Sequential Decision-making and Asymmetric Equilibria: An Application to Takeovers

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Abstract

With indivisible shareholdings and simultaneous shareholder decision-making, the existing takeover literature provides a reasonable profit only in asymmetric equilibria. We allow the raider to approach shareholders sequentially and thereby find a unique equilibrium that produces the same outcome. *J.E.L. classification*: L10, G34, H41. *Keywords:* takeovers, sequential, pivotal.

1. Introduction

Consider an unconditional offer to buy shares made by a raider, motivated by the raider's correct belief that he can improve the value of the firm, say from 0 to 1 per share. In the standard model decisions are made simultaneously by an infinite number of atomistic shareholders. No bid can succeed at a price below 1, so the raider cannot extract any value from the takeover. If the bid were going to succeed, then each atomistic shareholder (who can have no influence on the outcome), would refuse to sell in order to wait for the takeover to raise the value of the share to 1, leading to a contradiction. This provides the basic setup in Grossman and Hart (1980). Bagnoli and Lipman (1988) and Holstrom and Nalebuff (1992) consider a variation in which shareholders are finite in number, and so can exercise some influence over the probability of success. They find a unique symmetric mixed strategy equilibrium which gives very slight profits, tending to zero as the number of shareholders becomes

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large. They also find numerous asymmetric pure strategy equilibria which give greater profits to the raider, but which are indistinguishable but for the choice of who sells or and who does not - it is hard to motivate why some would sell and others not in the simultaneous case.

We find a symmetric equilibrium which reproduces the better profits for a raider under the asymmetric equilibria, by allowing sequential decision-making by shareholders. This completely pins down what each shareholder in the sequence will do and so restores a unique single equilibrium. While some takeover codes insist on the identical treatment of shareholders, and this may rule out a sequential approach, the method may be possible for non-listed companies such as private medical, legal, consultancy and accounting partnerships. We might further argue that takeover codes be revised in the light of our findings to improve the incentives for raiders to take over under-performing firms.

The method used in this note might also be usefully applied to other sequential decision-making problems. In particular, other related simultaneous decision-making problems with freeriding might be candidates for a unique solution under sequential decision-making.

2. The Basic Model

The scenario is simple, and is based on Bagnoli and Lipman (1988). A firm is viewed by a raider as badly managed and undervalued. The raider believes there is a gain to be made through takeover, rationalization and the exploitation of synergies. This gain per share we normalize to 1 while the current value of the firm per share we normalize to 0. Let there be $N \geq 3$ shareholders all holding an identical and indivisible share of the firm. The raider makes an offer $a \in [0, 1]$ to these shareholders. The set of actions available to each shareholder is $A_i \in \{\text{Sell, Refuse}\}$.

We assume that offers are unconditional on whether the takeover succeeds or fails, and that success requires at least an integer number of shares $K \in \left(\frac{N}{2}, N\right)$ to be sold to the raider. Payoffs for shareholder i are $\pi_i(\text{Sell}|\text{Takeover succeeds}) = \pi_i(\text{Sell}|\text{Takeover fails}) = a$; $\pi_i(\text{Refuse}|\text{Takeover succeeds}) = 1$; and $\pi_i(\text{Refuse}|\text{Takeover fails}) = 0$. A shareholder would like to sell if the takeover fails, but if it succeeds the shareholder would rather hang on to the share and see it rise in value to 1.

2.1. Simultaneous Decisions by Shareholders.

2.1. Simultaneous Decisions by Shareholders. Where shareholders all decide simultaneously whether to accept an offer, we can consider two possible *symmetric* equilibrium candidates in which the takeover succeeds (all sell and all mix with the same probability). First consider the "all sell" equilibrium. If all shareholders were selling at $a < 1$, then each shareholder would have an incentive to deviate by refusing to sell, thus guaranteeing 1 as the takeover would still succeed. Thus, there can be no "all sell" equilibrium in which the raider can extract positive surplus.

Bagnoli and Lipman show that the raider can always make a strictly positive profit in a symmetric mixed strategy equilibrium. However, the profit relies on the small difference between the probability of success relevant to a shareholder, namely the probability of success if he does not sell but the other shareholders mix, and the probability of success relevant to the raider, namely the probability of success if all shareholders mix. As $N \to \infty$, the difference between these probabilities tends to zero, so the expected profit to the raider vanishes.

The payoff can be improved by considering *asymmetric* pure strategy equilibria. The trick is to make shareholders pivotal. In an asymmetric equilibrium where precisely K shareholders sell and precisely $N - K$ do not, all those who sell are pivotal and hence accept the offer of a instead of forcing the takeover to fail and getting 0, while all those who do not sell are not pivotal and hence prefer $1 > a$, since the takeover succeeds. In such an equilibrium, with a close to zero, the raider will get a payoff close to $\frac{K}{N}$ per share. With the number of shares required for control defined as a proportion of the total number, the payoff per share will be constant in N, in contrast to the result for the unique symmetric equilibrium.

There are many equilibria with this property since for each permutation of shareholders such that precisely K accept and $N - K$ decline there exists an asymmetric equilibrium, so it is hard to motivate why the shareholders would play any specific one. We see in the next section that using this idea of pivotal shareholders gives us another way of inducing the same outcome as in this asymmetric equilibrium but where the outcome is uniquely defined.

2.2. **Sequential Decisions by Shareholders.** As shown we can have asymmetric equilibria that render takeovers possible and profitable. However, there are multiple equilibria which begs the question: why do some shareholders sell and others don't? We resolve this by moving away from simultaneous decision-making by shareholders to sequential decision-making. There are two major impacts of moving to a sequential framework: shareholders are now heterogeneous as they are all indexed by a unique number in the ordering; and each shareholder must consider the impact of his decision on others, and the impact of others' decisions on him. We now have a more strategic problem from the shareholders' perspective.

We denote the history of shareholder actions up to but not including j as $H_j = \{A_1,$ $A_2,$ ..., $A_{j\text{-}1}\}$, and the number of sales and refusals in this history as $\#H_j$ (Sell) and $\#H_j$ (Refuse) respectively. Before going on to the main proposition, we will define a concept which goes to the heart of the sequential model: refuse-pivotalness, and then state and prove a crucial lemma.

Definition. *Shareholder j is refuse-pivotal if $\#H_j\,(\text{Refuse}) = N - K$. Thus, if j refuses to sell, the takeover is certain to fail, while if j sells, the takeover may yet succeed.*

Lemma. *For any offer $a > 0$, if shareholder j is refuse-pivotal he will sell and so too will all shareholders $i > j$. The takeover must then succeed.*

Proof. If j is refuse-pivotal then $\pi_j\,(\text{Sell}) = a > \pi_j\,(\text{Refuse} \mid \text{Takeover fails}) = 0$, so j sells. Now $A_j = \text{Sell} \Rightarrow \#H_j\,(\text{Refuse}) = \#H_{j+1}\,(\text{Refuse}) = N - K$, so $j+1$ is also refuse-pivotal and sells. By induction this must also be the case for all $i > j$. The takeover succeeds as there were exactly $N - K$ refusals before j and so there must be K sales in total given j and all later shareholders sell. □

Proposition. *Under sequential decision-making, for any offer $a \in (0, 1)$ the unique equilibrium involves a successful takeover in which the first $N - K$ shareholders refuse to sell, gaining a payoff of 1, and the remaining K shareholders all sell, gaining a payoff of a.*

Proof. Take any shareholder in the sequence. There are four possibilities: (i) Shareholder j is refuse pivotal, i.e., $\#H_j(Refuse) = N - K$. (ii) Shareholder j is not refuse pivotal and $\#H_j(Sell) \geq K$. (iii) Shareholder j is not refuse pivotal and $\#H_j(Sell) < K$; $\#H_j(Refuse) > N - K$. (iv) Shareholder j is not refuse pivotal and $\#H_j(Sell) < K$; $\#H_j(Refuse) < N - K$.

In case (iv), from $j's$ perspective, there are only two possibilities. Either the takeover succeeds without anybody ever becoming refuse pivotal, or there are enough later refusals that a later shareholder k becomes refuse pivotal, in which case by the Lemma the takeover succeeds. Thus j knows that whatever he does, the takeover will succeed, so he refuses to sell to get $\pi_j\,(\text{Refuse} \mid \text{Takeover succeeds}) = 1 > \pi_j\,(\text{Sell}) = a$.

$\#H_1(Sell) = 0 < K$ and $\#H_1(Refuse) = 0 < N - K$. Thus, the first shareholder is in situation (iv) and so refuses to sell. Thus the second is in the same situation and also refuses, so long as $\#H_2(Refuse) = 1 < N - K$. The same argument applies by induction until the $(N - K + 1)\,th$ shareholder, for whom $\#H_{N-K+1}(Refuse) = N - K$. This shareholder is therefore refuse-pivotal, so by the Lemma $j = N - K + 1$ and all later shareholders sell, and the takeover succeeds. □

Thus, the raider can offer any $a \in (0, 1)$, and buy K shares from the last K shareholders approached, earning $(1-a)$ per share purchased. From the raider's perspective the sequential outcome produces an identical profit to the asymmetric outcome in Bagnoli and Lipman (1988), and a better outcome than the symmetric one where profits are small, vanishing to zero as the number of shareholders becomes large.

We of course require that the raider be able to commit to the order in which he approaches the shareholders; otherwise the later shareholders might try to hold out hoping that the raider will be forced to approach some of the earlier ones once more. However, a raider involved in multiple raids, e.g., a private equity firm, may well be able to build up such a reputation. The raider is most likely to be able to approach shareholders sequentially in this way where shareholdings are not too diffuse.

3. Conclusion

We have modified the basic framework of a takeover model to allow a raider to approach shareholders sequentially. We then have a unique equilibrium providing profits for the raider in contrast to the situation in Grossman and Hart (1980). The sequential decision-making framework closely duplicates the outcome in the multiple asymmetric equilibria (under simultaneous decision-making) in Bagnoli and Lipman (1988). Since in our model, shareholders are distinguished by their place in the ordering, we can eliminate all but one equilibrium and pin down which shareholders will gain most from the takeover. The intuition is quite clear: under sequential decision-making shareholders towards the end of the sequence will have to sell to obtain a strictly positive payoff. This allows the raider the freedom to make a much lower offer, and enables the earlier shareholders to earn a higher payoff than later decision-makers by holding onto their shares. Our model applies best to targets whose shareholdings are indivisible, not too diffuse and not publicly listed, e.g. professional partnerships.

Though the application to takeovers is particularly sharp, in principle, the general argument might help with any related simultaneous decision-making problem with freeriding. It might be that considering sequential decision-making can provide for a unique solution with reasonable payoffs, though in each application the feasibility of a sequential ordering of decision-making would need to be considered.

References

Grossman, S., Hart, O. D.: Takeover bids, the freerider problem, and the theory of the corporation. The Bell Journal of Economics 11, 42-64 (1980).

Bagnoli, M., Lipman, B. L.: Successful takeovers without exclusion. Review of Financial Studies 1, 89-110 (1988).

Holstrom, B., Nalebuff, B.: To the Raider Goes the Surplus? A Reexamination of the Free-rider Problem. Journal of Economics and Management Strategy 1, 37-62 (1990).