Holographic superconductors at low temperature

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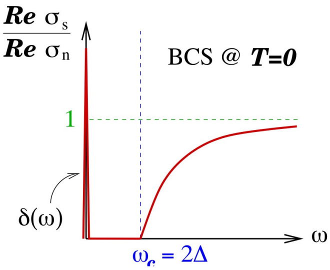
Outline

1. Introduction
   - BCS
   - High T_C
   - AdS/CFT
2. AdS/CFT Model
   - Bulk Geometry
   - Instability
   - Probe Limit
3. Low Temperatures
   - Motivation
   - Critical Temp.
   - Below T_C
   - Conductivity

Superconductivity

BCS theory of superconductivity uses a spontaneously broken gauge symmetry



[picture from Gubser group]

- 2nd order phase transition at T_C
- conductance contains delta function and is gapped
- Meissner Effect

Hich T_C

BCS theory is known to work well up to $T_C \sim 20K$
$\longrightarrow$ perhaps higher? - MgB_2 at $40K$

New(er) Materials

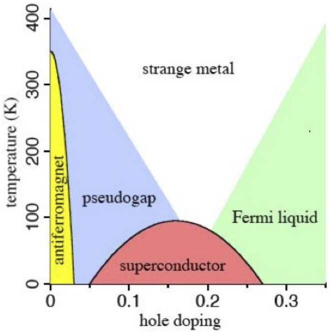
- cuprates, $T_C \sim 1.5 \times 100K$
- iron pnictides $T_C \sim .5 \times 100K$

superconducting phase difficult to describe

High T_C superconductors

Theoretical Difficulties...

- Perturbatve struggles with coupling
- Numerical simulation produces negative probabilities
    - time dependent path integral



AdS/CFT

some features found via gravitational duals

- allows for calculation of transport properties
  - σ, gap, Meissner *[Hartnoll, Herzog, Horowitz, . . .]*
  - Nernst effect *[Hartnoll, Kovtun, Muller, Sachdev]*

Differences

- global vs local symmetry breaking
- large N limit?

AdS/CFT

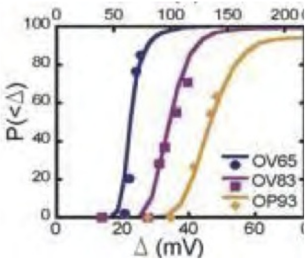
some features found via gravitational duals

- allows for calculation of transport properties
  - σ, gap, Meissner *[Hartnoll, Herzog, Horowitz, . . .]*
  - Nernst effect *[Hartnoll, Kovtun, Muller, Sachdev]*

Similarities

- finite chemical potential, charge density
- DC superconductivity
- gap, $\omega_g \sim 8T_C$ (large q)

Gap Measurement *[Gomes, et al.]*



Hairy black hole

Approach

- top-down: stringy solutions *[Denef, Hartnoll, Gubser, . . .]*
- bottom-up: pick a supergravity solution, assume it works (for the time being) and analyze

we use a gravity with Λ_{AdS}, scalar field ϕ of mass m and charge q coupled to $U(1)$ vector potential A_μ

$$S = \int d^4x\sqrt{-g}\left[\frac{R + 6/L^2}{16\pi G} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - |D_\mu\phi|^2 - m^2|\phi|^2\right]$$

with

$$D_\mu = \partial_\mu - iqA_\mu \ , \quad \phi = \frac{1}{\sqrt{2}}\psi e^{iq\theta}$$

$$S_\Psi = \frac{1}{2}\int d^4x\sqrt{-g}\left[\partial_\mu\Psi\partial^\mu\Psi + q^2\Psi^2(\partial_\mu\theta - A_\mu)^2 - m^2\Psi^2\right]$$

Hairy black hole

Ansatz:

$$
\begin{aligned}
ds^2 &= \frac{1}{z^2}\left[-g(z)e^{-\chi(z)}dt^2 + d\vec{x}^2 + \frac{dz^2}{g(z)}\right], \\
A_0 &= \Phi(z),\ \psi = \Psi(z)
\end{aligned}
$$

use scaling symmetries: $z_H = 1$, with $z \in [0, 1]$
$\longrightarrow$ AdS boundary at $z = 1$
Temperature:

$$T = -\frac{g'(1)}{4\pi}e^{-\chi(1)/2}$$

- look at only scale invariant quantities

Hairy black hole

Equations of Motion

$$\Psi'' + \left[\frac{g'}{g} - \frac{\chi'}{2} - \frac{2}{z}\right]\Psi' + \left[\frac{q^2\Phi^2 e^\chi}{g^2} - \frac{m^2}{z^2 g}\right]\Psi = 0,$$

$$\Phi'' + \frac{\chi'}{2}\Phi' - \frac{2q^2\Psi^2}{z^2 g}\Phi = 0,$$

$$-\chi' + z\Psi'^2 + \frac{zq^2\Phi^2\Psi^2}{g^2}e^\chi = 0,$$

$$\frac{g}{2}\Psi'^2 + \frac{z^2}{4}\Phi'^2 e^\chi - \frac{g'}{z} + \frac{3(g-1)}{z^2} + \frac{m^2\Psi^2}{2z^2} + \frac{q^2\Psi^2\Phi^2 e^\chi}{2g} = 0$$

[HHH]

in general these must be solved numerically

Hairy black hole

Asymptotics

To keep AdS boundary we require as $z \to 0$

$$\chi(0) \to 0\,, \quad g \to 1 \quad \Rightarrow \quad \Psi \sim \Psi^{\pm} z^{\Delta_\pm}\,, \quad \Phi \sim \mu - \rho z$$

$$\Delta_\pm = \frac{3}{2} \pm \sqrt{\frac{9}{4} + m^2}$$

- chemical potential, μ and charge density, ρ
- $\langle \mathcal{O}_{\Delta_\pm} \rangle = \sqrt{2}\Psi^{\pm}$

Horizon

as $z \to 1$

$$\Phi \to 0\,, \quad g \to 0$$

Hairy black hole

Analytic solution: Reissner-Nordstrom-AdS

$$g(z) = 1 - \left(1 + \frac{\rho^2}{4}\right) z^3 + \frac{\rho^2}{4} z^4, \; \Phi(z) = \rho\,(1 - z)\,, \; \chi = 0 = \Psi$$

Mechanism for Instability - I

- scalar develops effective mass

$$\mathcal{L}_\Psi = |\partial_r \Psi|^2 - \left(m^2 + g^{tt} q^2 \Phi^2\right) |\Psi|^2$$
$$\rightarrow m_{eff}^2 = m^2 + g^{tt} q^2 \Phi^2 = m^2 - 2q^2$$

[Gubser]

- Breitenlohner-Freedman bound near $z \sim 1$

$$m_{eff}^2 < m_{BF,3+1}^2 = -9/4$$

Hairy black hole

Analytic solution: Reissner-Nordstrom-AdS

$$g(z) = 1 - \left(1 + \frac{\rho^2}{4}\right) z^3 + \frac{\rho^2}{4} z^4, \ \Phi(z) = \rho\,(1 - z)\,, \ \chi = 0 = \Psi$$

Mechanism for Instability - II

Extremal black holes near horizon exhibit $AdS_2 \times \mathbb{R}^2$

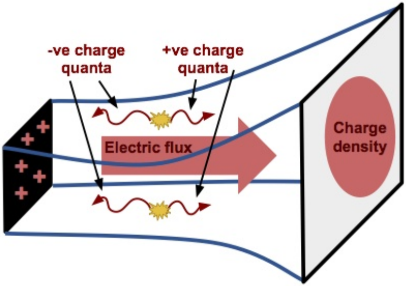
- effective mass can be below 2D m_{BF}^2

$$m_{eff}^2 = m^2 - 2q^2 < m_{BF,1+1}^2 = -1/4$$

Hairy black hole

Analytic solution: Reissner-Nordstrom-AdS

$$g(z) = 1 - \left(1 + \frac{\rho^2}{4}\right) z^3 + \frac{\rho^2}{4} z^4 , \; \Phi(z) = \rho(1 - z) , \; \chi = 0 = \Psi$$



picture from Hartnoll

instability for $q^2 \geq \frac{3+2m^2}{4}$ → including $q^2 = 0$

Probe limit

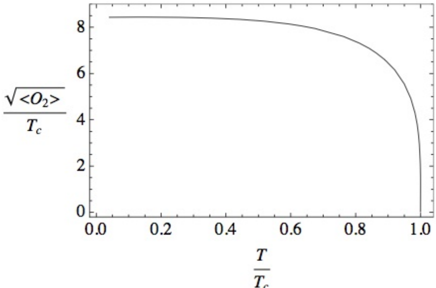
Start with the probe limit via

$$g = g_0 + \frac{1}{q^2} g_1 + \ldots , \quad \chi = \chi_0 + \frac{1}{q^2}\chi_1 + \ldots$$

$$\Psi = \frac{1}{q}\Psi_0 + \frac{1}{q^3}\Psi_1 + \ldots , \quad \Phi = \frac{1}{q}\Phi_0 + \frac{1}{q^3}\Phi_1 + \ldots$$

$$q \to \infty$$

Ψ condenses at some $T_C \propto \rho$

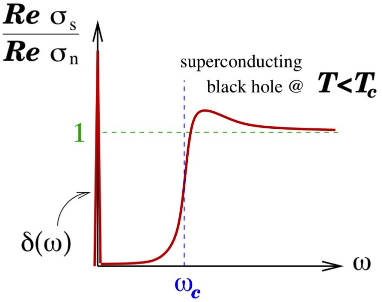


Probe limit

turn on a gauge field perturbation

$$A_x \sim \left(A_x^{(0)} + zA_x^{(1)} + \dots \right) e^{-i\omega t}$$

$$\sigma = \frac{\langle J_x \rangle}{-\partial_t A_x} = \frac{A_x^{(1)}}{i\omega A_x^{(0)}}$$



δ function from $\Im\sigma$

$\omega_C \sim 8T_C$

Free energy

Hair is Favored

- condensate

$$\langle O \rangle \sim (T_C - T)^{1/2}, \; T \lesssim T_C$$

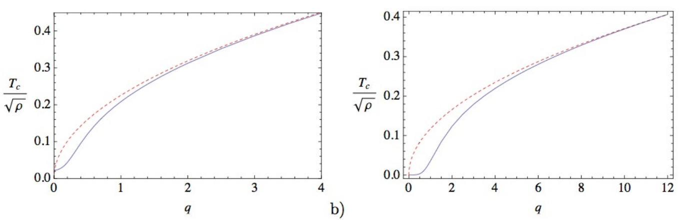
- free energy

$$\Delta F = F_{Hbh} - F_{RN} \begin{array}{ll} < & 0, \; T < T_C \\ > & 0, \; T > T_C \end{array}$$

Low temperatures

- must move beyond probe limit to see $T \to 0$
  - quantum critical point
  - hard gap? *[Horowitz, Roberts]*
  - S, ρ, C *[Basu]*
  - Fermi surfaces *[Faulkner, Liu, McGreevy, Vegh, ...*
    - emergent scaling symmetry

Small scalar charge



HHH

look at $|q^2| \lesssim 1$

Goal

Critical point in q?

instability qualitatively changes between $q \gg 1$ and $q^2 \lesssim 0$

- quantum critical point in q?

search in conjunction with small Temperature

critical temperature $T_C \to$ solve wave equation for Ψ

$$\Psi'' + \left[\frac{g'}{g} - \frac{\chi'}{2} - \frac{2}{z}\right]\Psi' + \left[\frac{q^2\Phi^2 e^{\chi}}{g^2} - \frac{m^2}{z^2 g}\right]\Psi = 0$$

in two regions

- far from horizon, $1 - z \gtrsim \epsilon$
- near horizon, $1 - z \lesssim \sqrt{\epsilon}$

Ψ

Away from horizon, use extremal background

$$\rho^2 = 12\,, \quad T = 0$$

far solution

$$\Psi_{far} = \mathcal{C}\frac{z}{z - z_0}\left(\frac{z - z_0^*}{z - z_0}\right)^{\frac{2\sqrt{2} - i}{2\sqrt{3}}q}\left(\frac{1 - z}{z - z_0}\right)^{\delta_+} F\left(\{\delta_-, q\}; 2z_0^2\frac{1 - z}{z - z_0}\right) + c.c.$$

Near horizon, let $z = 1 - \frac{\epsilon}{6}\zeta \longrightarrow T = \epsilon/4\pi$

$$\zeta(1 + \zeta)\Psi'' + (2\zeta + 1)\Psi' + \frac{1}{3}\left[1 + q^2\frac{\zeta}{1 + \zeta}\right]\Psi = 0$$

near solution

$$\Psi_{near} = \mathcal{A}(1 + \zeta)^{-iq/\sqrt{3}} F\left(\{\delta_-, q\}; -\zeta\right)$$

Matching

match $z \to 1$ with $\zeta \to \infty$ ($\epsilon \ll 1$)

$$\begin{aligned} &\mathcal{C}(1-z_0)^{\delta_-}\left(\frac{1-z_0^*}{1-z_0}\right)^{\frac{2\sqrt{2}-i}{2\sqrt{3}}q}(1-z)^{\delta_+} + c.c. \\ &= \mathcal{A}\frac{\Gamma(-1-2\delta_-)}{\Gamma(-\delta_- - \frac{iq}{\sqrt{3}})\Gamma(-\delta_- - \frac{iq}{\sqrt{3}})}\zeta^{\delta_+} + c.c. \end{aligned} \tag{1}$$

- the phase of $\mathcal{C}$ is fixed by regularity at the horizon

▶ also choose condensed operator by falloff for $\Delta = 2$

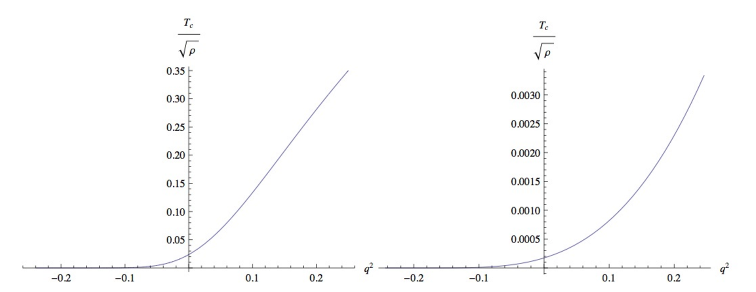
$$\Psi_{far}(z \to 0) \sim 0 \cdot z + (\ )z^2$$

Matching

constraint on $\epsilon \to T_C$

- $m^2 = -2$

$$\frac{T_C^{\Delta=2}}{\sqrt{\rho}} = 1.7\times 10^{-4}\,, \quad \frac{T_C^{\Delta=1}}{\sqrt{\rho}} = .024\,, \quad T_C = 0, q^2 = q_c^2 = -\frac{1}{4}$$



$\Delta = 1$ $\qquad$ $\Delta = 2$

Trial function

compare matching result with a trial function (α parameter):

$$\Psi \sim z^{\Delta}\left(1 - \frac{\alpha z^2}{\sqrt{z_+ - z}}\right)$$

$z_+ > 1$ is the outer horizon

- instability in action at $T = 0$ for

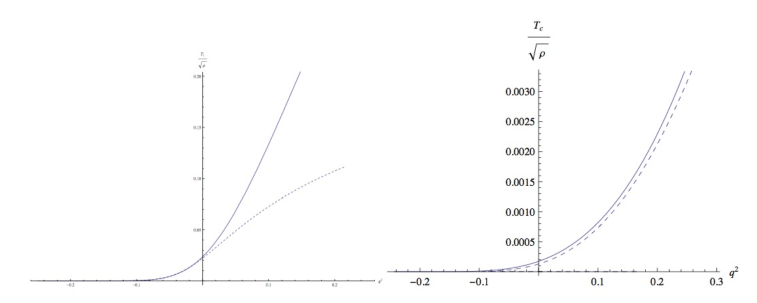
$$q_c^2 = \frac{2m^2 + 3}{4}$$

otherwise

- S admits instability for $q^2 > q_c^2$
- no instability for $q^2 < q_c^2$

Trial function

Trial vs Matching

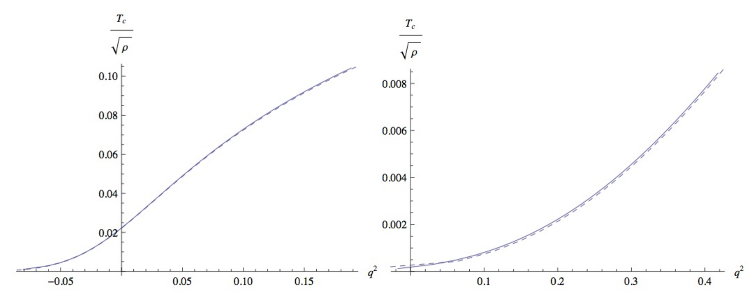


$\Delta = 1$ $\Delta = 2$

- good agreement for $q^2 \sim q_c^2$

Trial function

Trial vs Numeric



$\Delta = 1$ $\Delta = 2$

- excellent agreement with numerics

Below T_C

Expand the fields as

$$g = g_0 + \langle\mathcal{O}_\Delta\rangle^2 g_1 + \dots\,, \quad \chi = \chi_0 + \langle\mathcal{O}_\Delta\rangle^2 \chi_1 + \dots$$
$$\Psi = \langle\mathcal{O}_\Delta\rangle \Psi_0 + \langle\mathcal{O}_\Delta\rangle^3 \Psi_1 + \dots\,, \quad \Phi = \Phi_0 + \langle\mathcal{O}_\Delta\rangle^2 \Phi_1 + \dots \quad (2)$$

correction to temperature

$$\frac{T}{\sqrt{\rho}} = \frac{T_0}{\sqrt{\rho}}\left[1 - \langle\mathcal{O}_\Delta\rangle^2 \mathcal{T}_1\right], \; \mathcal{T}_1 = -\frac{g_1'}{g_0'} + \frac{1}{2}\chi_1 + \frac{\rho_1}{2\rho_0}\Big|_{z=1}$$

- ρ_1 is determined from first order correction to Ψ equation

$$\int_0^1 \frac{dz}{z^2} g_0 \Psi_0 \mathcal{H}_1 \Psi_0 = 0$$

Condensate at low temperature

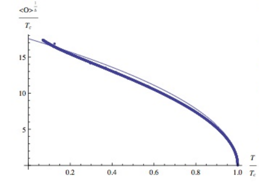
Near the critical temperature

$$\langle \mathcal{O}_\Delta \rangle = \frac{1}{\sqrt{\mathcal{T}_1}} \sqrt{1 - \frac{T}{T_C}}$$

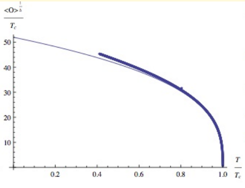
at $T = 0$, to first order, we obtain the energy gap

$$\frac{\langle \mathcal{O}_\Delta \rangle^{1/\Delta}}{T_C} = \frac{1}{T_0 \mathcal{T}_1^{1/2\Delta}}$$

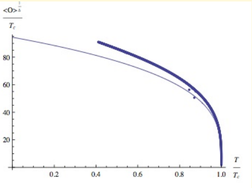
Energy gap



$\Delta = 1$



$\Delta = 1.5$



$\Delta = 1.7$

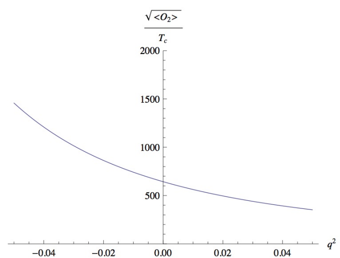
Condensate at low temperature

Near the critical temperature

$$\langle \mathcal{O}_\Delta \rangle = \frac{1}{\sqrt{\mathcal{T}_1}} \sqrt{1 - \frac{T}{T_C}}$$

at $T = 0$, to first order, we obtain the gap

$$\frac{\langle \mathcal{O}_\Delta \rangle^{1/\Delta}}{T_C} = \frac{1}{T_0 \mathcal{T}_1^{1/2\Delta}}$$



continuous across $q^2 = 0$

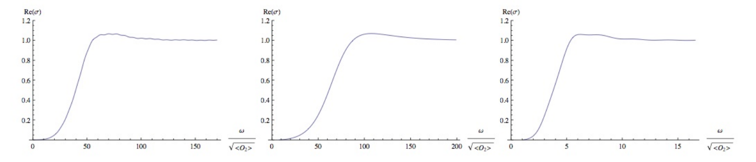
diverges as $q^2 \to q_c^2$

Conductivity

- add $A_x, g_{tx} \sim e^{-i\omega t}$ perturbation

$$-\frac{d^2 A_x}{dz_*^2} + VA_x = \omega^2 A_x, \;\; V = g\left(2q^2 \frac{\Psi^2}{z^2} e^{-\chi} + z^2 \Phi'^2\right)$$

$\Delta = 2$



$q^2 = -1/8$ $\qquad$ $q^2 = 0$ $\qquad$ $q^2 = 1/4$

Clear deviation from previous $\omega_g \propto \langle q\mathcal{O}_\Delta\rangle^{1/\Delta}$
$\Rightarrow \omega \propto \langle \mathcal{O}_\Delta\rangle^{1/\Delta}$

Summary

- gravity provides tools to strongly coupled superconductors
- At low temperature, understand how to vary q
  - matching and trial function T_C
  - energy gap
  - conductance
- no discontinuities at $q^2 \to 0$ or any other (small) q^2
- energy gap diverges at q_c^2

work to be done
$\Rightarrow$ spatial dependence, lattice, ...