A semi-classical realization of IR renormalons

work w/ M. Ünsal, arXiv:1203... to appear.

Outline:

1. Motivation: Borel resummation
2. QCD on $\mathbb{R}^4$: Instantons and renormalons
3. QCD(adj) on $\mathbb{R}^3 \times S^1$: Multi-instanton "molecules"
4. Some conjectures: IR renormalon ~ bion-bion

Background: 't Hooft's renormalons, Polyakov 3d confinement, Bogomolny & Zinn-Justin bounces, Ünsal QCD on $\mathbb{R}^3 \times S^1$.

1. MOTIVATION / BOREL RESUMMATION / BZJ PRESCRIPTION

- When is there a continuum def'n of QFT?
- Can we make sense of the semiclassical expansion of QFT?

$$F(g^2) = \underbrace{(p_0 + p_1 g^2 + p_2 g^4 + \dots)}_{P} + e^{-\frac{8\pi^2}{g^2}}\underbrace{(i_0 + i_1 g^2 + \dots)}_{I} + e^{\frac{-16\pi^2}{g^2}}\underbrace{(b_0 + b_1 g^2 + \dots)}_{B} + \dots$$

↳ instantons ↳ inst.-$\overline{\text{inst}}$ = "bounce"

but $P, I, B, \dots$ not convergent.

- Borel resummation idea: if P has convergent Borel tranfrm

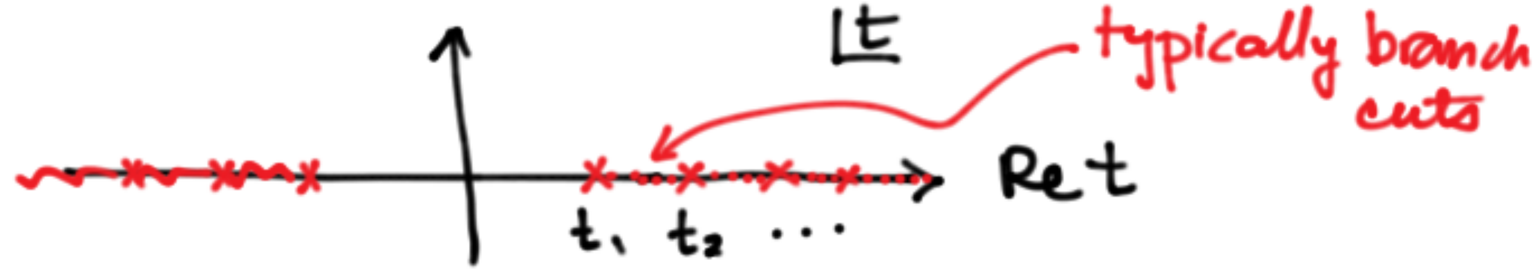
$$\mathcal{B}P(t) := \frac{p_0}{0!} + \frac{p_1}{1!}t^1 + \frac{p_2}{2!}t^2 + \dots$$

in neighborhood of $t=0$, then

$$\mathbb{P}(g^2) := \frac{1}{g^2}\int_0^\infty \mathcal{B}P(t) \cdot e^{-t/g^2}\, dt$$

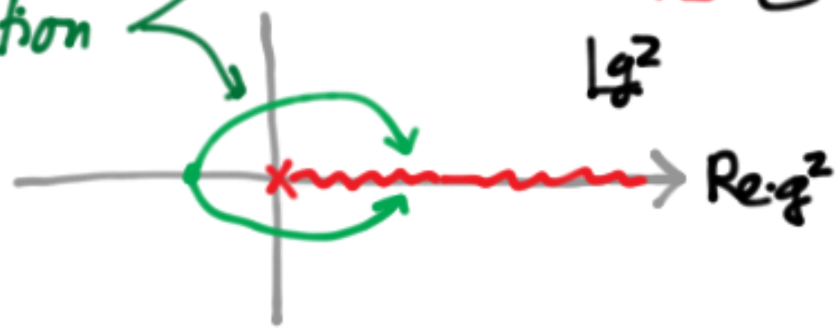
formally gives back $P(g^2)$ but is ambiguous if $\mathcal{B}P(t)$ has singularities @ $t \in \mathbb{R}^+$: pick contour / $g^2 \to g^2 \pm i\epsilon$.

"Borel plane": complex t-plane

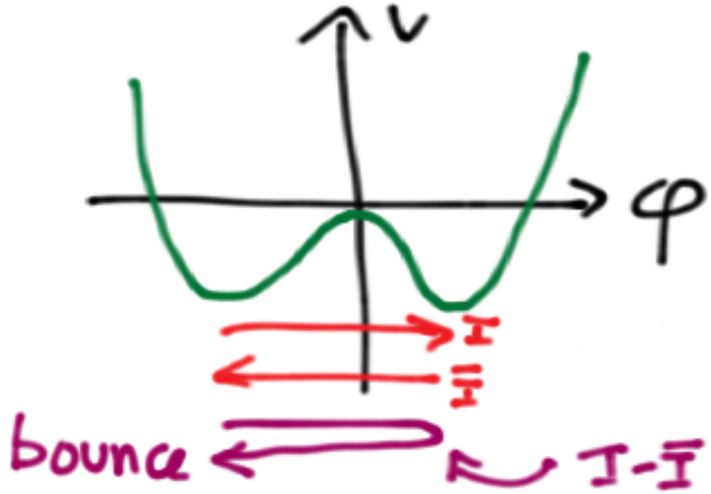


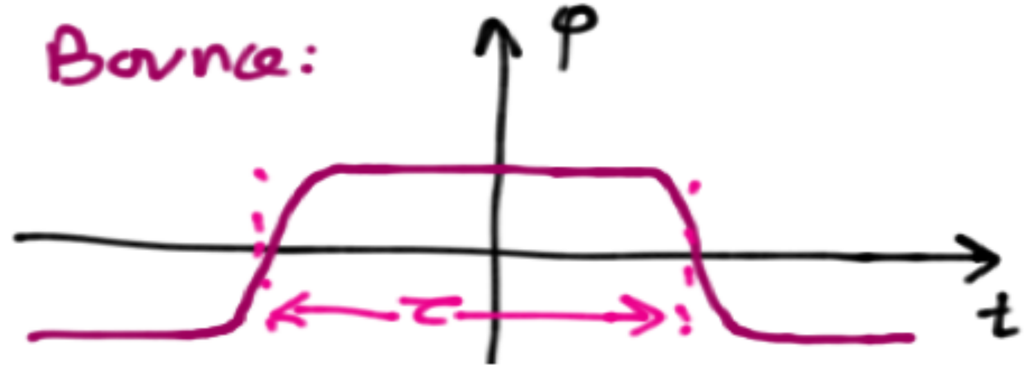
$$\Rightarrow \quad \mathbb{P}(g^2) = \mathrm{Re}\,\mathbb{P}(g^2) \pm i\, \underbrace{\mathrm{Im}\,\mathbb{P}(g^2)}_{\simeq\, e^{-t_1/g^2} + e^{-t_2/g^2} + \dots}$$

analytic continuation is ambiguous



- Ambiguity in $\mathbb{P}(g^2)$ has same form as inst. contrib $\sim e^{-\frac{8\pi^2}{g^2}}$
- (~1980) Bogomolnyi, Zinn-Justin: 'bounce' in double-well QM
  $\mathbb{B}(g^2)$ <u>evaluated @ $g^2<0$ & continued to $g^2>0$ in same way as</u> $\mathbb{P}(g^2)$ gives unambiguous, real answer:
  Im part cancels in $\mathbb{P}(g^2) + \mathbb{B}(g^2)$.



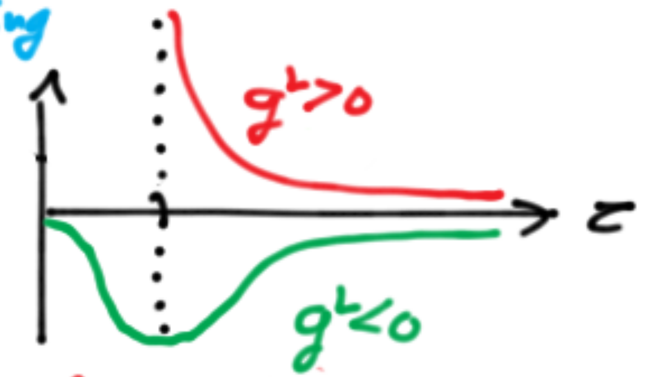


τ = quasi-0-mode
(must be integrated to calc. I-Ī contrib.)

I-Ī interaction:

$$a_{I\bar{I}} \sim \int_0^\infty d\tau \left[\underbrace{\exp(g^{-2}e^{-\tau})}_{e^{-V}} - 1\right]$$

non-interacting D.I.G.



$$a(g^2_{<0}) = c + \ln(-g^2) \xRightarrow{\text{continue}} a(g^2_{>0}) = c + \ln g^2 \pm i\pi$$

$\Rightarrow \mathbb{B} = \mathrm{Re} \pm i\pi e^{-2S_I}$. Also, high orders p.t. $\Rightarrow$ sing. in Borel plane @ $t = 2\cdot 8\pi^2 \Rightarrow \mathbb{P} = \cdots \mp i\pi e^{-2S_I}$.

QUESTION: Can this idea work for QFT? (eg QCD?)

- ('t Hooft): <u>no</u> on $\mathbb{R}^4$
- (us): <u>yes</u> on $\mathbb{R}^3 \times S^1$ (for some QFTs)

A. Yung '88 sQCD

2. QCD on $\mathbb{R}^4$ / RENORMALONS

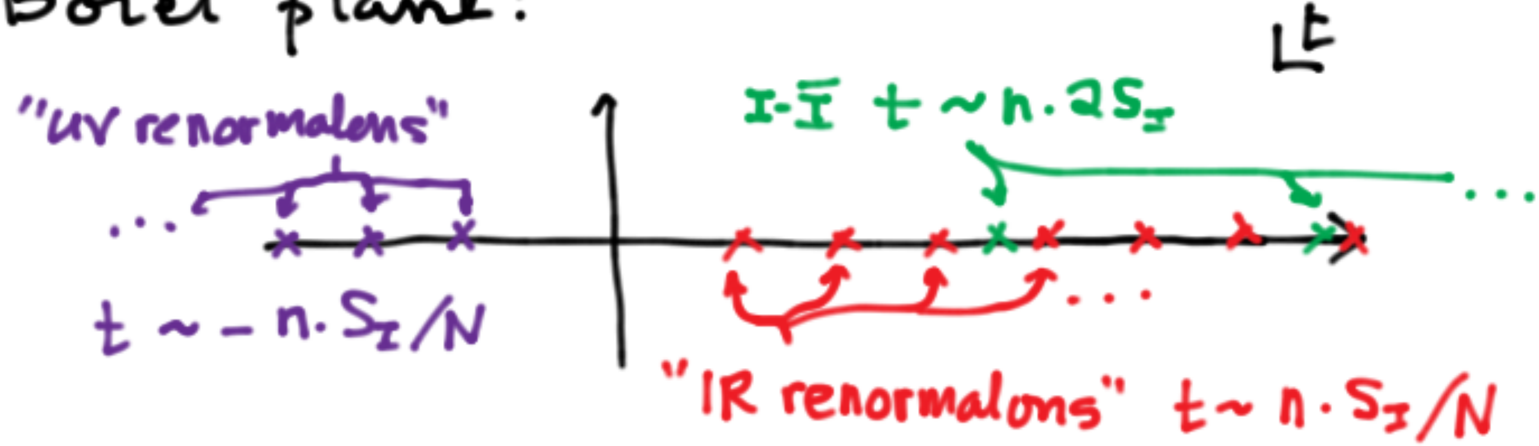
– Why doesn't this work for QCD on $\mathbb{R}^4$?

- Inst.-$\overline{\text{inst.}}$ contribution calculated in same way gives $\mathbb{B} \sim \pm i e^{-2S_I}$ ambiguity. ← cancel ✓
- Lipatov: Borel transform $\mathbb{B}P(t)$ has singularities at $t = t_n \equiv 2nS_I g^2 \Rightarrow \mathbb{P} \sim \mp i e^{-t_n/g^2} = \mp i e^{-2nS_I}$
- BUT: $\mathbb{B}P(t)$ has other singularities closer to origin of Borel plane @ $t = \tilde{t}_n := n \cdot \frac{16\pi^2}{\beta_0}$ $n = 2, 3, 4 \ldots$

  $\sim n S_I g^2 / N$ $N = \mathrm{rank}(G)$
- Comes from leading (?) divergence of pert. theory due to class of (g. inv.) diagrams $\sim \left(\frac{n}{2}\right)!$ @ n^{th} order:

$|q| \sim \Lambda \Leftrightarrow$ "IR"

- Borel plane:



- 't Hooft called these singularities "renormalons" in the hope/expectation that they would be shown to be associated to a semiclassical saddlepoint like the instanton (e.g. "meron"...?)

- But: no such configuration is known.

- This is a real problem in pert. QCD: so-called "power-law corrections" are invoked (=guessed) to remove renormalon pole ambiguity in $P(g^2)$.
(See Beneke "Renormalon" review '99)

3. QCD(adj) on $\mathbb{R}^3 \times S^1$ / TOPOLOGICAL MOLECULES

- Studied extensively by M. Ünsal & collaborators...

- S^1: periodic BCs for fermions (not thermal S^1!)
- On small S^1, $G \to U(1)^r$ semiclassically for large class.
  $\Rightarrow$ low energy theory is 3d compact $U(1)$ w/ fermions
- 3d $U(1) \xleftrightarrow{\text{EM dual}}$ periodic scalar σ : "dual photon"

On $\mathbb{R}^3$: (e.g. 3d $SU(2) \to U(1)$)
- $\exists$ "monopole-instanton" $\mathcal{M}$
- Polyakov ('77): w/ no fermions, dilute "gas" of $\mathcal{M}$ generates eff. pot'l $V(\sigma)$: mass gap & confinement.
  $$\mathcal{M} + \overline{\mathcal{M}} \Rightarrow V(\sigma) \sim \cos(\sigma)\, e^{-S_M} \Rightarrow m_\sigma \sim e^{-S_M}.$$
- Affleck, Harvey, Witten ('82): w/ fermions, $\mathcal{M}$ has fermionic 0-modes $\Rightarrow V(\sigma)\cdot \psi^n + \not\Rightarrow m_\sigma$.

On $\mathbb{R}^3 \times S^1$:

- Lee & Yi ('97): for $G \to U(1)^N$ $\exists$ "extra" BPS monopole $\mathcal{M}_i$ $i = 1 \dots N+1$.
- Davies, Hollowood, Khoze ('00): ($N=1$ SYM on S^1) $S_{\mathcal{M}_i} \sim \frac{1}{N} S_I$ $\quad$ $8\pi^2/g^2$
- Ünsal ('07): w/ fermions, $\exists$ $\mathcal{M}_i \overline{\mathcal{M}}_j = B_{ij}$ bound-state "bions" w/ net magnetic charge

$$B + \bar{B} \Rightarrow V_B(\sigma) \sim \cos(\sigma) \cdot e^{-2S_M}$$

generates mass gap. confinement.

- PCA, MÜ : We look at $B_{ij}\overline{B_{ij}}$ (bion-$\overline{\text{bion}}$) topological molecules. Performing the Bogomolny $-g^2 \to g^2$ analytic continuation, find that in semi-classical expansion they give

$$B_{ij}\overline{B_{ij}} \Rightarrow \pm i\pi e^{-4S_M} \text{ ambiguity.}$$

$\Rightarrow$ $\boxed{\text{Then, by Lipatov, predict poles in Borel plane @ } t_n = 4nS_M g^2 \sim 4n S_I g^2/N}$

4. CONJECTURE: BION-$\overline{\text{BION}}$ ≅ IR-RENORMALON

$$t_{\text{IR-renormalon}} = n \cdot S_I g^2/\beta_0 \sim n \cdot \frac{8\pi^2}{N} \sim 4nS_I g^2/h^\vee = t_{B\bar{B}}$$

So we have found semiclassical saddlepoint config. for QCD on $\mathbb{R}^3 \times S^1$ giving Borel-plane singularity $O(N)$ times closer to origin than BPST instanton singularity, just like 't Hooft's mysterious IR renormalon singularity.

Conjecture

(1) Same set of bubble diagrams in 4d which give IR renormalon singularity also give $B\bar{B}$ singularity in perturbation theory around $U(1)^N$ vacuum.

(2) Abelianizing gauge theories on $\mathbb{R}^3 \times S^1$ have no singularities closer to the Borel plane origin.