Networked Unknown Input Observer Analysis and Design for Time-Delay Systems

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Abstract—**The insertion of communication networks in the feedback loops of control systems is a defining feature of modern control systems. These systems are often subject to unknown inputs in a form of disturbances, perturbations, or attacks. The objective of this paper is to analyze and design an observer for networked systems with unknown disturbances and inputs. The network effect can be viewed as either a perturbation or time-delay to the exchanged signals. In this paper, we focus on the time-delay representation of the network. First, we review an Unknown Input Observer (UIO) design for non-networked system from the UIO literature. Second, we derive the dynamics of the UIO-based NCS, also referred to as Networked Unknown Input Observer (N_{et}UIO). Third, we design the N_{et}UIO such that the effect of higher delay order terms are nullified, assuring that the effect of the unknown inputs on the plant state estimation is minimized. Fourth, we derive a bound on the maximum allowable time-delay for the N_{et}UIO. Finally, a numerical example is shown to illustrate the usefulness of the proposed model.**

Keywords*—*Networked Control Systems, Unknown Input Observers, N_{et}UIO, Time-Delay Systems.

I. Introduction and Literature Review

THE objective of this paper is to analyze and design an observer for Networked Control Systems (NCS) with unknown disturbances and inputs. Many modern control systems are becoming networked, where often a band-limited network is used as a mean of communication between sensors, actuators and controllers [1]. Estimators in general, and observers in specific, use the known plant's inputs and outputs to generate estimates for the state of the plant. The closed loop system is then controlled through a controller that often use the estimated plant state to generate control commands.

State-estimators and observers are used in power networks to precisely estimate the plant state (i.e., the bus voltages and phase angles), which is crucial for successful control and operation of the modern smart-grid. The generated real-time dynamic estimates of the bus voltages and angles facilitate calculating optimal power flows for transmission lines [2]. One of the main objectives behind utilizing state estimators and observers for dynamical systems is to augment or replace expensive sensors in a control system [3]. For that and various reasons, the analysis and design of dynamic robust observers for linear and nonlinear systems, for systems with known and unknown inputs and disturbances have received noticeable attention in the past few decades.

Luenberger was the first to propose, analyze and design observers [4]–[6]. The well-known Luenberger Observer is still utilized for various engineering applications. Furthermore, observers for systems with unknown inputs and disturbances, also called Unknown Input Observers (UIO), have been extensively studied since the late seventies. The following are some well known research efforts on UIOs: Bhattacharyya [7], Chang *et al.* [8], Chen *et al.* [9], Chen and Patton [10], Corless and Tu [11], Darouach *et al.* [12], Hui and Żak [13]. For more references on different UIO architectures, we direct the reader to [3, p. 431].

The study of UIOs is becoming more crucial to large-scale systems, as these systems are becoming more susceptible to high-disturbances, faults, cyber and physical attacks. The design of robust UIOs for systems under attacks would result in a better estimation of the state of the plant and thus better control and performance. UIOs can be used to employ Fault Detection and Isolation (FDI) mechanisms as proposed by Chen *et al.* [9]. Teixeira *et al.* [14] applied UIOs to design an FDI scheme to analyze power networks under cyber attacks.

The basic idea behind most FDI schemes is to generate weighted residual functions for each subsystem or node which is defined as the difference between the actual system outputs and the estimated ones [3], as in power networks. After choosing a suitable dynamic or static estimation threshold for the error of the residual functions, faulty nodes are isolated. For example, if a bus in a power network is continuously generating higher residuals than the threshold, this bus would be geographically identified and then physically isolated from the overall network.

II. Research Gaps and Paper Overview

As mentioned in Section I, observers use the known plant's inputs and outputs to generate estimates for the state of the plant. The closed loop system is then controlled through a controller that often uses the estimated plant state to generate control commands. Observers in different large-scale dynamical systems such as transportation networks, power plants, and remotely controlled mobile agents are often distributed. Hence, the UIO's inputs (i.e., the plant's input and output) are transmitted through a communication network, which is a key component in modern NCSs. Thus, most observers for systems with unknown inputs are Networked Unknown Input Observers (N_{et}UIO).

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Most of the developed UIOs in the literature are designed for non-networked systems as in [7]–[13]. The UIO's input is assumed to be transmitted without any disturbances, perturbations, or time-delays. Since communication networks are inserted in most decentralized control systems, the analysis and design observers for *networked* systems with unknown inputs becomes a necessity. The network effect can be either modeled as a perturbation or time-delay to the transmitted signals. In [15] and [16], we analyzed the effect of perturbation of the signals exchanged through communication networks for decentralized observer-based control for systems with only known inputs. In [17], we applied a time-delay analysis for NCSs with applications to power networks, also for systems with only known inputs.

In this paper, the proposed design targets the time-delay representations of a network. In Section III, we review an Unknown Input Observer (UIO) design for non-networked system from the UIO literature as in [3]. In Section IV we derive the dynamics of the UIO-based NCS and we design the $\mathrm{N_{et}}$UIO such that the effect of higher delay order terms are nullified, assuring that the effect of the unknown inputs to the plant is minimized. In Section V, we derive a bound on the maximum allowable time-delay for the $\mathrm{N_{et}}$UIO. Section VI includes numerical examples to illustrate the usefulness and applicability of the proposed model. Closing remarks, conclusions, and future work are presented in Section VII.

III. System Modeling and Problem Formulation

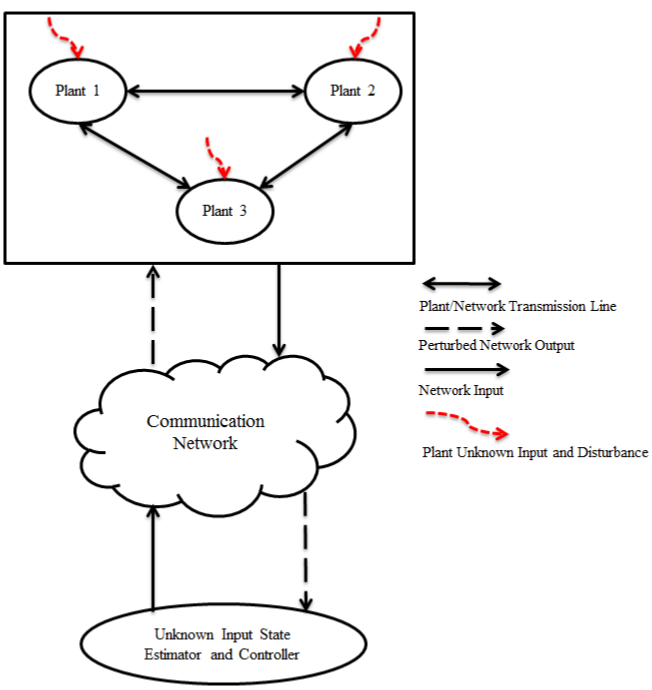

Fig. 1. Networked Unknown Input Observer ($\mathrm{N_{et}}$UIO) Generic Architecture

The objective of the research presented in this paper is to study the effect of Unknown Input Observers (UIO) architecture for networked systems with unknown disturbances. The non-networked UIO architecture used in this paper is presented in [3]. The generic architecture considered in this paper is depicted in Figure 1. The input to the UIO block is the delayed or perturbed version of the plant's output ($\boldsymbol{y}$), that is $\hat{\boldsymbol{y}}$, and the delayed version of the known input $\boldsymbol{u}_1$, that is $\hat{\boldsymbol{u}}_1$. These two quantities are assumed to be known to the UIO block. The output of the UIO is the estimate of the plant state ($\boldsymbol{x}_p$), that is $\hat{\boldsymbol{x}}_p$. The local controller takes the a reference input ($\boldsymbol{v}_{ref}$) and $\hat{\boldsymbol{x}}_p$ as inputs. The control law is computed through a linear state feedback, but this could be changed according to the application under consideration. The unknown input for the plant is $\boldsymbol{u}_2$ (unknown plant disturbances, nonlinearities and actuator faults).

A. Observer Review for Non-Networked Systems with Unknown Inputs

The observer design and the state estimations for non-networked systems with unknown inputs used in this paper is based on a projector operator approach used in [3]. In this paper, we assume a Linear Time-Invariant (LTI) class of systems. The modeled plant can be a linearized representation of a nonlinear plant.

The linearized plant dynamics can be written as:

$$\begin{aligned}\dot{\boldsymbol{x}}_p &= \boldsymbol{A}_p\boldsymbol{x}_p + \boldsymbol{B}_p^{(1)}\boldsymbol{u}_1 + \boldsymbol{B}_p^{(d)}\boldsymbol{u}_d + \boldsymbol{B}_p^{(a)}\boldsymbol{f}_a\\ \boldsymbol{y} &= \boldsymbol{C}_p\boldsymbol{x}_p,\end{aligned}$$

where $\boldsymbol{A} \in \mathbb{R}^{n\times n}$ is the state matrix, $\boldsymbol{B}_p^{(1)} \in \mathbb{R}^{n\times m_1}$ is the input matrix, $\boldsymbol{B}_p^{(d)} \in \mathbb{R}^{n\times m_d}$ is the disturbance matrix, $\boldsymbol{B}_p^{(a)} \in \mathbb{R}^{n\times m_a}$ represents the actuator fault's matrix, and $\boldsymbol{C}_p \in \mathbb{R}^{p\times n}$ is the output matrix. The unknown inputs to the given system are $\boldsymbol{u}_d$ (representing the unknown plant disturbances and nonlinearities) and $\boldsymbol{f}_a$ (actuator fault). We assume that $\boldsymbol{A}_p, \boldsymbol{B}_p^{(1)}, \boldsymbol{B}_p^{(d)}, \boldsymbol{B}_d^{(a)}$ and $\boldsymbol{C}_p$ are all known system parameters. For simplicity, we can combine the unknown inputs $\boldsymbol{u}_d$ and $\boldsymbol{f}_a$ into one unknown input quantity $\boldsymbol{u}_2$. More precisely, $\boldsymbol{u}_2 = \begin{bmatrix}\boldsymbol{u}_d^\top & \boldsymbol{f}_a^\top\end{bmatrix}^\top \in \mathbb{R}^{m_2}$. In addition, the unknown input matrices $\boldsymbol{B}_p^{(d)}$ and $\boldsymbol{B}_p^{(a)}$ are combined into $\boldsymbol{B}_p^{(2)} \in \mathbb{R}^{n\times m_2}$. The plant dynamics can be re-written as:

$$\dot{\boldsymbol{x}}_p = \boldsymbol{A}_p\boldsymbol{x}_p + \boldsymbol{B}_p^{(1)}\boldsymbol{u}_1 + \boldsymbol{B}_p^{(2)}\boldsymbol{u}_2. \quad (1)$$

The dynamics of the UIO presented in [3] for non-networked systems are

$$\dot{\boldsymbol{q}} = (\boldsymbol{I} - \boldsymbol{M}\boldsymbol{C}_p)\Big(\boldsymbol{A}_p\boldsymbol{q} + \boldsymbol{A}_p\boldsymbol{M}\boldsymbol{y} + \boldsymbol{B}_p^{(1)}\boldsymbol{u}_1 + \boldsymbol{L}(\boldsymbol{y} - \boldsymbol{C}_p\boldsymbol{q} - \boldsymbol{C}_p\boldsymbol{M}\boldsymbol{y})\Big) \quad (2)$$

$$\hat{\boldsymbol{x}}_p = \boldsymbol{q} + \boldsymbol{M}\boldsymbol{y}, \quad (3)$$

where $\boldsymbol{M} \in \mathbb{R}^{n\times p}$ is chosen such that $(\boldsymbol{I} - \boldsymbol{M}\boldsymbol{C}_p)\boldsymbol{B}_p^{(2)} = \boldsymbol{O}$ and $\boldsymbol{L}$ is an added gain to improve the convergence of the estimated state ($\hat{\boldsymbol{x}}_p$). The initial conditions for the observer are $\boldsymbol{q}(0) = (\boldsymbol{I} - \boldsymbol{M}\boldsymbol{C}_p)\hat{\boldsymbol{x}}(0)$, where $\hat{\boldsymbol{x}}(0)$ is an estimate of the initial plant state. Under the assumption that the pair $(\boldsymbol{C}_p, \boldsymbol{A}_p)$ is detectable, this observer for non-networked control systems guarantees that the estimation error $(\boldsymbol{e}(t) = \boldsymbol{x}_p(t) - \hat{\boldsymbol{x}}_p(t))$ converges to zero as $t \to \infty$ under mild conditions [3].

As mentioned in the introduction, the objective of the paper is to analyze the effect of the communication network on the state and unknown input estimation for an UIO architecture. To do so, we first rewrite the dynamics of the observer so that it matches the typical setup of controllers/observers from the NCS literature. Letting $\boldsymbol{x}_c = \boldsymbol{q}$, the dynamics of the UIO can be rewritten as follows:

$$\dot{\boldsymbol{x}}_c = (\boldsymbol{I} - \boldsymbol{M}\boldsymbol{C}_p)\bigg(\boldsymbol{A}_p\boldsymbol{x}_c + \boldsymbol{A}_p\boldsymbol{M}\boldsymbol{y} + \boldsymbol{B}_p^{(1)}\boldsymbol{u}_1 + \boldsymbol{L}(\boldsymbol{y} - \boldsymbol{C}_p\boldsymbol{x}_c - \boldsymbol{C}_p\boldsymbol{M}\boldsymbol{y})\bigg)$$

$$\dot{\boldsymbol{x}}_c = \boldsymbol{A}_c\boldsymbol{x}_c + \boldsymbol{B}_c^{(1)}\boldsymbol{y} + \boldsymbol{B}_c^{(2)}\boldsymbol{u}_1,$$

where

$$\boldsymbol{A}_c = (\boldsymbol{I} - \boldsymbol{M}\boldsymbol{C}_p)(\boldsymbol{A}_p - \boldsymbol{L}\boldsymbol{C}_p), \boldsymbol{B}_c^{(2)} = (\boldsymbol{I} - \boldsymbol{M}\boldsymbol{C}_p)\boldsymbol{B}_P^{(1)}$$

$$\boldsymbol{B}_c^{(1)} = (\boldsymbol{I} - \boldsymbol{M}\boldsymbol{C}_p)(\boldsymbol{A}_p\boldsymbol{M} + \boldsymbol{L} - \boldsymbol{L}\boldsymbol{C}_p\boldsymbol{M}).$$

The addition of the communication network perturbs the UIO's inputs (which are $\boldsymbol{y}$ and $\boldsymbol{u}_1$), as the observer uses the plant's input and output to estimate the state of the plant. Hence, the dynamics of the UIO are as follows:

$$\dot{\boldsymbol{x}}_c = \boldsymbol{A}_c\boldsymbol{x}_c + \boldsymbol{B}_c^{(1)}\hat{\boldsymbol{y}} + \boldsymbol{B}_c^{(2)}\hat{\boldsymbol{u}}_1, \quad (4)$$

$$\hat{\boldsymbol{x}}_p = \boldsymbol{x}_c + \boldsymbol{M}\hat{\boldsymbol{y}}. \quad (5)$$

In this section, we assume that state-feedback control is used.

$$\begin{aligned} \boldsymbol{u}_1 &= -\boldsymbol{K}\hat{\boldsymbol{x}}_p + \boldsymbol{v}_{ref} \\ \boldsymbol{u}_1 &= -\boldsymbol{K}\boldsymbol{x}_c - \boldsymbol{K}\boldsymbol{M}\hat{\boldsymbol{y}} + \boldsymbol{v}_{ref}. \end{aligned} \quad (6)$$

IV. Network Effect as Pure Time Delay

A. Closed Loop Dynamics

In this section, we model the communication network by a pure-time delay. Precisely, we assume that $\hat{\boldsymbol{y}} = \boldsymbol{y}(t - \tau)$ and $\hat{\boldsymbol{u}}_1 = \boldsymbol{u}_1(t - \tau)$, where τ is the time delay due to the presence of the network in the feedback loops. To simplify the derivations, we assume that the communication network is only inserted between UIO and its inputs and that the reference input $(\boldsymbol{v}_{ref})$ is set to zero.

Assuming that the innovation function of the observer is embedded in the UIO dynamics, we can rewrite the dynamics of the observer and the controller as in the typical form of an NCS controller/observer:

$$\begin{cases} \dot{\boldsymbol{x}}_c(t) = \boldsymbol{A}_c\boldsymbol{x}_c(t) + \boldsymbol{B}_c^{(1)}\boldsymbol{y}(t - \tau) + \boldsymbol{B}_c^{(2)}\boldsymbol{u}_1(t - \tau) \\ \boldsymbol{u}_1(t) = \boldsymbol{C}_c\boldsymbol{x}_c(t) + \boldsymbol{D}_c\boldsymbol{y}(t - \tau), \end{cases} \quad (7)$$

where $\boldsymbol{C}_c = -\boldsymbol{K}$ and $\boldsymbol{D}_c = -\boldsymbol{K}\boldsymbol{M}$. The plant and the controller state dynamics can be written as:

$$\begin{aligned} \dot{\boldsymbol{x}}_p(t) &= \boldsymbol{A}_p\boldsymbol{x}_p(t) + \boldsymbol{B}_p^{(1)}\boldsymbol{C}_c\boldsymbol{x}_c(t) \\ &\quad + \boldsymbol{B}_p^{(1)}\boldsymbol{D}_c\boldsymbol{C}_p\boldsymbol{x}_p(t - \tau) + \boldsymbol{B}_p^{(2)}\boldsymbol{u}_2(t) \\ \dot{\boldsymbol{x}}_c(t) &= \boldsymbol{A}_c\boldsymbol{x}_c(t) + \boldsymbol{B}_c^{(1)}\boldsymbol{C}_p\boldsymbol{x}_p(t - \tau) + \boldsymbol{B}_c^{(2)}\boldsymbol{u}_1(t - \tau) \\ &= \boldsymbol{A}_c\boldsymbol{x}_c(t) + \boldsymbol{B}_c^{(1)}\boldsymbol{C}_p\boldsymbol{x}_p(t - \tau) \\ &\quad + \boldsymbol{B}_c^{(2)}\boldsymbol{C}_c\boldsymbol{x}_c(t - \tau) + \boldsymbol{B}_c^{(2)}\boldsymbol{D}_c\boldsymbol{C}_p\boldsymbol{x}_p(t - 2\tau) \end{aligned}$$

Combining $\dot{\boldsymbol{x}}_p(t)$ and $\dot{\boldsymbol{x}}_c(t)$ to find

$$\dot{\boldsymbol{x}}(t) = \left[\dot{\boldsymbol{x}}_p^\top(t) \ \dot{\boldsymbol{x}}_c^\top(t)\right]^\top,$$

we get,

$$\begin{bmatrix}\dot{\boldsymbol{x}}_p(t) \\ \dot{\boldsymbol{x}}_c(t)\end{bmatrix} = \boldsymbol{\Gamma}_0\begin{bmatrix}\boldsymbol{x}_p(t) \\ \boldsymbol{x}_c(t)\end{bmatrix} + \boldsymbol{\Gamma}_1\begin{bmatrix}\boldsymbol{x}_p(t-\tau) \\ \boldsymbol{x}_c(t-\tau)\end{bmatrix} + \boldsymbol{\Gamma}_2\begin{bmatrix}\boldsymbol{x}_p(t-2\tau) \\ \boldsymbol{x}_c(t-2\tau)\end{bmatrix} + \boldsymbol{\Gamma}_3\boldsymbol{u}_2(t),$$

where $\boldsymbol{\Gamma}_0 = \begin{bmatrix}\boldsymbol{A}_p & \boldsymbol{B}_p^{(1)}\boldsymbol{C}_c \\ \boldsymbol{O} & \boldsymbol{A}_c\end{bmatrix}$, $\boldsymbol{\Gamma}_1 = \begin{bmatrix}\boldsymbol{B}_p^{(1)}\boldsymbol{D}_c\boldsymbol{C}_p & \boldsymbol{O} \\ \boldsymbol{B}_c^{(1)}\boldsymbol{C}_p & \boldsymbol{O}\end{bmatrix}$, $\boldsymbol{\Gamma}_2 = \begin{bmatrix}\boldsymbol{O} & \boldsymbol{O} \\ \boldsymbol{B}_c^{(2)}\boldsymbol{D}_c\boldsymbol{C}_p & \boldsymbol{O}\end{bmatrix}$ and $\boldsymbol{\Gamma}_3 = \begin{bmatrix}\boldsymbol{B}_p^{(2)} \\ \boldsymbol{O}\end{bmatrix}$.

We can now write $\dot{\boldsymbol{x}}(t)$ as:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{\Gamma}_0\boldsymbol{x}(t) + \boldsymbol{\Gamma}_1\boldsymbol{x}(t-\tau) + \boldsymbol{\Gamma}_2\boldsymbol{x}(t-2\tau) + \boldsymbol{\Gamma}_3\,\boldsymbol{u}_2(t). \quad (8)$$

With longer delays, we can consider the following approximation as in [19] and [20, p. 62]:

$$\dot{\boldsymbol{x}}_p(t - \tau) = \dot{\boldsymbol{x}}_c(t - \tau) = 0, \text{ a.e..}$$

Taking the second derivative of $\boldsymbol{x}(t)$ and substituting the above approximation, we have,

$$\ddot{\boldsymbol{x}}(t) = \boldsymbol{\Gamma}_0\dot{\boldsymbol{x}}(t) + \boldsymbol{\Gamma}_3\,\dot{\boldsymbol{u}}_2(t). \quad (9)$$

We use the following Taylor series expansion for $\boldsymbol{x}(t-\tau)$:

$$\boldsymbol{x}(t - \tau) = \sum_{n=0}^{\infty}(-1)^n\frac{\tau^n}{n!}\boldsymbol{x}^{(n)}(t).$$

Neglecting the higher order terms, we get an approximated expression of $\dot{\boldsymbol{x}}(t)$ in terms of only $\boldsymbol{x}(t)$ and τ as follows:

$$\boldsymbol{x}(t - \tau) = \boldsymbol{x}(t) - \tau\dot{\boldsymbol{x}}(t) + \frac{\tau^2}{2}\ddot{\boldsymbol{x}}(t) \quad (10)$$

$$\boldsymbol{x}(t - 2\tau) = \boldsymbol{x}(t) - 2\tau\dot{\boldsymbol{x}}(t) + 2\tau^2\ddot{\boldsymbol{x}}(t) \quad (11)$$

Substituting (9) into (10) and (11):

$$\begin{aligned} \boldsymbol{x}(t - \tau) &= \boldsymbol{x}(t) + (\frac{\tau^2}{2}\boldsymbol{\Gamma}_0 - \tau)\dot{\boldsymbol{x}}(t) \\ &\quad + \frac{\tau^2}{2}\boldsymbol{\Gamma}_3\dot{\boldsymbol{u}}_2(t) \end{aligned} \quad (12)$$

$$\begin{aligned} \boldsymbol{x}(t - 2\tau) &= \boldsymbol{x}(t) + (2\tau^2\boldsymbol{\Gamma}_0 - 2\tau)\dot{\boldsymbol{x}}(t) \\ &\quad + 2\tau^2\boldsymbol{\Gamma}_3\dot{\boldsymbol{u}}_2(t) \end{aligned} \quad (13)$$

By substituting (12) and (13) into (8), we can write $\dot{\boldsymbol{x}}(t)$ as:

$$\begin{aligned} \dot{\boldsymbol{x}} &= (\boldsymbol{\Gamma}_0 + \boldsymbol{\Gamma}_1 + \boldsymbol{\Gamma}_2)\,\boldsymbol{x} + \left(\frac{\tau^2}{2}\boldsymbol{\Gamma}_1\boldsymbol{\Gamma}_3 + 2\tau^2\boldsymbol{\Gamma}_2\boldsymbol{\Gamma}_3\right)\dot{\boldsymbol{u}}_2 \\ &\quad + \left(-\tau\boldsymbol{\Gamma}_1 + \frac{\tau^2}{2}\boldsymbol{\Gamma}_1\boldsymbol{\Gamma}_0 + 2\tau^2\boldsymbol{\Gamma}_2\boldsymbol{\Gamma}_0 - 2\tau\boldsymbol{\Gamma}_2\right)\dot{\boldsymbol{x}} + \boldsymbol{\Gamma}_3\,\boldsymbol{u}_2. \end{aligned}$$

Rearranging, we get

$$\boxed{\dot{\boldsymbol{x}}(t) = \boldsymbol{\Psi}_0\boldsymbol{x}(t) + \boldsymbol{\Psi}_1\dot{\boldsymbol{u}}_2(t) + \boldsymbol{\Psi}_2\boldsymbol{u}_2(t)} \tag{14}$$

where

$$\boldsymbol{\Psi}_0 = \boldsymbol{\Theta}\left(\boldsymbol{\Gamma}_0 + \boldsymbol{\Gamma}_1 + \boldsymbol{\Gamma}_2\right),$$

$$\boldsymbol{\Psi}_1 = \boldsymbol{\Theta}\left(\frac{\tau^2}{2}\boldsymbol{\Gamma}_1\boldsymbol{\Gamma}_3 + 2\tau^2\boldsymbol{\Gamma}_2\boldsymbol{\Gamma}_3\right), \boldsymbol{\Psi}_2 = \boldsymbol{\Theta}\boldsymbol{\Gamma}_3$$

$$\boldsymbol{\Theta} = \left(\boldsymbol{I} - \left(-\tau\boldsymbol{\Gamma}_1 + \tfrac{\tau^2}{2}\boldsymbol{\Gamma}_1\boldsymbol{\Gamma}_0 + 2\tau^2\boldsymbol{\Gamma}_2\boldsymbol{\Gamma}_0 - 2\tau\boldsymbol{\Gamma}_2\right)\right)^{-1}.$$

B. Time-Delay Based Networked UIO Design

In this section, we design the controller and the observer to minimize the effect of the unknown input from the global dynamics of the closed loop system as well as the higher order time-delay terms.

Recall that

$$\boldsymbol{\Gamma}_2 = \begin{bmatrix} \boldsymbol{O} & \boldsymbol{O} \\ \boldsymbol{B}_c^{(2)}\boldsymbol{D}_c\boldsymbol{C}_p & \boldsymbol{O} \end{bmatrix} \text{ and } \boldsymbol{\Gamma}_3 = \begin{bmatrix} \boldsymbol{B}_p^{(2)} \\ \boldsymbol{O} \end{bmatrix}.$$

Then, $\boldsymbol{\Gamma}_2\boldsymbol{\Gamma}_3 = \begin{bmatrix} \boldsymbol{O} \\ \boldsymbol{B}_c^{(2)}\boldsymbol{D}_c\boldsymbol{C}_p\boldsymbol{B}_p^{(2)} \end{bmatrix}$. The matrices $\boldsymbol{B}_p^{(1)}, \boldsymbol{B}_p^{(2)}$ and $\boldsymbol{C}_p$ are given matrices. Also, $\boldsymbol{B}_c^{(2)} = (\boldsymbol{I} - \boldsymbol{M}\boldsymbol{C}_p)\boldsymbol{B}_p^{(1)}$ and $\boldsymbol{D}_c = -\boldsymbol{K}\boldsymbol{M}$. In addition, the UIO for the non-networked system is designed with the following restriction on $\boldsymbol{B}_p^{(2)}$ as in [3]:

$$(\boldsymbol{I} - \boldsymbol{M}\boldsymbol{C}_p)\boldsymbol{B}_p^{(2)} = \boldsymbol{O}.$$

We can design the observer such that the higher order delay terms are nullified and the effect of unknown input is minimized. Precisely, we can set $\boldsymbol{\Gamma}_2\boldsymbol{\Gamma}_3 = \boldsymbol{O}$, then we must have $\boldsymbol{B}_c^{(2)}\boldsymbol{D}_c\boldsymbol{C}_p\boldsymbol{B}_p^{(2)} = \boldsymbol{O}$, or

$$\boldsymbol{B}_c^{(2)}\boldsymbol{D}_c = -(\boldsymbol{I} - \boldsymbol{M}\boldsymbol{C}_p)\boldsymbol{B}_p^{(1)}\boldsymbol{K}\boldsymbol{M} = \boldsymbol{O}.$$

Hence, we have the following matrix equations to solve:

$$(\boldsymbol{I} - \boldsymbol{M}\boldsymbol{C}_p)\boldsymbol{B}_p^{(1)}\boldsymbol{K}\boldsymbol{M} = \boldsymbol{O} \tag{15}$$
$$(\boldsymbol{I} - \boldsymbol{M}\boldsymbol{C}_p)\boldsymbol{B}_p^{(2)} = \boldsymbol{O}. \tag{16}$$

We don't have much control over $\boldsymbol{K}$ as this depends on what needs to be achieved through the linear state feedback. Hence, the design variable here is $\boldsymbol{M}$. All other matrices involved in the above system of equations is assumed to be given. Letting $\boldsymbol{R} = \boldsymbol{B}_p^{(1)}\boldsymbol{K} \in \mathbb{R}^{n\times n}$, we can rewrite the above system as:

$$\boldsymbol{R}\boldsymbol{M} = \boldsymbol{M}\boldsymbol{C}_p\boldsymbol{R}\boldsymbol{M} \tag{17}$$
$$\boldsymbol{B}_p^{(2)} = \boldsymbol{M}\boldsymbol{C}_p\boldsymbol{B}_p^{(2)}. \tag{18}$$

Assuming that $\boldsymbol{M}$ has full rank, then $\boldsymbol{M}^\dagger\boldsymbol{M} = \boldsymbol{I}$, and (17) can be written as $\boldsymbol{R} = \boldsymbol{M}\boldsymbol{C}_p\boldsymbol{R}$. Rearranging, we have:

$$\begin{bmatrix} \boldsymbol{M} & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{M} \end{bmatrix}\begin{bmatrix} \boldsymbol{W}_1 \\ \boldsymbol{C}_p\boldsymbol{R} \end{bmatrix} = \begin{bmatrix} \boldsymbol{E}_1 \\ \boldsymbol{R} \end{bmatrix}, \tag{19}$$

or

$$\boldsymbol{I}_2 \otimes \boldsymbol{M}\begin{bmatrix} \boldsymbol{W}_1 \\ \boldsymbol{C}_p\boldsymbol{R} \end{bmatrix} = \begin{bmatrix} \boldsymbol{E}_1 \\ \boldsymbol{R} \end{bmatrix},$$

where $\boldsymbol{O}_{a\times b}$ denotes a matrix of zeros $\in \mathbb{R}^{a\times b}$ and,

$$\boldsymbol{W}_1 = \begin{bmatrix} \boldsymbol{C}_p\boldsymbol{B}_p^{(2)} & \boldsymbol{O}_{p\times(n-m_2)} \end{bmatrix}, \boldsymbol{E}_1 = \begin{bmatrix} \boldsymbol{B}_p^{(2)} & \boldsymbol{O}_{p\times(n-m_2)} \end{bmatrix}.$$

We can write (19) as,

$$(\boldsymbol{I}_2 \otimes \boldsymbol{M})\boldsymbol{W} = \boldsymbol{G}, \tag{20}$$

where $\boldsymbol{M} \in \mathbb{R}^{n\times p}$, $(\boldsymbol{I}_2 \otimes \boldsymbol{M}) \in \mathbb{R}^{2n\times 2p}$, $\boldsymbol{W} = \begin{bmatrix} \boldsymbol{W}_1 \\ \boldsymbol{C}_p\boldsymbol{R} \end{bmatrix} \in \mathbb{R}^{2p\times 2n}$ and $\boldsymbol{G} = \begin{bmatrix} \boldsymbol{E}_1 \\ \boldsymbol{R} \end{bmatrix} \in \mathbb{R}^{2n\times 2n}$. Solving (20) provides a solution for (15) and (16).

V. Stability Analysis of the N_{ET}UIO

After designing the N_{et}UIO, the simplified dynamics of the closed loop system with a feedback controller can be written as:

$$\boxed{\dot{\boldsymbol{x}}(t) = \boldsymbol{\Psi}_0\boldsymbol{x}(t) + \boldsymbol{\Psi}_1\dot{\boldsymbol{u}}_2(t) + \boldsymbol{\Psi}_2\boldsymbol{u}_2(t)} \tag{21}$$

where

$$\boldsymbol{\Psi}_0 = \boldsymbol{\Theta}\left(\boldsymbol{\Gamma}_0 + \boldsymbol{\Gamma}_1\right), \boldsymbol{\Psi}_1 = \boldsymbol{\Theta}\left(\frac{\tau^2}{2}\boldsymbol{\Gamma}_1\boldsymbol{\Gamma}_3\right),$$

$$\boldsymbol{\Psi}_2 = \boldsymbol{\Theta}\boldsymbol{\Gamma}_3,\ \boldsymbol{\Theta} = \left(\boldsymbol{I} - \left(-\tau\boldsymbol{\Gamma}_1 + \frac{\tau^2}{2}\boldsymbol{\Gamma}_1\boldsymbol{\Gamma}_0\right)\right)^{-1}.$$

In this section, we analyze the stability of the UIO-based NCS with the feedback controller. First, recall that the non-networked UIO is designed such that the closed loop system is stable. Hence, the non-networked system (i.e., with $\tau = 0$) is asymptotically stable for any bounded unknown input. The dynamics of the non-networked system for the UIO can be written as:

$$\dot{\boldsymbol{x}}(t) = \left(\boldsymbol{\Gamma}_0 + \boldsymbol{\Gamma}_1\right)\boldsymbol{x}(t) + \boldsymbol{\Gamma}_3\boldsymbol{u}_2(t) = \boldsymbol{\Gamma}\boldsymbol{x}(t) + \boldsymbol{\Gamma}_3\boldsymbol{u}_2(t). \tag{22}$$

where $\boldsymbol{\Gamma}$ is Hurwitz by the design assumption of the non-networked UIO.

Theorem 1. *For the UIO-based NCS in (21) and for a Hurwitz $\boldsymbol{\Gamma}$, we have $\boldsymbol{P} = \boldsymbol{P}^\top \succ \boldsymbol{O}$, is the solution to the Lyapunov matrix equation*

$$\boldsymbol{\Gamma}^\top\boldsymbol{P} + \boldsymbol{P}\boldsymbol{\Gamma} = -2\boldsymbol{Q},$$

for a given $\boldsymbol{Q} = \boldsymbol{Q}^\top \succ \boldsymbol{O}$ and if the UIO design parameter ($\boldsymbol{M}$) satisfies (20), and if $\|\dot{\boldsymbol{u}}_2\| \leq \rho\|\boldsymbol{x}\|$ and $\|\boldsymbol{u}_2\| < \mu\|\boldsymbol{x}\|$, where $\rho, \mu > 0$, then if the network induced delay satisfies the following inequality,

$$\Big(\mu\|\boldsymbol{P}\boldsymbol{\Gamma}_1\boldsymbol{\Gamma}_0\boldsymbol{\Gamma}_3\| + \mu\|\boldsymbol{P}\boldsymbol{\Gamma}_1^2\boldsymbol{\Gamma}_3\| + \|\boldsymbol{P}\boldsymbol{\Gamma}_1\boldsymbol{\Gamma}_0\boldsymbol{\Gamma}\| + 2\|\boldsymbol{P}\boldsymbol{\Gamma}_1^2\boldsymbol{\Gamma}\|$$
$$+\rho\|\boldsymbol{P}\boldsymbol{\Gamma}_1\boldsymbol{\Gamma}_3\|\Big)\tau^2 + \Big(-2\mu\|\boldsymbol{P}\boldsymbol{\Gamma}_1\boldsymbol{\Gamma}_3\| - 2\|\boldsymbol{P}\boldsymbol{\Gamma}_1\boldsymbol{\Gamma}\|\Big)\tau$$
$$+\Big(2\mu\|\boldsymbol{P}\boldsymbol{\Gamma}_3\| - 2\lambda_{min}(\boldsymbol{Q})\Big) < 0$$

then the origin is a globally exponentially stable equilibrium point of the N_{et}UIO.

For brevity, we omit the proof of Theorem 1.

VI. Numerical Example

In this section, we illustrate the usefulness of our proposed UIO-based NCS design with a numerical example. We show the UIO design for non-networked (Section VI-A) and networked systems (Section VI-B) for a single input single output (SISO) system.

A. Single Input Single Output UIO Design Example for A Non-Networked System

In this example, we review the UIO design based on [3]. We follow the exact algorithm to build the non-networked UIO. Then, we simulate the networked system with the UIO. The given system example is a simple one known input, one unknown input, one output LTI system (i.e., $n = 3, m_1 = 1, m_2 = 1, p = 1$). The system is modeled by:

$$\boldsymbol{A} = \boldsymbol{A}_p = \begin{bmatrix} -5 & 3 & 0 \\ 4 & -10 & 4 \\ 0 & 0 & -4 \end{bmatrix}, \boldsymbol{B}_1 = \boldsymbol{B}_p^{(1)} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix},$$

$$\boldsymbol{B}_2 = \boldsymbol{B}_p^{(2)} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, \& \ \boldsymbol{C} = \boldsymbol{C}_p = [2 \ 4 \ -1].$$

Before analyzing the networked UIO, we first follow the design algorithm for the UIO in [3] and find the design parameters for the non-networked system. After that, we map the non-networked UIO to a general NCS form by computing $\boldsymbol{A}_c, \boldsymbol{B}_c^{(1)}, \boldsymbol{B}_c^{(2)}$ as follows:

$$\boldsymbol{A}_c = \tilde{\boldsymbol{P}}(\boldsymbol{A}_p - \boldsymbol{L}\boldsymbol{C}_p), \boldsymbol{B}_c^{(2)} = \tilde{\boldsymbol{P}}\boldsymbol{B}_P^{(1)}$$

$$\boldsymbol{B}_c^{(1)} = \tilde{\boldsymbol{P}}(\boldsymbol{A}_p\boldsymbol{M} + \boldsymbol{L} - \boldsymbol{L}\boldsymbol{C}_p\boldsymbol{M}).$$

We set the initial plant state to $\boldsymbol{x}_p(0) = [-10 \ 10 \ 8]^\top$, the initial UIO state to

$$\boldsymbol{x}_c(0) = (\boldsymbol{I}_n - \boldsymbol{M}\boldsymbol{C}_p)\,[12 \ 15 \ 20]^\top = [-10 \ 4 \ 4]^\top,$$

the unknown input $u_2(t) = 0.5\sin(t)$. Figure 2 shows a very good estimation for the plant states for the non-networked UIO.

B. UIO Design Example for the Networked System

After finding the UIO parameters for the non-networked system as in Section VI-A, we follow the steps mentioned in Sections III and IV to map the UIO to the typical NCS configuration and then follow Algorithm 1.

Since $\boldsymbol{A}_p$ is already stable, a simple solution for (20) would be setting $\boldsymbol{K} = \boldsymbol{O}$, hence the design matrix $\boldsymbol{M}$ would be:

$$\boldsymbol{M} = \boldsymbol{B}_p^{(2)}(\boldsymbol{C}_p\boldsymbol{B}_p^{(2)})^\dagger = [0.3333 \ 0.1667 \ 0.3333]^\top.$$

Choosing the same known and unknown inputs and initial states for the networked observer and plant as in Section VI-A, we simulate the N_{et}UIO. Figure 3 shows the state trajectories and estimation error of the networked system for $\tau = 0.17$ sec. The plots show that the networked system is stable for this small value of the time-delay. The estimation error for the networked system converges to zero, albeit exhibiting more transient response than the non-networked case due to the network effect. Other simulations have showed that as τ increases, the estimation error diverges. Setting a threshold for the estimation error to be $\boldsymbol{e}_{\text{max}} = 10$, the minimum value of τ that would violate this estimation threshold constraint is $\tau_{\text{exper.}}^{\text{max}} = 0.18$ sec.

Recall that the bound on τ would guarantee the stability of the UIO-based NCS is given by Theorem 1. Evaluating the coefficients for the second degree polynomial (in terms of τ) for the bound, we get:

$$(2.308 \cdot 10^7)\tau^2 - (4.4712 \cdot 10^6)\tau - 0.4410 < 0.$$

For this inequality to hold, we need

$$0 < \tau < \tau_{\text{theor.}}^{\text{max}} = 0.1937 \text{ sec.}$$

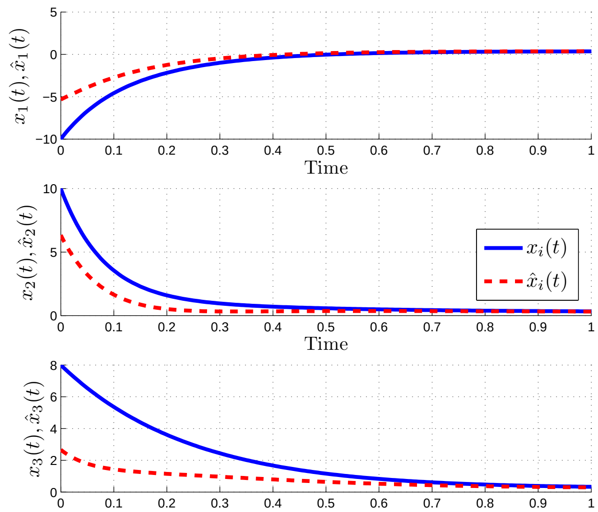

Fig. 2. UIO State Estimation For Non-Networked System ($\tau = 0$)

Algorithm 1 N_{et}UIO Design and Stability Analysis

1: Solve for $\boldsymbol{M}$ in (20):
$$(\boldsymbol{I}_2 \otimes \boldsymbol{M})\boldsymbol{W} = \boldsymbol{G},$$
where $\boldsymbol{W}$ and $\boldsymbol{G}$ are defined in Section IV-B.
2: Given $\boldsymbol{A}_p, \boldsymbol{A}_c, \boldsymbol{B}_p^{(1)}, \boldsymbol{B}_p^{(2)}, \boldsymbol{B}_c^{(1)}, \boldsymbol{B}_c^{(2)}, \boldsymbol{C}_p, \boldsymbol{C}_c$ and $\boldsymbol{D}_c$, compute $\boldsymbol{\Gamma}, \boldsymbol{\Gamma}_0, \boldsymbol{\Gamma}_1, \boldsymbol{\Gamma}_2$ and $\boldsymbol{\Gamma}_3$
3: Find a matrix $\boldsymbol{P} = \boldsymbol{P}^\top \succ \boldsymbol{O}$, a solution to the Lyapunov matrix equation
$$\boldsymbol{\Gamma}^\top\boldsymbol{P} + \boldsymbol{P}\boldsymbol{\Gamma} = -2\boldsymbol{Q}$$
4: Analyze the stability of the networked system:
$$\dot{\boldsymbol{x}}(t) = \boldsymbol{\Psi}_0\boldsymbol{x}(t) + \boldsymbol{\Psi}_1\dot{\boldsymbol{u}}_2(t) + \boldsymbol{\Psi}_2\boldsymbol{u}_2(t)$$
by varying the time-delay (τ)
5: Establish an experimental bound on τ that guarantees the stability of the UIO-baed NCS
6: Compare the theoretical bound on τ given by the quadratic polynomial in Theorem 1 and the experimental one computed in Step 5

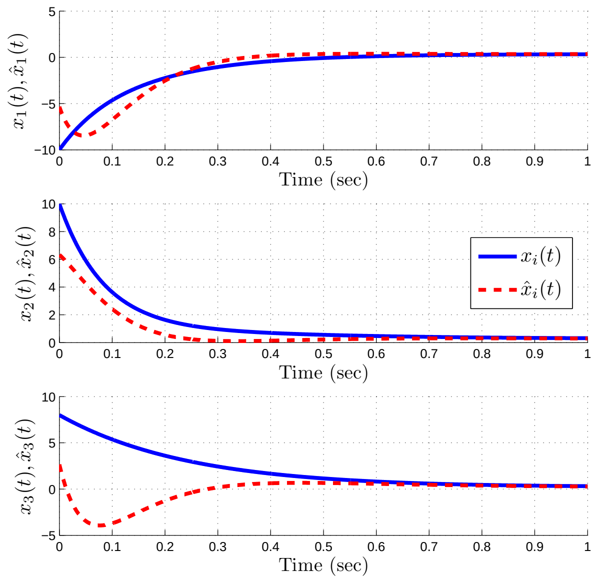

Fig. 3. UIO State Estimation for the $\mathrm{N_{et}}$UIO ($\tau = 0.17$ sec)

Hence, the derived upper bound ($\tau^{\max}_{\text{theor.}} = 0.1937$ sec.) for the time-delay that guarantees the convergence of the estimation error for the $\mathrm{N_{et}}$UIO is close to the actual one ($\tau^{\max}_{\text{exper.}} = 0.18$ sec.)

VII. Conclusions and Future Work

Albeit it provides many advantages such as the ease of use, flexibility, and utilization of more efficient control laws, the addition of a communication network in the feedback loops of control systems complicates their analysis and design. In this paper, we discuss an Unknown Input Observer-based design for Networked Control Systems. The determination of an upper bound on the network induced time-delay is significantly important in the design of an NCS so that a suitable sampling period is chosen. When the time-delay is greater than the sampling period in an NCS, then the global stability of the overall NCS can not be guaranteed as discussed by Kim *et al.* [23]. The results show that the derived bound for UIO-based NCS is accurate. In our future work, we plan to consider other UIO architectures and Sliding Mode Observers for networked systems. We also want to apply the proposed model of the $\mathrm{N_{et}}$UIO for power networks where fault detection and isolation techniques can be employed to better monitor the usually networked power systems.

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