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Comparison of the Performances of Two Aggregate Air Traffic Models

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Inspired by an existing linear dynamic system air traffic model, a discrete-time Link Transmission Model is developed for air traffic flow prediction and optimization in this paper. Compared with a Large-capacity Cell Transmission Model, this model reduces the number of state variables by approximately an order of 10, which decreases the computing time in prediction and optimization of air traffic flow. Comparison and analysis regarding the prediction accuracy and computational efficiency of the new model are performed based on two benchmark scenarios, one including a simple case with two cross flight paths, the other including the entire National Airspace System. Numerical results of this study show that the Link Transmission Model achieves a same level of accuracy as Large-capacity Cell Transmission Model in air traffic prediction , and the computational efficiency is significantly improved with an equally optimal solution in air traffic optimization.

Nomenclature

$C_s(t)$	=	maximum number of aircraft allowed in sector s at time t
$f^k(t)$	=	departures into path k at time t
K	=	number of paths in air traffic prediction and optimization
n^k	=	the number of links on path k
$q_i^k(t)$	=	equals $\beta_i^k(t)x_i^k(t)$, optimal number of aircraft transitioning from link i to link $i+1$ at time t
S	=	number of sectors in National Airspace System
T	=	planning time horizon of simulation and optimization
T_i	=	length of link i, scaled in minute
$x_i^k(t)$	=	aircraft count in link i on path k at time t
$x_j^{k,i}(t)$	=	aircraft count in j th cell of link i on path k at time t
$u_j^{k,i}(t)$	=	delay control of aircraft in jth cell of link i on path k at time t
$\lambda_s(t)$	=	Lagrange multiplier for sector s at time t
$\beta_i^k(t)$	=	transition fraction of aircraft that move from link i to link $i+1$ at time t
i	=	index of link
j	=	index of cell
k	=	index of path
s	=	sector index
s_i	=	sector that link i lies in
t	=	time step

I. Introduction

Part of the responsibility of the air traffic control (ATC) authority is to guarantee the safe operation for the National Airspace System (NAS) and to take appropriate control actions ahead before possible congestion

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occurs in an airspace. Aggregate aircraft count in an area reflects traffic condition in the region and indicates potential safety problem, which is the major concern from the perspective of air traffic controller. As shown in literature, the prediction of aircraft count in an area and optimization for Traffic Flow Management (TFM) helps to develop optimal strategies to increase efficiency while maintaining safety in the NAS.

Among various approaches, aggregate models have been of particular interest in TFM studies. The first aggregate model reported in literature[1] formulates the traffic flow using linear discrete-time dynamic equations and employs latitude-longitude tessellation in space. Its dimension linearly depends on the spacial discretization of the NAS rather than the number of aircraft involved in it, enabling simulation and analysis with well-developed control theory and optimization tools. Linear Dynamic System Model[2] (LDSM) aggregates air traffic in each Air Route Traffic Control Center (ARTCC, hereafter denoted as Center). The dimension of the state space is determined by the number of Centers, thus it is fixed and in a low order.[3] Streams flowing between Centers are sorted by four types of input and output, which is further simplified in a linear dynamic form. More recently, inspired by the Cell Transmission Model used in highway traffic,[4,5] an Eulerian-Lagrangian Large-capacity Cell Transmission Model (CTM(L)) was developed for TFM.[6] It models the NAS as a huge network involving thousands of paths connecting hundreds of sectors.[7] Besides aggregate aircraft count in each sector, the origin-destination informations of flights are also taken into consideration. Each path consists of several links, and each link is subdivided into a number of cells according to the length of the link (scaled in minute). The state variable is defined as the aggregate aircraft count in each cell. Its linear formulation enables ease of detailed analysis and optimization of a single path as well as aggregate properties for a region (Center or sector). Therefore, CTM(L) offers flexibility for the decision making of air traffic management (ATM) at both microscopic and macroscopic levels.

However, as pointed out by several researchers, there is a trade-off between accuracy (detailed flight information) and computational efficiency for aggregate model. For example, CTM(L) entails large numbers of state variables to generate detailed flight information. In a global formulation for the entire NAS, billions of variables are involved which inevitably incurs a great challenge to the programming and computational efficiency. Commercial optimization software available nowadays hardly offers the ability to handle optimization problem with such a high order of variables (and constraints). In order to make the CTM(L)-based TFM optimization problem tractable and solvable with existing software, sophisticated optimization methods, such as dual decomposition,[8] has to be introduced. In the dual decomposition method the traffic flow optimization for the entire NAS is decomposed into collection of subproblems path by path, which are handled as lower order optimization problems respectively. A global optimal solution is asymptotically approximated as the subproblems converge in an iterative manner.

The dimension of the subproblems depends on the number of cells on each path, which is determined by the average flight time scaled in minutes. A long path introduces considerable cells. However, the path may pass only tens of sectors. Since decision-making can be operated at a sector level, traffic information in an individual cell does not help much for traffic flow management. It is desirable to aggregate the traffic to a further level (such as a link level) to lower down the order of state variables, which motivated the work presented in this paper.

Inspired by the LDSM,[2] a Link Transmission Model (LTM) is developed in this paper, which reduces the order of CTM(L) without sacrificing the accuracy of prediction and facilitate the Integer Programming. The computational efficiency is therefore improved.

The rest of this paper is organized as follows. Section II reviews the LDSM and CTM(L). Then the LTM is introduced. A dual decomposition algorithm based on LTM for TFM optimization is also addressed. Section III contains two parts. The first part uses a Two-cross-paths scenario to validate the LTM-based TFM optimization in order to fully demonstrate how the algorithm works in a simplified and low-order case. In the second part, the LTM is applied to the entire NAS. Recorded flight data from Aircraft Situation Display to Industry (ASDI) and Enhanced Traffic Management System (ETMS)[9] are used as input for air traffic prediction and TFM optimization. The LTM will be compared with the CTM(L) with respect to prediction accuracy and computational efficiency in Sector IV. In the final section, concluding remarks are addressed

II. Aggregate Air Traffic Model

A. Review of Linear Dynamic System Model and the Large-capacity Cell Transmission Model

1. Linear Dynamic System Model

In LDSM, the state variable is defined as the aggregate aircraft count in a Center. The dimension of LDSM is fixed, namely 23, representing 22 Centers in U.S. and one for the international airspace. Each state variable is propagated forward as a function of departures and arrivals in corresponding Center along with inflow and outflow of traffic between contiguous Centers (as shown in Fig. 1). Equation (1) describes the dynamics of LDSM.

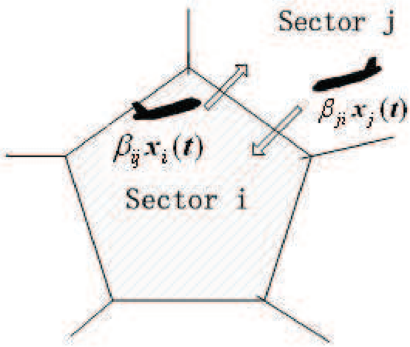


Figure 1. Illustration of Linear Dynamic System Model

$$x_i(t+1) = x_i(t) - \sum_{j=1}^{N} \beta_{ij}(t)x_i(t) + \sum_{j=1, j\neq i}^{N} \beta_{ji}(t)x_j(t) + f_i(t) \tag{1}$$

where N is the number of Centers. $f_i(t)$ is the departures within Center i, which can be modeled to include both stochastic and deterministic components.[10] It is the forcing input into the system. $\beta_{ij}(t)$ and $\beta_{ji}(t)$ are the fractions of aircraft transitioning between Center i and Center j at time t. In air traffic prediction, $\beta_{ij}(t)$ can be statistically obtained from historical flight data using data mining technique.

Basically, LDSM focuses on aggregate aircraft count at a Center level, therefore it involves a relatively small set of equations. The accuracy of the model depends on the transition fractions $\beta_{ji}(t)$ and aggregation interval. The former describes the transition pattern of the NAS. The latter influences the accuracy of the model errors. In general, the shorter the aggregation interval, the more accurate the predicted aircraft count is.

2. Large-capacity Cell Transmission Model

CTM(L) is a graph-theoretic multicommodity model, as shown in Fig. 2. The NAS is based on a huge network of *Paths* connecting the airports across the continent. Each path passes through several sectors. The segments of a path lying inside a sector are termed *Links*. The average flight time that flights pass through a link is a measure of its length, usually scaled in minute. It can be calculated from historical flight data. Each link is further subdivided into small segments called *Cells*. The number of cells in a link is equal to the length of the link. For example, a link averaged 30 minutes of flight time has 30 cells in it. A sector may contain several links belonging to different paths. Equation (2) presents the dynamics of CTM(L).

$$\begin{aligned} x_j^{k,i}(t+1) &= x_{j-1}^{k,i}(t) - u_{j-1}^{k,i}(t) + u_j^{k,i}(t) \\ x_0^{k,0}(t+1) &= f^k(t) + u_0^{k,0}(t) \end{aligned} \tag{2}$$

The state variable is defined as the aggregate aircraft count in each cell. $u_j^{k,i}(t)$ represents the number of aircraft held in the jth cell of link i. Unlike LDSM in which each Center has input, only the first cell of a path

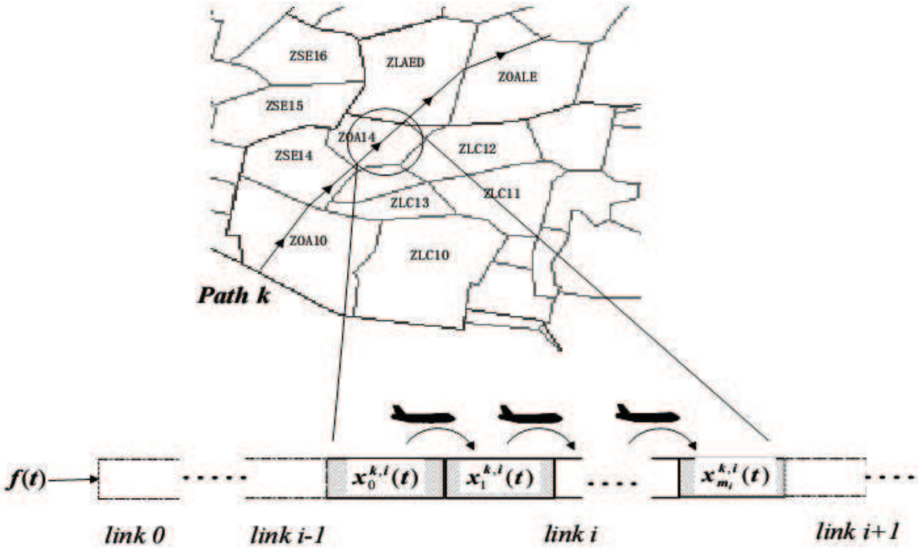


Figure 2. Illustration of Cell Transmission Model

has input in CTM(L). In air traffic prediction where delay policy is not applied (there is no $u_j^{k,i}(t)$ in the Eq. (2)), aircraft count of upstream cell is simply transmitted to the downstream cell during each time interval, the traffic flow therefore moves forward cell by cell as time evolves. In comparison with LDSM, CTM(L) involves far more state variables. For example, an air route from east coast to west coast averaged about five hours in length leads to approximately 300 cells on the path. In TFM optimization, the number of state variables is also proportional to the planning time horizon T. For a two-hour optimization, approximately 300×120 state variables are defined for the path. Moreover, the delay control $u_j^{k,i}(t)$ is also variable to be optimized, the dimension of the state variable doubles as a result. In practice, thousands of paths are identified across the entire NAS, resulting billions of state variables along with considerable constraints in CTM(L). Hence, the size of the TFM problem is a formidable challenge.

B. Link Transmission Model

1. *Dynamics*

The advantage of CTM(L) lies in its deterministic dynamics. Eq. (2) describes a highly linear discrete-time system. The vector form of its dynamics has a transition matrix with 1 on its diagonal and -1 on the subdiagonal,[8] which is easy for Linear Programming (LP). However, it brings tremendous state variables as a tradeoff.

The lower-order LDSM inspired the ongoing effort to reduce the number of states variables in CTM(L). A Link Transmission Model is developed by aggregating the aircraft count at the link level, as shown in Fig. 3. Generally, a flight can not pass through a link within one minute, hence only a fraction of the aircraft

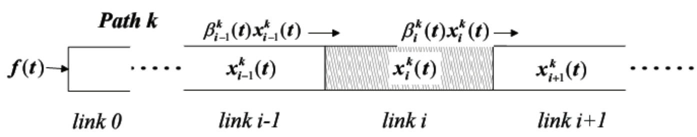


Figure 3. Combination of CTM(L) and LDSM to form Link Transmission Model

count in a link is transmitted to the downstream link during one time interval. A transition fraction $\beta_i^k(t)$

which is similar to the transition probability $\beta_{ij}(t)$ in LDSM is introduced into LTM . In essence, LTM is a combination of LDSM and CTM(L). Based on the graph-theoretic model of CTM(L), each link can be viewed as a Center with only two contiguous Centers, the traffic flow pass through it unidirectionally. Hence, the dynamics of LTM can be formulated as:

$$\boldsymbol{X^k}(t+1) = \boldsymbol{A^k}(t)\boldsymbol{X^k}(t) + \boldsymbol{B^k} f^k(t) \tag{3}$$

where

$$\boldsymbol{X^k}(t) = [x_0^k(t), x_1^k(t), \ldots, x_{n^k}^k(t)]^T$$

$$\boldsymbol{A^k}(t) = \begin{bmatrix} (1-\beta_0^k(t)) & & & \\ \beta_0^k(t) & (1-\beta_1^k(t)) & & \\ & & \ddots & \\ & & \beta_{n^k-1}^k(t) & (1-\beta_{n^k}^k(t)) \end{bmatrix} \boldsymbol{B^k} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$\boldsymbol{X^k}(t)$ is the state vector for path k. $\beta_i^k(t)x_i^k(t)$ represents the fraction of aircraft transitioning from link i to link $i+1$ at time t. Equation (3) is a general linear state space representation of a discrete-time system, whose dynamics is straightforward. Aircraft count in a link at time $t+1$ consists of two components: a fraction of aircraft transmitted from its upstream link, and a fraction of aircraft that stay in it. Like the CTM(L), the transition matrix of LTM is also a sparse matrix with non-zero entries on diagonal and subdiagonal, which is suitable for implementation.

Compared with CTM(L), LTM significantly reduces the number of state variables. The dimension of state space now depends on the number of links that a path has rather than the number of cells. A path of 300 minutes in length may comprise no more than ten links. Another feature is that the delay control is not explicitly formulated but incorporated into the transition fraction $\beta_i^k(t)$ (see Appendix B). In air traffic prediction, $\beta_i^k(t)$ is a determined parameter calculated from historical flight data, whereas it is a variable to be optimized in TFM optimization. With the dynamics of LTM, the TFM problem can be formulated.

2. *Optimization Formulation*

The cost function can be formulated in various forms according to specified objectives. In this paper, the objective is defined as the minimum flight time of all flights in the planning time horizon.

$$\min \sum_{t=0}^{T}\sum_{k=0}^{K}\sum_{i=0}^{n^k} c_i^k x_i^k(t) \tag{4}$$

where c_i^k represents the weight imposed on link i of path k.

The minimization is subject to the following constraints:

1) *Initial conditions*

$$\begin{aligned} x_i^k(0) &= 0, \qquad i \in \{1, \ldots, n^k\} \\ x_0^k(0) &= f^k(0) \end{aligned} \tag{5}$$

Initially, all links are assumed to be empty except for the first link. This can be easily modified to adjust to practical condition.

2) *LTM dynamics*

$$\begin{aligned} x_i^k(t+1) &= [1-\beta_i^k(t)]x_i^k(t) + \beta_{i-1}^k(t)x_{i-1}^k(t), \qquad i \in \{1, \ldots, n^k\} \\ x_0^k(t+1) &= [1-\beta_0^k(t)]x_0^k(t) + f^k(t) \\ 0 \leq \beta_i^k(t) &\leq 1 \end{aligned} \tag{6}$$

3) *Sector capacity constraint*

$$0 \leq \sum_{(i,k)\in Q_{s_i}} x_i^k(t) \leq C_{s_i}(t) \tag{7}$$

This constraint restricts the number of aircraft in a sector to be below the allowed maximum number of aircraft.[11]

4) *Departure and arrival constraint*

$$\sum_{t=0}^{T}\beta_0^k(t)x_0^k(t)=\sum_{t=0}^{T}\beta_{n^k}^k(t)x_{n^k}^k(t)=\sum_{t=0}^{T}f^k(t) \tag{8}$$

This constraint guarantees that all flights passing through their paths within the given planning time horizon. In other words, no aircraft is held in the air at the end of the planning time horizon. This constraint is important as it is the impetus that drives the traffic flow moving forward.

5) *Minimum dwell time constraint*

$$\begin{aligned}\sum_{t=0}^{T_0+T_1\cdots+T_i-1}\beta_i^k(t)x_i^k(t) &= 0 \\ \sum_{t=T_0+T_1\cdots+T_i}^{T^*}\beta_i^k(t)x_i^k(t) &\leq \sum_{t=T_0+T_1\cdots+T_{i-1}}^{T^*-T_i}\beta_{i-1}^k(t)x_{i-1}^k(t)\end{aligned} \tag{9}$$

This constraint guarantees that a flight must stay in a link for a minimum time of the link length. This is a key constraint that regulates the speed of the flights, ensuring that the traffic flow does not move "too fast." Detailed discussion about this constraint can be found in Appendix B.

6) *Integer constraint*

$$x_i^k(t)\in Z_+,\qquad \beta_i^k(t)x_i^k(t)\in Z_+ \tag{10}$$

This constraint guarantees that the optimal solution can be applied to integer number of aircraft.

***Remark*:** (1) subscripts and superscripts above are: $t\in\{0,\ldots,T\}, k\in\{0,\ldots,K\}, s_i\in\{0,\ldots,S\}$. (2) Figuring out the optimal $\beta_i^k(t)$ may require nonlinear programming.[13] Fortunately, $\beta_i^k(t)$ always appears together with $x_i^k(t)$, a new variable can be defined as $q_i^k(t)=\beta_i^k(t)x_i^k(t)$. Then the problem is converted to a linear optimization problem.

C. Dual Decomposition Method based on Link Transmission Model

Although LTM reduces the dimension, it is challenging to calculate the global optimum for the large scale NAS-wide TFM problem with conventional programming methods. On one hand, formulating the NAS-wide TFM problem requires large block of memory of the computer. One the other hand, handling optimization problem with up to millions of state variables hobbles the available optimization tools. Therefore, the solution must resort to advanced mathematical algorithm. Ref. 12 introduces a dual decomposition method that decomposes the NAS-wide TFM problem into small subproblems path by path.[14] Each subproblem merely involves a single path and is solved by LP, making the optimization problem technically tractable. The decomposition algorithm based on LTM is summarized in Table 1. Detailed derivation of the algorithm is in Appendix A.

III. Model validation

In this section, both air traffic prediction and TFM optimization are presented using the LTM. A Two-cross-paths scenario is used to evaluate the performance of optimization of LTM in comparison with CTM(L) in the first part. In the second part, the LTM is extended to the NAS-wide TFM application. In both cases, the optimization formulation and dual decomposition algorithm described in Section II are applied to optimize the TFM problem. The model validation is implemented with C++ code on a 2.8 GHz CPU, 16G RAM PC running LINUX. The optimization tool used is CPLEX11.0.[15] The airspace under study is constructed by 485 high altitude sectors defined by the Future ATM Concept Evaluation Tool (FACET).[16]

Table 1. Dual decomposition algorithm based on LTM

Subproblem:

$d^{k^\star}(\lambda) = \min \sum_{t=0}^{T} \sum_{i=0}^{n^k} [c_i^k + \lambda_{s_i}(t)] x_i^k(t),$

$s.t.\ \ \boldsymbol{X^k}(0) = \boldsymbol{B^k} f^k(0),$

$\boldsymbol{X^k}(t+1) = \boldsymbol{A^k}(t)\boldsymbol{X^k}(t) + \boldsymbol{B^k} f^k(t),$

$0 \leq x_i^k(k) \leq C_{s_i}(t),$

$\sum_{t=0}^{T} \beta_0^k(t) x_0^k(t) = \sum_{t=0}^{T} \beta_{n^k}^k(t) x_{n^k}^k(t) = \sum_{t=0}^{T} f^k(t),$

$\sum_{t=0}^{T_0+T_1\ldots+T_i-1} \beta_i^k(t) x_i^k(t) = 0,$

$\sum_{t=T_0+T_1\ldots+T_i}^{T^*} \beta_i^k(t) x_i^k(t) \leq \sum_{t=T_0+T_1\ldots+T_{i-1}}^{T^*-T_i} \beta_{i-1}^k(t) x_{i-1}^k(t), \qquad T^* \in \{T_0 + T_1 \cdots + T_i, \ldots, T\}$

Master problem:

$d^\star(\lambda) = \max\{-\sum_{t=0}^{T} \sum_{s_i=1}^{S} \lambda_{s_i}(t) C_{s_i}(t) + \sum_{k=0}^{|K|} d^{k^\star}(\lambda)\},$

Update:

$g_{s_i}(t) = -(\sum_{(k,i)\in Q_{s_i}} x_i^k(t) - C_{s_i}(t)),$

$\lambda_{s_i}(t) := (\lambda_{s_i}(t) - \alpha_i g_{s_i}(t))_+,$

where

$k \in \{0, \ldots, K\}, t \in \{0, \ldots, T\}, i \in \{1, \ldots, n^k\}, s_i \in \{0, \cdots, S\}.$

$g_{s_i}(t)$ is the subgradient of dual function.

$\lambda_{s_i}(t)$ is the Lagrange multiplier, $(\cdot)_+$ denotes the non-negative part of a vector.

$\alpha_i = \frac{1}{i+1}$ is the subgradient step and i is the iteration index.

Historical nonmilitary flight data are acquired from ASDI/ETMS, which provides latitude, longitude, altitude and time information of all airborne aircraft in the United States. The ASDI data serve as a baseline to the simulation and optimization results.

The flight path and link length informations are obtained and stored in the database by mining one month ASDI data, as done in Ref. 6. To set up the dual decomposition algorithm, flight informations are extracted from the ASDI data, including flight identification, departure $f^k(t)$ and the flight path by which the flights are clustered. This process decomposes the large-scale problem into small subproblems path by path.

A. A Two-cross-paths scenario

For illustration and validation purpose, a simple scenario of Two-cross-paths is formulated as a small TFM optimization problem, as shown in Fig. 4. Two paths are chosen from the flight path database. Path A comprises four links and Path B comprises five links. For simplicity, segments of the path in origin and destination sectors are ignored. The links in Path A are ZID99-ZDC18-ZDC19, ZDC18-ZDC19-ZDC58, ZDC19-ZDC58-ZDC54, ZDC58-ZDC54-ZDC34. Links in Path B are ZDC04-ZDC12-ZDC18, ZDC12-ZDC18-ZDC19, ZDC18-ZDC19-ZDC58, ZDC19-ZDC58-ZNY56, ZDC58-ZNY56-ZBW49. Departures are obtained from database as well between 1:00 PM and 3:00 PM UTC on May 1, 2005. In this case, it is stipulated that a congestion happen when the aircraft count in a sector exceed seven, namely $C_{s_i}(t) = 7$ which can be adapted to reflect practical capacities. Optimization results are shown in Fig. 5 and Fig. 6.

As expected, the optimized control by both LTM and CTM(L) alleviate the congestion in sectors ZNY58 and ZDC19 where the two paths intersect. For example, the actual aircraft count in ZDC58 exceed seven (Fig. 5(b)) for more than 30 minutes (ASDI data). In contrast,sector capacities are well respected when optimized delay control is applied based on LTM or CTM(L). However, compliance with the capacity restriction

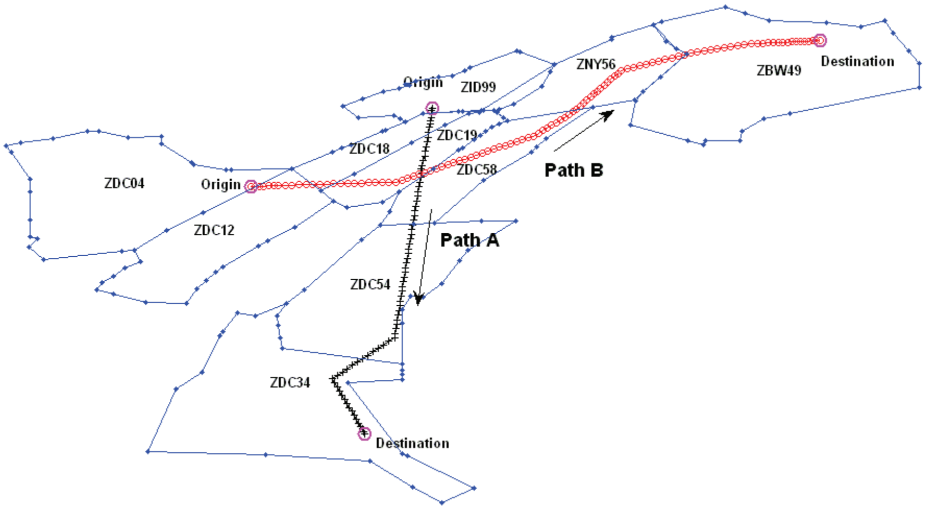


Figure 4. A Two-cross-paths scenario

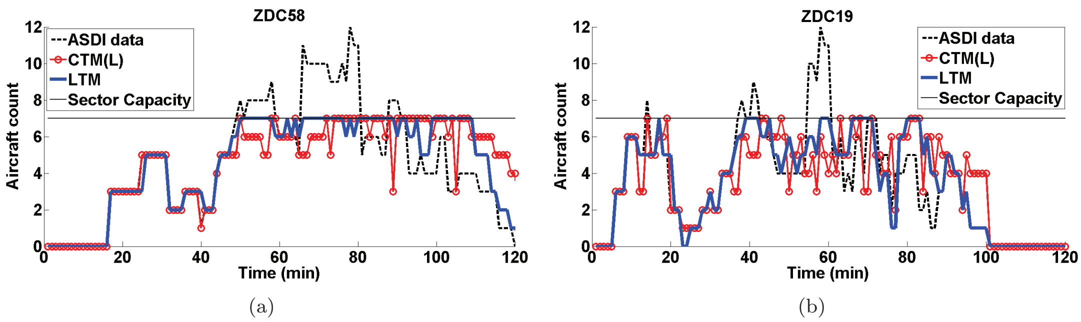


Figure 5. Optimization of traffic flow using LTM and CTM(L)

inevitably incurs delay as a tradeoff. As can be seen in Fig. 5(a), some flights are still airborne at the end of the two hours planning time horizon, which suggests that a fraction of flights are subject to delay in order to avoid the congestion.[17,18]

To verify the result of dual decomposition algorithm based on LTM, the resulting parameters are compared with those of CTM(L) in Fig. 6. The dual decomposition method approximates the global optimum in iterative manner. It takes about 150 iterations to converge in this case. Master problem objectives and the total flight time by both models are consistent with a slight difference. Therefore, LTM produces a equally optimal solution as CTM(L) in optimization terms.

B. NAS-wide Simulation and Optimization for TFM

To assess the use of LTM in practical applications, NAS-wide air traffic prediction and optimization for TFM based on LTM and CTM(L) are presented in this subsection. Flights position data extracted from ASDI/ETMS are used as baseline to evaluate accuracy of prediction as well as optimization.

1. *Air traffic prediction*

A full day (1440 minutes) NAS-wide air traffic prediction between 0:00 AM and 12:00 PM UTC on May 1, 2005 is shown in Fig. 7. Both LTM-based and CTM(L)-based aircraft count predictions share a similar aircraft count profile as ASDI data. The trend of these curves reflect the daily traffic condition pattern in each sector, namely the workload is light during nighttime and heavy during the daytime. Although the

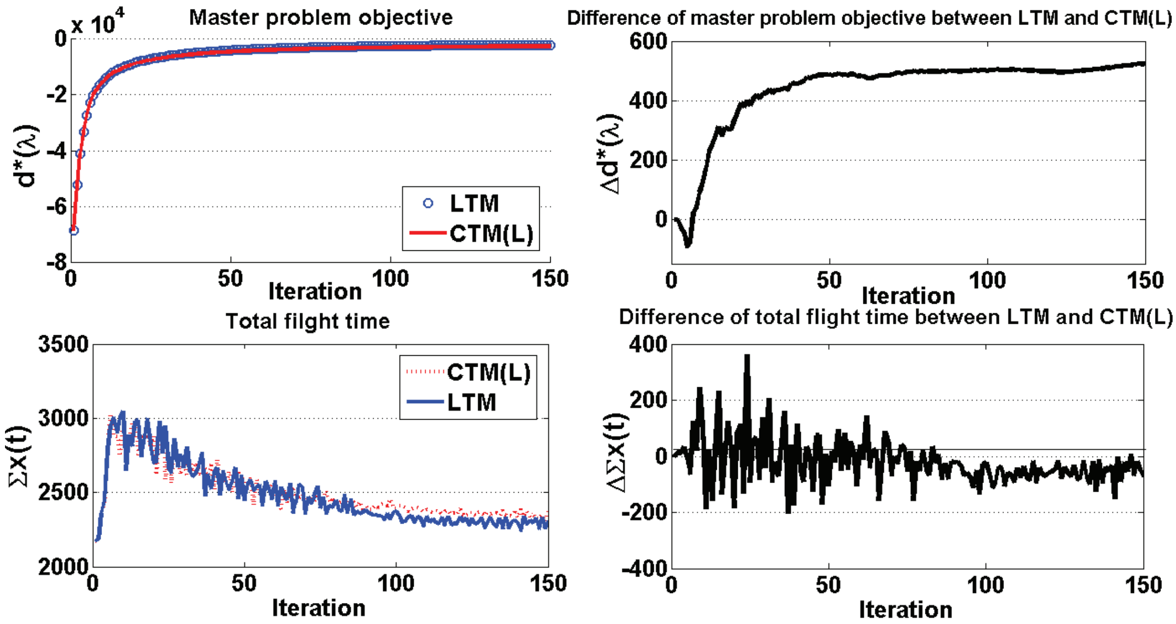


Figure 6. Comparison of dual decomposition algorithm parameters

predictions do not exactly match the actual aircraft count in a relatively short time interval, the trend and change of the predicted traffic condition are accurate enough for ATM forecast in the 24 hours horizon. Another important observation is the computing time. The prediction is finished in five seconds even though the NAS-wide TFM problem involves thousands of paths, implying that this model is able to be used in real-time application.

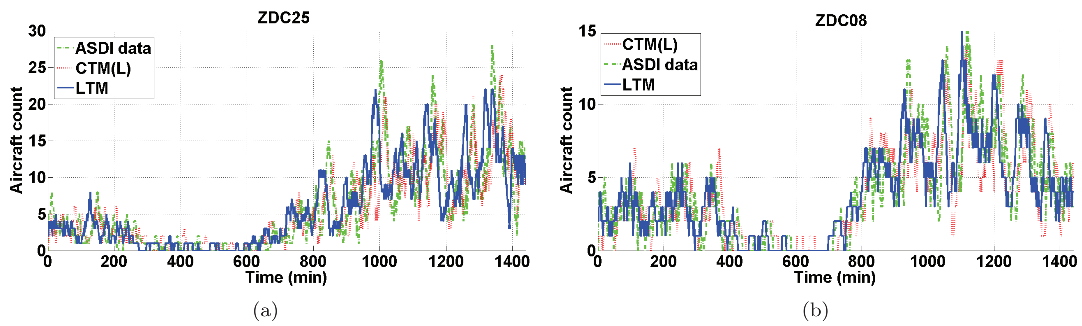
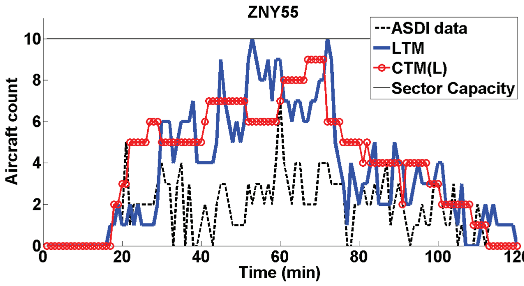


Figure 7. NAS-wide TFM prediction using LTM and CTM(L)

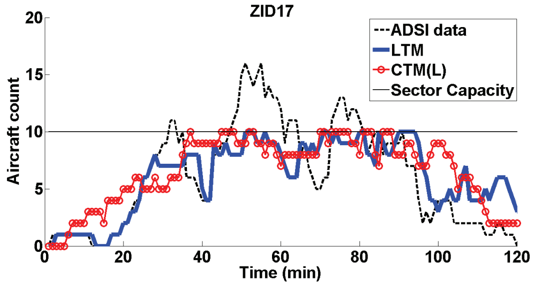
2. *Optimization*

The optimization is extended to a two-hour NAS-wide TFM problem. 2326 paths and 2796 flights are involved which amounts to 2326 subproblems are identified in the dual decomposition algorithm. Given that each subproblem is solved at a same path-optimization level as the Two-cross-paths scenario, the complexities for both cases are the same, with the only difference that the NAS-wide problem has a much larger number of subproblems in each iteration. Figure 8 shows the optimization results.

As expected, sector capacities are well respected for both LTM and CTM(L). Like the Two-cross-paths scenario, delay occurs at the end of planning time horizon when delay control is applied. In sector ZID17 and ZJX16, aircraft are restricted to enter the sector when the aircraft count exceeds the workload. In contrast, workload of sector ZNY55 is light without control (see ADSI data). In optimization, some aircraft are held in ZNY55, its workload increases as a result. This is the desirable effect of optimization. Workloads of sectors are leveraged in order to achieve a minimum overall flight time while maximizing the utilization of airspace in each sector. Although the optimal delay control for LTM and CTM(L) are not exactly the same, they both keep the aircraft count in each sector below the maximum number allowed .



(a)



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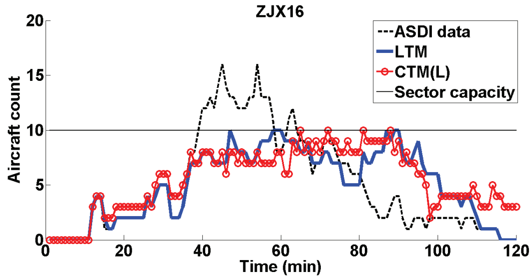

(c)

Figure 8. NAS-wide TFM optimization using LTM and CTM(L)

IV. Discussion of Computational Efficiency

The primary advantage of employing LTM for TFM optimization lies in the high computational efficiency. Figure 9 shows the statistics of both cases. From subfigure (a) and (b) which are the comparisons of computing time, the LTM-based optimization consumes far less CPU time than CTM(L)-based optimization to finish the iterations of dual decomposition algorithm. The ratio is approximately 6:1. As the computing time almost linearly increases as the dual decomposition iteratively progresses, it is clear that more iteration it runs, the more CPU time is saved by LTM. Subfigure (c) and (d) presents the numbers of internal iterations needed by SIMPLEX algorithm. In each update of dual decomposition algorithm, LTM-based optimization needs roughly one eighth of SIMPLEX iterations needed by CTM(L)-based optimization. There are two reasons for this. First, LTM involves fewer variables than CTM(L). Suppose a path composed of m links and totally n cells, the planning time horizon is T minutes, then the number of variables for CTM(L) is $2 \times n \times T$ (delay control $u_j^{k,i}(t)$ is also treated as variable to be optimized). In contrast, there are only $2 \times m \times T$ variables for LTM. Generally n is greater than m. In the Two-cross-paths scenario in which $n = 87, m = 9, T = 120$, the difference of numbers of variable between the two model is 18960. In the NAS-wide TFM problem in which totally 10323 links and 98781 cells are involved, the number of variable is almost decreased by an order of 10 by using LTM. Hence, there are fewer vertices to search in the SIMPLEX algorithm for LTM.[19] Second, the number of constraints also account for the time saving. To set up the optimization problem with constraints described in Section II, CTM(L) involves approximately $[2 + 2n + m] \times T$ constraints, whereas LTM involves $[2+3m] \times T$ constraints. The difference is 18960 in the Two-cross-paths scenario. The reduced-dimension state variable together with the reduced-size constraints significantly decrease the complexity of the TFM optimization problem, and contribute to the improved computational efficiency as a result.

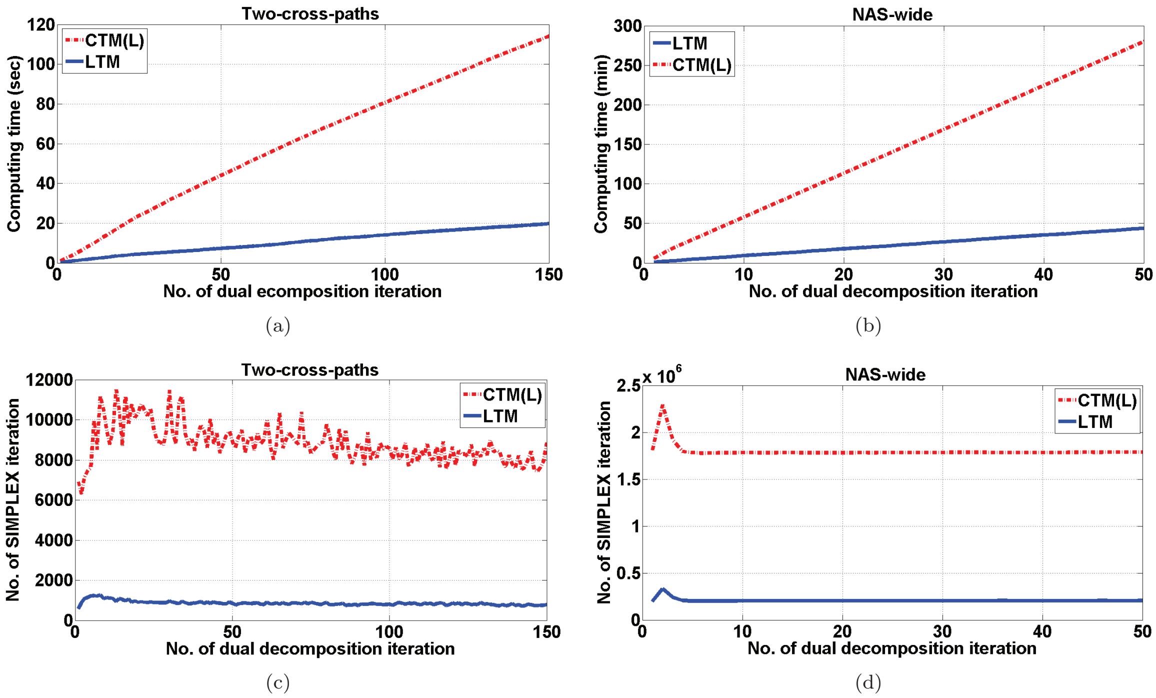

Figure 9. Comparison of computational efficiency

V. Conclusion

This paper develops a Link Transmission Model for air traffic prediction and TFM optimization based on a Linear Dynamic System Model and a Large-capacity Cell Transmission Model. Performance of the proposed model is evaluated in the context of Two-cross-paths scenario and the entire NAS TFM problem respectively. In air traffic prediction, numerical results indicate that LTM predicts the aircraft count in each sector as precisely as CTM(L) dose. In TFM optimization, LTM achieves the equally optimal solution with higher computational efficiency due to less number of state variables and constraints. In terms of computer implementation, LTM lowers down the hardware requirements. With the dual decomposition algorithm, the large-scale optimization problem is decomposed and technically tractable with regular optimization tools. Given the independence between subproblems, computational efficiency of the NAS-wide TFM optimization is expected to be further improved by employing parallel computing.

Appendix

A. Derivation of the subproblem in the dual decomposition algorithm

The dual decomposition method based on CTM(L) is summarized as follows:[8]

$$Subproblem: \quad d^{k^\star}(\lambda) = \min_{x,u,Z^k} \sum_{t=0}^{T} c^k x^k(t) + \sum_{t=0}^{T} \sum_{i=0}^{n^k} \lambda_{s_i}(t) Z^k_{s_i}(t) \tag{11}$$

$$s.t. \quad \boldsymbol{X^k}(0) = \boldsymbol{B^k} f^k(0) \tag{12}$$

$$\boldsymbol{X^k}(t+1) = \boldsymbol{A^k}(t)\boldsymbol{X^k}(t) + \boldsymbol{B_1^k}\boldsymbol{U^k}(t) + \boldsymbol{B_2}^{\boldsymbol{k}} f^k(t) \tag{13}$$

$$0 \le u_j^{k,i}(t) \le x_j^{k,i}(t) \tag{14}$$

$$\sum_{(i,k)\in Q_s} x_j^{k,i}(t) = Z^k_{s_i}(t) \tag{15}$$

$$Masterproblem:$$

$$d^\star(\lambda) = \max_{\lambda_{s_i}\ge 0} \{-\sum_{t=0}^{T}\sum_{s=0}^{S} \lambda_{s_i}(t) C_{s_i}(t) + \sum_{k=0}^{K} d^{k^\star}(\lambda)\} \tag{16}$$

$$Update:$$

$$g_{s_i}(t) = -(\sum_{k=1}^{K} Z^k_{s_i}(t) - C_{s_i}(t)) \tag{17}$$

$$\lambda_{s_i}(t) := (\lambda_{s_i}(t) - \alpha_i g_{s_i}(t))_+ \tag{18}$$

$$\alpha_i \to 0, \sum_{i=1}^{\infty} \alpha_i = \infty \tag{19}$$

The objective function of the subproblem in LTM is the same as CTM(L). The derivation is outlined as follows:

$$\begin{aligned}
\sum_{t=0}^{T} c^k x^k(t) + \sum_{t=0}^{T}\sum_{i=0}^{n^k} \lambda_{s_i}(t) Z^k_{s_i}(t) &= \sum_{t=0}^{T}\sum_{i=0}^{n^k} c_i^k \sum_{j}^{n_i} x_j^{k,i}(t) + \sum_{t=0}^{T}\sum_{i=0}^{n^k} \lambda_{s_i}(t) \sum_{j}^{n_i} x_j^{k,i}(t) \\
&= \sum_{t=0}^{T}\sum_{i=0}^{n^k} (c_i^k + \lambda_{s_i}(t)) \sum_{j}^{n_i} x_j^{k,i}(t) \\
&= \sum_{t=0}^{T}\sum_{i=0}^{n^k} (c_i^k + \lambda_{s_i}(t)) x_i^k(t)
\end{aligned}$$

where n_i represents the number of cells in link i. $\sum_{j=1}^{n_i} x_j^{k,i}(t)$ is the aggregate aircraft count in link i, denoted as $x_i^k(t)$ in LTM.

B. Derivation of LTM dynamics

Detailed explanation of the LTM dynamics is given here. In Eq. (2), flights move forward cell by cell, hence the coefficient for $x_{j-1}^{k,i}(t)$ is 1. In LTM, flight has to stay in link i for at least T_i minutes, then the number of aircraft in the link are not totally transmitted to the downstream link in one time interval. So the aircraft in a link split into two parts, as shown in Eq. (20), part A represents the fraction that transmitted from upstream link, part B represents the fraction that still stay in the same link.

$$x_i^k(t+1) = \underbrace{\beta_{i-1}^k(t) x_{i-1}^k(t) - u_{i-1}^k(k)}_{A} + \underbrace{(1-\beta_i^k(t)) x_i^k(t) + u_i^k(t)}_{B}, \tag{20}$$

the effect of delay control $u_i^k(t)$ can be replaced by the transition fraction $\beta_i^k(t)$, which is capable of regulating the aircraft count in optimization. Then the dynamics of LTM is simplified as:

$$x_i^k(t+1) = \beta_{i-1}^k(t)x_{i-1}^k(t) + (1-\beta_i^k(t))x_i^k(t)$$

To obtain a linear model, define a new variable $q_i^k(t) = \beta_i^k(t)x_i^k(t)$ and substitute for $\beta_i^k(t)x_i^k(t)$ in Eq. (20):

$$x_i^k(t+1) = x_i^k(t) + q_{i-1}^k(t) - q_i^k(t)$$

The minimum dwell time is presented by the following constraint:

$$\sum_{t=0}^{T_1+T_1\ldots+T_i-1} \beta_i^k(t)x_i^k(t) = 0, \qquad i \in \{1,\cdots,n^k\} \tag{21}$$

$$\sum_{t=T_0+T_1\ldots+T_i}^{T^*} \beta_i^k(t)x_i^k(t) \le \sum_{t=T_0+T_1\ldots+T_{i-1}}^{T^*-T_i} \beta_{i-1}^k(t)x_{i-1}^k(t) \qquad T^* \in \{T_0+T_2\cdots+T_i,\ldots,T\} \tag{22}$$

Eq. (21) is equivalent to guaranteeing that the flights do enter the link $i+1$ not earlier than $t = T_1+T_2\cdots+T_i$. In other words, flight must sequentially pass through its air route link by link. Eq. (22) describes the fact that accumulated output of a link at time T^* is less than the accumulated input of it at time T^*-T_i. Which means that a flight must stay in a link for a period no less than T_i. Without this constraint, optimization result would be all zero, which is equivalent to that all flights fleet instantaneously through the path resulting in a minimum flight time of zero without violating any constraints. Therefore, this constraint is critical to obtain a correct optimization result.

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