED



DISCRETE - TIME FILTERING

X(0) REMOVED INTO X(1)

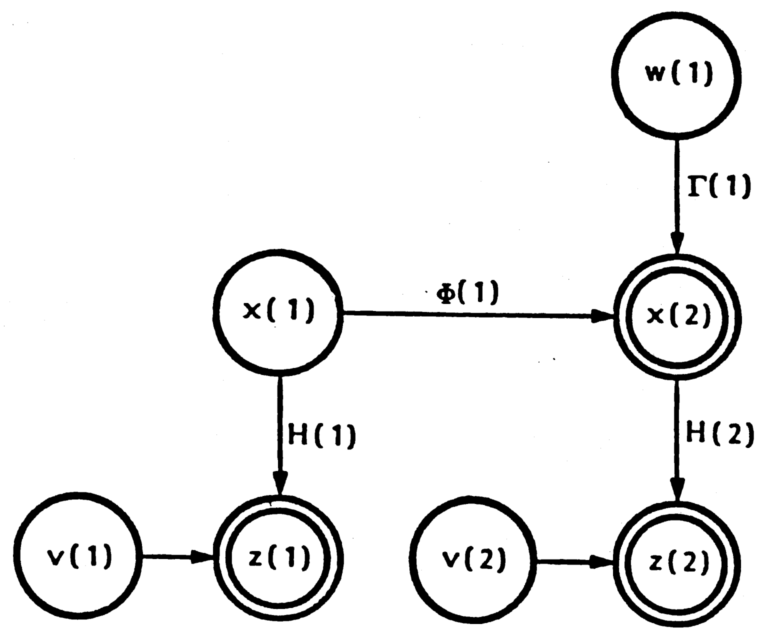


○ = VECTOR CHANCE NODE

◎ = VECTOR DETERMINISTIC NODE

DISCRETE - TIME FILTERING

W(0) REMOVED INTO X(1)



○ = VECTOR CHANCE NODE

◎ = VECTOR DETERMINISTIC NODE

DISCRETE - TIME FILTERING

OPERATION COUNTS FOR PROCESSING A VECTOR OF p MEASUREMENTS

	ALGORITHM	ADDITIONS	MULTIPLICATIONS	DIVISIONS	SQUARE ROOTS
SCALAR PROCESSING ALGORITHMS	INFLUENCE DIAGRAM	$(1.5n^2 + 1.5n)p$	$(1.5n^2 + 4.5n)p$	np	0
	CONVENTIONAL KALMAN	$(1.5n^2 + 1.5n)p$	$(1.5n^2 + 4.5n)p$	p	0
	U-D COVARIANCE	$(1.5n^2 + 3.5n)p$	$(1.5n^2 + 5.5n)p$	np	0
	SQUARE ROOT COVARIANCE	$(1.5n^2 + 3.5n)p$	$(2n^2 + 5n)p$	2np	np
	POTTER SQUARE ROOT	$(3n^2 + 3n)p$	$(3n^2 + 4n)p$	2p	p
	KALMAN STABILIZED	$(4.5n^2 + 5.5n)p$	$(4n^2 + 7.5n)p$	p	0
BATCH PROCESSING ALGORITHMS	SRIF, R TRIANGULAR	$(n^2 + 2n)p +$ $1.5n^2 + 2.5n$	$(n^2 + 3n)p +$ $2n^2 + 3n$	2n	n
	NORMAL EQUATION	$(0.5n^2 + 1.5n)p +$ $(n^3 + 6n^2 + 5n)/6$	$(0.5n^2 + 2.5n)p$ $(n^3 + 9n^2 - 4n)/6$	2n	n
	SRIF, R GENERAL	$n^2p +$ $(4n^3 + 3n^2 + 5n)/6$	$n^2p +$ $(4n^3 + 9n^2 - n)/6$	2n	n

DISCRETE - TIME FILTERING

WEIGHTED OPERATION COUNTS FOR PROCESSING A VECTOR OF p MEASUREMENTS

ALGORITHM	WEIGHTED OPERATION COUNTS
INFLUENCE DIAGRAM	$(3.6n^2 + 12.3n)p$
CONVENTIONAL KALMAN	$(3.6n^2 + 7.8n + 4.5)p$
U-D COVARIANCE	$(3.6n^2 + 15.7n)p$
SQUARE ROOT COVARIANCE	$(4.3n^2 + 40.9n)p$
POTTER SQUARE ROOT	$(7.2n^2 + 8.6n + 30.4)p$
KALMAN STABILIZED	$(10.1n^2 + 16n + 4.5)p$
SRIF, R TRIANGULAR	$(2.4n^2 + 6.2n)p + 4.3n^2 + 37.1n$
NORMAL EQUATION	$(1.2n^2 + 5.0n)p + 0.4n^3 + 3.1n^2 + 30.3n$
SRIF, R GENERAL	$2.4n^2p + 1.6n^3 + 2.6n^2 + 31n$

OPERATION	WEIGHT
$+$	1
$\times$	1.4
$\div$	4.5
$\sqrt{}$	21.4

DISCRETE - TIME FILTERING

OPERATION COUNTS FOR TIME UPDATE

ALGORITHM	ADDITIONS	MULTIPLICATIONS	DIVISIONS	SQUARE ROOTS
INFLUENCE DIAGRAM	$1.17n^3 + 0.17n^2 - 3n + 3 +$ $(2.5n^2 - 2.5n + 1)r +$ $(n - 1)r^2$	$1.17n^3 + 1.3n^2 - 4.5n + 5 +$ $(2.5n^2 + 0.5n - 1)r +$ $(n - 1)r^2$	$0.5n^2 + 0.5n +$ $(n - 1)r$	0 0
CONVENTIONAL KALMAN	$1.5n^3 + 2n^2 + 0.5n +$ $(0.5n^2 + 0.5n)r$	$1.5n^3 + 1.5n^2 +$ $(0.5n^2 + 1.5n)r$	0	0
U-D COVARIANCE	$1.5n^3 + 0.5n^2 +$ $n^2 r$	$1.5n^3 + 2.5n^2 - n +$ $(n^2 + 2n - 1)r$	$n - 1$	0
SQUARE ROOT COVARIANCE	$1.7n^3 + 2n^2 + 0.3n +$ $(n^2 + n)r$	$1.7n^3 + 2n^2 + 0.3n +$ $(n^2 + n)r$	n	n

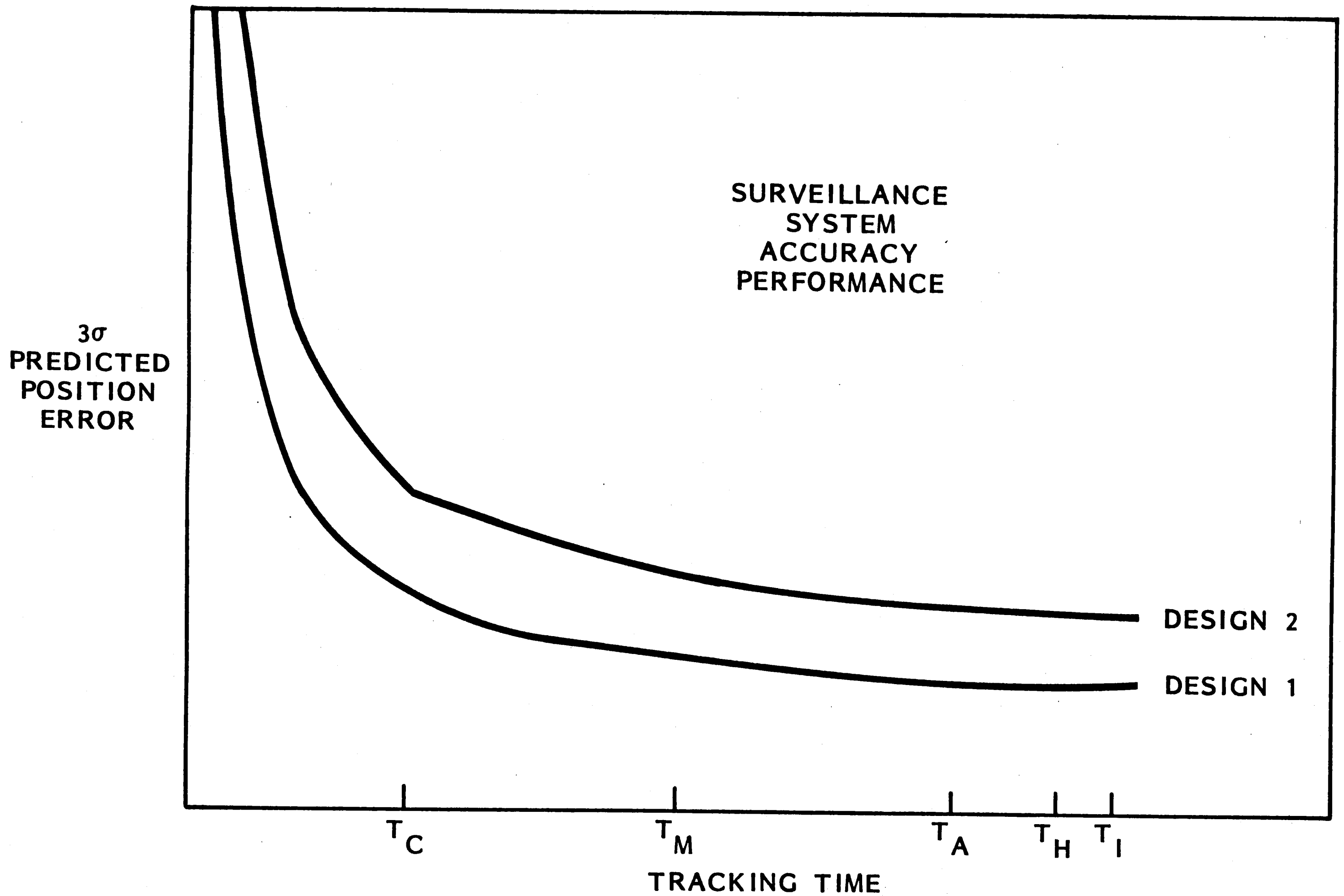
DISCRETE - TIME FILTERING

WEIGHTED OPERATION COUNTS FOR TIME UPDATE

ALGORITHM	WEIGHTED OPERATION COUNTS
INFLUENCE DIAGRAM	$2.8n^3 + 3.95n^2 - 11.55n + 10 + (6n^2 + 2.7n - 4.9)r + (2.4n - 2.4)r^2$
CONVENTIONAL KALMAN	$3.6n^3 + 4.1n^2 + 0.5n + (1.2n^2 + 2.6n)r$
U-D COVARIANCE	$3.6n^3 + 4n^2 + 3.1n - 4.5 + (2.4n^2 + 4.2n - 2.8)r$
SQUARE ROOT COVARIANCE	$4n^3 + 4.8n^2 + 26.7n + (2.4n^2 + 2.4n)r$

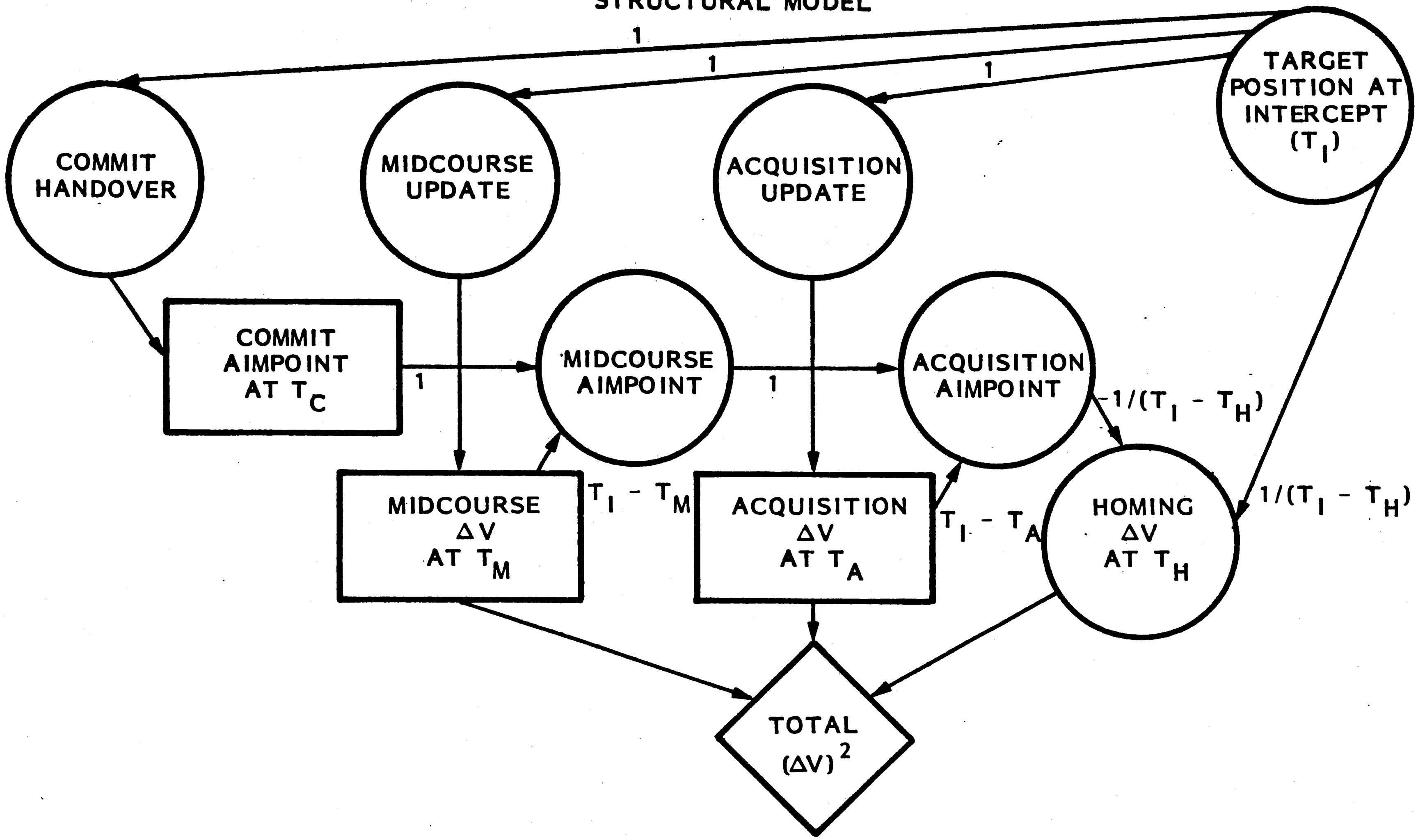
ΔV-ACCURACY ANALYSIS

TYPICAL ACCURACY ANALYSIS OUTPUT



ΔV-ACCURACY ANALYSIS

STRUCTURAL MODEL



Δ V-ACCURACY ANALYSIS

SURVEILLANCE-INTERCEPTOR INTERFACE SENSITIVITY