Quantum Quenches

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$\Rightarrow$ Consider quantum mechanics with Hamiltonian dependent on an external parameter λ,

$$H_\lambda = H(\hat{p}, \hat{x}; \lambda)\,.$$

The dynamics of the system induced by variation in λ is well-understood:

- For a stationary state $|n\rangle$ with energy $E_n = \hbar\omega_n$, the *slow* changes in λ, *i.e.,* $\frac{d\ln\lambda(t)}{\omega_n dt} \ll 1$, are adiabatic: the system continues to be in the state $|n\rangle$ with time-dependent energy $E_n = E_n(\lambda(t))$ tracing the change in λ.
- A *fast* (abrupt) change in λ, *i.e.,* $\frac{d\ln\lambda(t)}{dt} = C\cdot\delta(t)$ results in the evolution of the wave-function ψ_n of $|n\rangle$ for $t > 0$ as a mixed state of *quenched* Hamiltonian

$$H_\lambda \to H_{e^C\cdot\lambda}\,.$$

What about QFT?

The behavior of quantum quenches in QFT is a much more difficult question, *i.e.,* the dynamics of the four dimensional quantum field theory under time-dependent variation of one of its coupling constants,

$$\mathcal{L}_0 \ \to \mathcal{L}_\lambda = \mathcal{L}_0 + \lambda(t)\, \mathcal{O}\,.$$

Here, $\mathcal{L}_0$ is the undeformed Lagrangian of the theory, and $\lambda(t)$ is a time-dependent coupling constant of a relevant operator $\mathcal{O}$ in the theory. A textbook example in QFT — an interaction picture — is when $\mathcal{L}_0$ is a Lagrangian of a free theory, and the (small) coupling constant λ is turned-on adiabatically so that

$$\lim_{t\to-\infty} \lambda(t) = 0\,, \qquad \lim_{t\to+\infty} \frac{d\ln\lambda}{dt} = 0\,.$$

Description of quantum quenches in strongly interactive systems, or with non-adiabatic profile of a coupling constant, has been studied to a lesser extent.

Some questions one can be interested in:

- How transition between the adibatic and non-adiabatic regimes occur?
- What are the observables of a non-stationary QFTs?
- Are instantanuous quenches in QFT well-defined?
- How does a system relaxes as a result of a quench?
- Is there a difference in relaxtion of one-point and many-point correlation functions?
- How does non-local obsevables (Wilson lines) relax?
- $\cdots$

Outline of the talk:

- Description of the model
- Holographic renormalization and ambiguities
- Results:
  - typical response of the system to a quench;
  - non-abiabaticity of the quench;
  - no instantanuous quenches;
  - renormalization scheme-dependence and divergences in $\langle T_{\mu\nu} \rangle$ and $\mathcal{O}_\Delta$;
  - renormalization scheme-dependence and the relaxation time;
  - contsructing renormalization scheme-independent observables.
- Future directions

Consider quenching the coupling λ_Δ in the deformation of large-N $SU(N)$ $\mathcal{N}=4$ supersymmetric Yang-Mills by a (gauge invariant) relevant operator $\mathcal{O}_\Delta$

$$\mathcal{L}_{SYM} \qquad \to \qquad \mathcal{L}_{SYM} + \lambda_\Delta \mathcal{O}_\Delta \,.$$

- We focus on two cases when $\Delta = 2, 3$
- The initial state is a thermal state of the gauge theory plasma.
- We discussed *perturbative* quenches, *i.e.,* during the quench the coupling constant λ_Δ is always small compare to the temperature of the initial state T_i:

$$\frac{|\lambda_\Delta|}{T_i^{4-\Delta}} \ll 1 \,.$$

- We allow for *non-perturbative* rates of change of $\lambda_\Delta = \lambda_\Delta(t)$:

$$\lambda_\Delta(t) = \lambda_\Delta^0 \left(\frac{1}{2} \pm \frac{1}{2} \tanh \frac{t}{\mathcal{T}} \right), \qquad \mathcal{T} = \frac{\alpha}{T_i} \,,$$

i.e., , we do not restrict values of α.

- We are interested in the basic gauge invariant observables of the theory undergoing the quantum quench: the stress-energy tensor T_{ij} and the VEV of $\mathcal{O}_\Delta$.

The gravitational dual to the above quench:

$$S_5 = \frac{1}{16\pi G_5} \int d^5\xi \sqrt{-g} \left(R + 12 - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2} m^2 \phi^2 + \mathcal{O}(\phi^4) \right) ,$$

with

$$m^2 = \begin{cases} -3\,, \iff \text{corresponding operator } \mathcal{O}_3\,, \\ -4\,, \iff \text{corresponding operator } \mathcal{O}_2\,. \end{cases}$$

Since our quenches are homogeneous and isotropic in the boundary spatial directions, we assume that both the background metric and the scalar field depend only on a radial coordinate r and a time v. With the background ansatz

$$ds_5^2 = -A(v,r)\, dv^2 + \Sigma(v,r)^2\, (d\vec{x})^2 + 2drdv\,, \qquad \phi = \phi(v,r)\,,$$

From the effective gravitational action we obtain the following:

- evolution equations:

$$0 = \Sigma(\dot{\Sigma})' + 2\Sigma'\dot{\Sigma} - 2\Sigma^2 + \frac{1}{12}m^2\phi^2\Sigma^2$$

$$0 = A'' - \frac{12}{\Sigma^2}\Sigma'\dot{\Sigma} + 4 + \phi'\dot{\phi} - \frac{1}{6}m^2\phi^2$$

$$0 = \frac{2}{A}\,(\dot{\phi})' + \frac{3\Sigma'}{\Sigma A}\,\dot{\phi} + \frac{3\phi'}{\Sigma A}\,\dot{\Sigma} - \frac{m^2}{A}\,\phi$$

- the constraint equations:

$$0 = \ddot{\Sigma} - \frac{1}{2}A'\dot{\Sigma} + \frac{1}{6}\Sigma(\dot{\phi})^2$$

$$0 = \Sigma'' + \frac{1}{6}\Sigma(\phi')^2$$

In above, for any function $h(r, v)$,

$$h' \equiv \partial_r h\,, \qquad \dot{h} \equiv \partial_v h + \frac{1}{2}A\partial_r h\,.$$

When $m^2 = -3$,

$$\phi = \frac{1}{r}\,p_0 + \frac{1}{r^2}\,\left(p_0'\right) + \frac{1}{r^3}\left(p_2 - \left(\frac{1}{2}p_0'' + \frac{1}{6}p_0^3\right)\ln r\right) + \mathcal{O}(r^{-4}\ln r)$$

$$\Sigma = r + \mathcal{O}(r^{-1})$$

$$A = r^2 - \frac{1}{6}p_0^2 + \frac{1}{r^2}\left(a_4 + \left(\frac{1}{6}p_0 p_0'' + \frac{1}{36}p_0^4 - \frac{1}{6}(p_0')^2\right)\ln r\right) + \mathcal{O}(r^{-3}\ln r)$$

where $\{p_0, p_2, a_4\}$ are functions of v.

In addition, a constraint equation implies:

$$0 = -2a_4' + \frac{5}{27}p_0^3 p_0' + \frac{2}{3}p_0' p_2 - \frac{2}{3}p_0 p_2' - \frac{1}{9}p_0' p_0'' + \frac{4}{9}p_0 p_0'''$$

Physical meaning of $\{p_0, p_2, a_4\}$:

- a 'source' [non-normalizable component],

$$p_0 \; \propto \; \lambda_3$$

- a 'response' [normalizable compotent]

$$p_2 \; \sim \mathcal{O}_3$$

- Note that in the absence of the source/response the constraint implies

$$a_4' = 0 \qquad \Rightarrow \qquad \text{energy density} = constant$$

In general, the constraint equation can be integrated to quantify the change of $\mathcal{E}$ during the quench:

$$a_4 = \mathcal{C} + \frac{5}{216} p_0(v)^4 - \frac{5}{36}(p_0(v)')^2 + \frac{2}{9} p_0(v) p_0(v)'' - \frac{1}{3} p_0(v) p_2(v) + \frac{2}{3} \int_{-\infty}^{v} ds\, p_0(s)' p_2(s)$$

where $\mathcal{C}$ is a constant, related to the energy density in the infinite past.

Comment on numerical procedure (all to quadratic order in the source inclusive):

- Numerically solve the PDE for the scalar $\phi(v, r)$ for a given profile of the non-normalizable component

$$p_0 = p_0(v)$$

- Numerical solution determines normalizable component

$$p_2 = p_2(v)$$

- Given $\{p_0, p_2\}$ we can integrate the constraint equation to obtain

$$a_4 = a_4(v)$$

- Once $\{p_0, p_2, a_4\}$ are determined, we translate them in QFT observables:

$$\mathcal{E} = \mathcal{E}(v)\,, \qquad \mathcal{P} = \mathcal{P}(v)\,, \qquad \mathcal{O}_3 = \mathcal{O}_3(v)$$

To compute correlation functions of gauge-invariant observables, the theory has to be regularized and renormalized:

$$S_{ct} = S_{ct}^{divergent} + S_{ct}^{finite}$$

$$S_{ct}^{divergent} = \frac{1}{16\pi G_5} \int_{\partial\mathcal{M}_5,\, \frac{1}{r}=\epsilon} d^4x\, \sqrt{-\gamma} \Big(6 + \frac{1}{2}\phi^2 + \frac{1}{12}\phi^4 \ln\epsilon + \frac{1}{2}\, \gamma^{ij}\partial_i\phi\partial_j\phi\, \ln\epsilon + \frac{1}{12} R^\gamma \phi^2\, \ln\epsilon \Big)$$

$$S_{ct}^{finite} = \frac{1}{16\pi G_5} \int_{\partial\mathcal{M}_5,\, \frac{1}{r}=\epsilon} d^4x\, \sqrt{-\gamma} \Big(\delta_1\, \phi^4 + \delta_2\, \gamma^{ij}\partial_i\phi\partial_j\phi + \delta_3\, R^\gamma \phi^2 \Big)$$

where we have separated the counterterm which diverges in the limit $\epsilon = \frac{1}{r} \to 0$ from the finite counterterms.

The finite counterterms are parametrized by:

$$\delta_1\,, \qquad \delta_2\,, \qquad \delta_3$$

Once the theory is renormalized, we can compute 1-point correlation functions:

$$8\pi G_5\, \mathcal{E} = -\frac{3}{2}a_4 - \frac{1}{12}(p_0')^2 + \frac{1}{8}p_0^2 a_1^2 - \frac{1}{2}p_0 p_2 + \frac{1}{3}p_0 p_0'' + \frac{7}{288}p_0^4 + \mathcal{E}^{ambiguity}$$

$$8\pi G_5 \mathcal{P} = -\frac{1}{2}a_4 - \frac{1}{36}(p_0')^2 + \frac{1}{6}p_0 p_2 - \frac{1}{18}p_0 p_0'' + \frac{7}{864}p_0^4 + \mathcal{P}^{ambiguity}$$

$$16\pi G_5\, \langle \mathcal{O}_3 \rangle = \frac{1}{2}p_0'' - \frac{1}{12}p_0^3 - 2a_1 p_0' + \frac{1}{2}p_0 a_1^2 - 2p_2 + \mathcal{O}_3^{ambiguity}$$

where we employ the label ^{ambiguity} to denote renormalization scheme ambiguities:

$$\mathcal{E}^{ambiguity} = \frac{1}{2}\delta_1 p_0^4 + \frac{1}{2}\delta_2 (p_0')^2\,,$$

$$\mathcal{P}^{ambiguity} = -2\delta_3 (p_0')^2 - 2\delta_3 p_0 (p_0'') - \frac{1}{2}\delta_1 p_0^4 + \frac{1}{2}\delta_2 (p_0')^2$$

$$\mathcal{O}_3^{ambiguity} = 4\delta_1 p_0^3 + 2\delta_2 p_0''\,.$$

Note that for arbitrary δ_i, the following (diffeomorphism) Ward identity,

$$\partial_i \langle T_{ij} \rangle = -\langle \mathcal{O}_3 \rangle \, \partial_j p_0 \, ,$$

is equivalent to the constraint

$$0 = -2a_4' + \frac{5}{27} p_0^3 p_0' + \frac{2}{3} p_0' p_2 - \frac{2}{3} p_0 p_2' - \frac{1}{9} p_0' p_0'' + \frac{4}{9} p_0 p_0'''$$

$\Rightarrow$ We focus on the quences of the type

$$\lim_{\tau \to \pm\infty} p_0(\tau) = constant$$

so, provided that the same is true for $p_2(\tau)$, *i.e.,*

$$\lim_{\tau \to \pm\infty} p_2(\tau) = constant$$

(numerically we verified that this is indeed the case), we have a thermal equilibrium state in the infinite past, and a thermal equilibrium state in the infinite future.

For example, if

$$\lim_{\tau\to-\infty} p_0 = 0\,, \qquad \lim_{\tau\to+\infty} p_0 = 1$$

i.e., we quench from a thermal state of a **CFT** to a thermal state of a **massive** gauge theory,

$$\mathcal{E} = \frac{3}{8}\pi^2 N^2 T_i^4 \left(1 - \left(2a_4 + \frac{1}{3}(p_0')^2\ \ln\frac{\pi T_i}{\Lambda_2} + \frac{1}{9}(p_0')^2 + \frac{2}{3}p_0 p_2\right)\frac{(m_f^0)^2}{\pi^2 T_i^2} + \mathcal{O}\left(\frac{(m_f^0)^4}{T_i^4}\right)\right)$$

$$\mathcal{P} = \frac{1}{8}\pi^2 N^2 T_i^4 \left(1 - \left(2a_4 + \frac{1}{9}(p_0')^2 - \frac{2}{3}p_0 p_2 + \frac{2}{9}p_0 p_0'' - \frac{2}{3}\left(p_0 p_0'' + (p_0')^2\right)\ \ln\frac{\pi T_i}{\Lambda_3}\right.\right.$$
$$\left.\left. + (p_0')^2\ \ln\frac{\pi T_i}{\Lambda_2}\right)\frac{(m_f^0)^2}{\pi^2 T_i^2} + \mathcal{O}\left(\frac{(m_f^0)^4}{T_i^4}\right)\right)$$

$$\mathcal{O}_3 = -\frac{\sqrt{2}}{2} N^2 T_i^2 m_f^0 \left(p_2 - \frac{1}{4}p_0'' + \frac{1}{2}\ln\frac{\pi T_i}{\Lambda_2}\ p_0'' + \mathcal{O}\left(\frac{(m_f^0)^2}{T_i^2}\right)\right)$$

where

$$\delta_2 = \frac{1}{2}\ln\Lambda_2\,, \qquad \delta_3 = \frac{1}{12}\ln\ \Lambda_3$$

Note that the number of ambiguities in renormalization scheme is precisely what is needed to make sense of $\ln(T)$ terms once the gravity data is translated into gauge theory data.

Similarly, we can analyse the quenches

$$\lim_{\tau\to-\infty} p_0 = 1\,, \qquad \lim_{\tau\to+\infty} p_0 = 0$$

i.e., we quench from a thermal state of a **massive** gauge theory to a thermal state of a **CFT**.

Another interesting observables are:

$$\frac{T_f}{T_i} = \left(1 + \left(\pm\frac{\Gamma\left(\frac{3}{4}\right)^4}{3\pi^2} - \frac{1}{2}a_4^\infty\right)\frac{(m_f^0)^2}{\pi^2 T_i^2} + \mathcal{O}\left(\frac{(m_f^0)^4}{T_i^4}\right)\right)$$

$$\frac{\mathcal{E}_f}{\mathcal{E}_i} = \left(1 + \left(\pm\frac{2\Gamma\left(\frac{3}{4}\right)^4}{3\pi^2} - 2a_4^\infty\right)\frac{(m_f^0)^2}{\pi^2 T_i^2} + \mathcal{O}\left(\frac{(m_f^0)^4}{T_i^4}\right)\right)$$

$$\frac{\mathcal{P}_f}{\mathcal{P}_i} = \left(1 - \left(\pm\frac{2\Gamma\left(\frac{3}{4}\right)^4}{3\pi^2} + 2a_4^\infty\right)\frac{(m_f^0)^2}{\pi^2 T_i^2} + \mathcal{O}\left(\frac{(m_f^0)^4}{T_i^4}\right)\right)\,.$$

where

$$a_4^\infty = \lim_{\tau\to+\infty} a_4(\tau)$$

Consider quenches of the type

$$p_0 = \frac{1}{2} + \frac{1}{2}\tanh\frac{\pi T_i\,\tau}{\alpha}$$

where T_i is the initial temperature.

$\Rightarrow$ For $\alpha \gg 1$ the quenches are slow compare to a characteristic thermal scale $\propto \frac{1}{T_i}$, we expect an “adiabatic” response

$$p_2(\tau)\Big|_{adiabatic} = -\frac{\Gamma\left(\frac{3}{4}\right)^4}{\pi^2}\,p_0(\tau)$$

$\Rightarrow$ note that for the adiabative response, from

$$0 = -2a_4' + \frac{2}{3}p_0'p_2 - \frac{2}{3}p_0p_2' - \frac{1}{9}p_0'p_0'' + \frac{4}{9}p_0p_0''' \qquad \Longrightarrow \qquad a_4' \approx 0 + \mathcal{O}(\alpha^{-3})$$

$\Rightarrow$

$$\frac{T_f}{T_i} = \left(1 + \left(\pm\frac{\Gamma\left(\frac{3}{4}\right)^4}{3\pi^2} + \mathcal{O}(\alpha^{-2})\right)\frac{(m_f^0)^2}{\pi^2 T_i^2} + \mathcal{O}\left(\frac{(m_f^0)^4}{T_i^4}\right)\right)$$

and similarly for $\mathcal{E}$, $\mathcal{P}$

Typical response of the system:

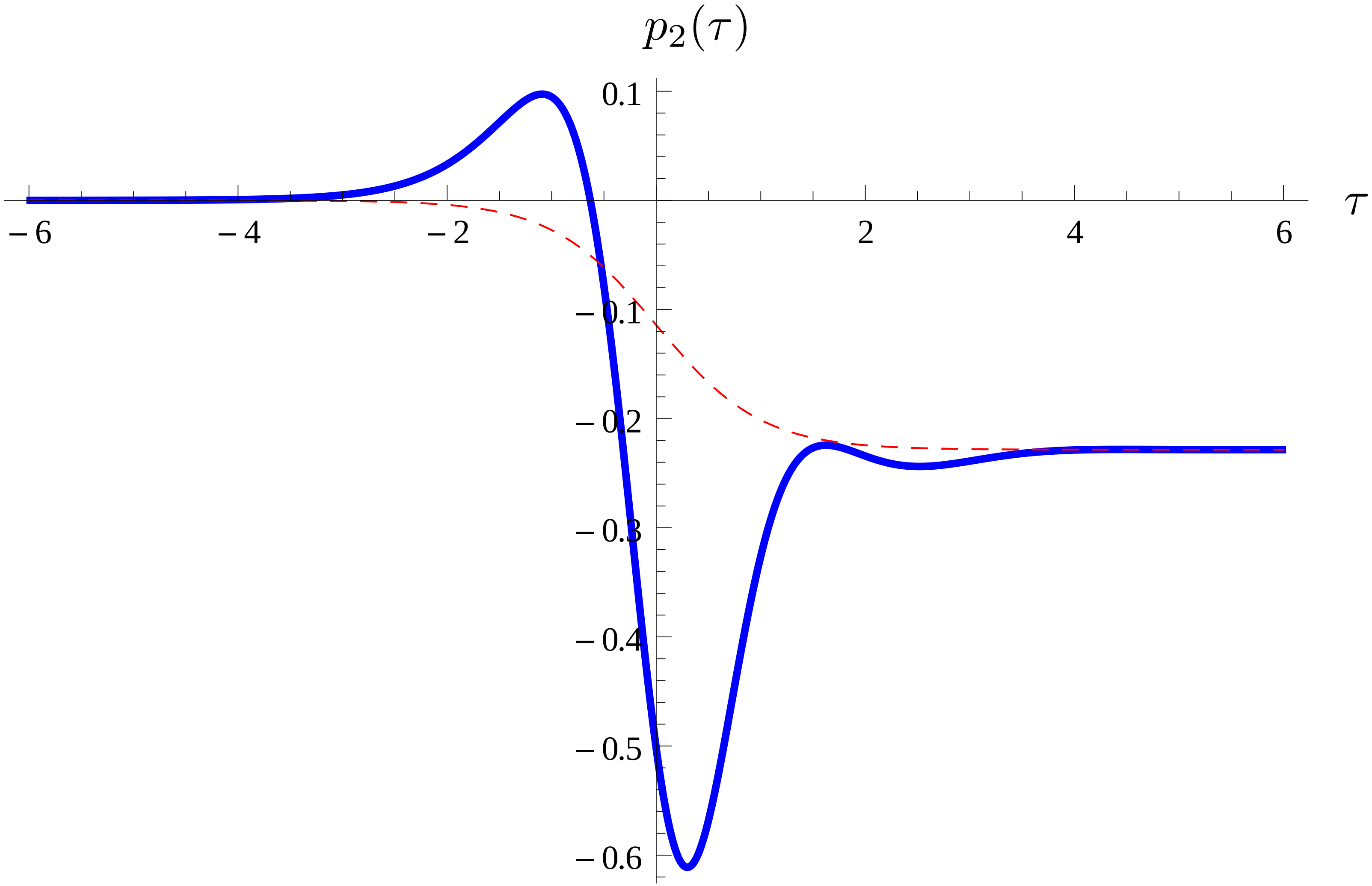
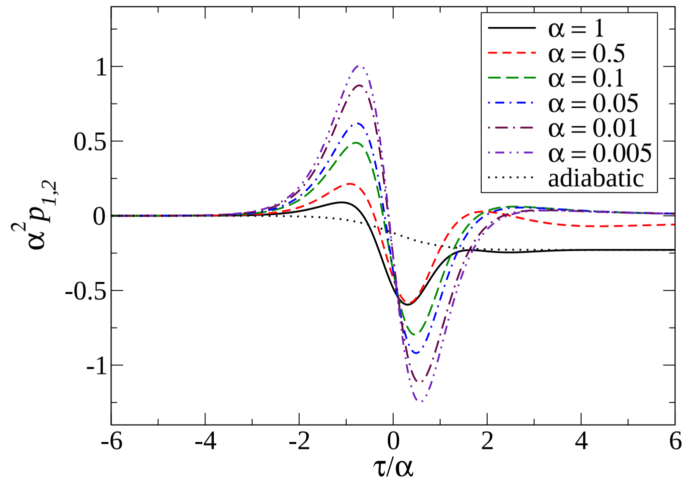
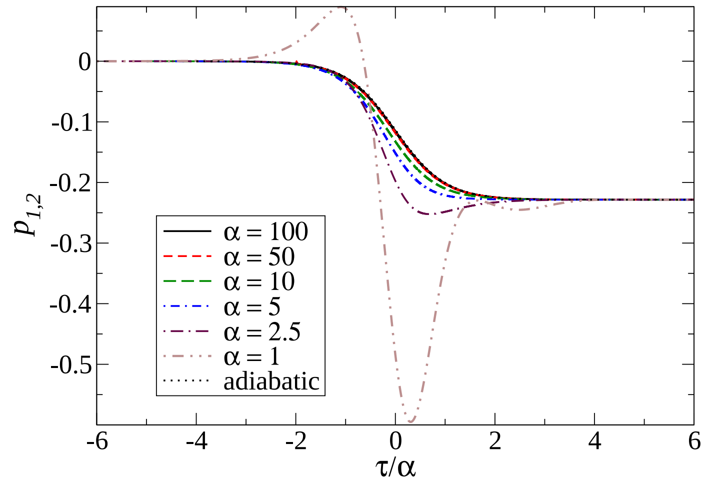
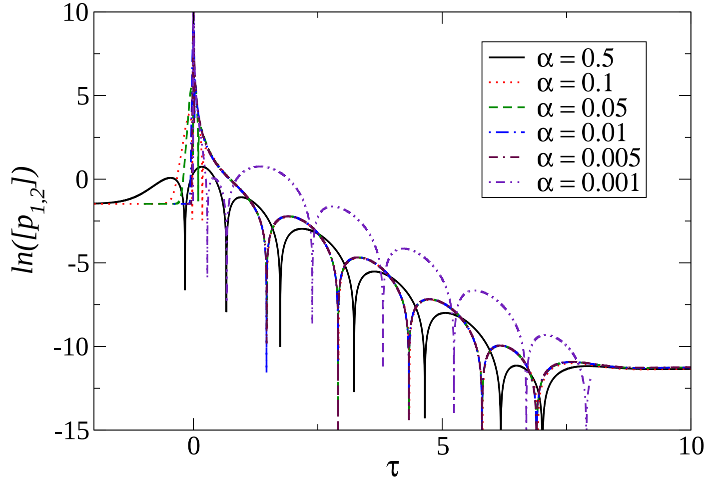


Figure 1: Evolution of the normalizable component p_2 during the quench with $\alpha = 1$. The dashed red lines represent the adiabatic response.

More evolutions:

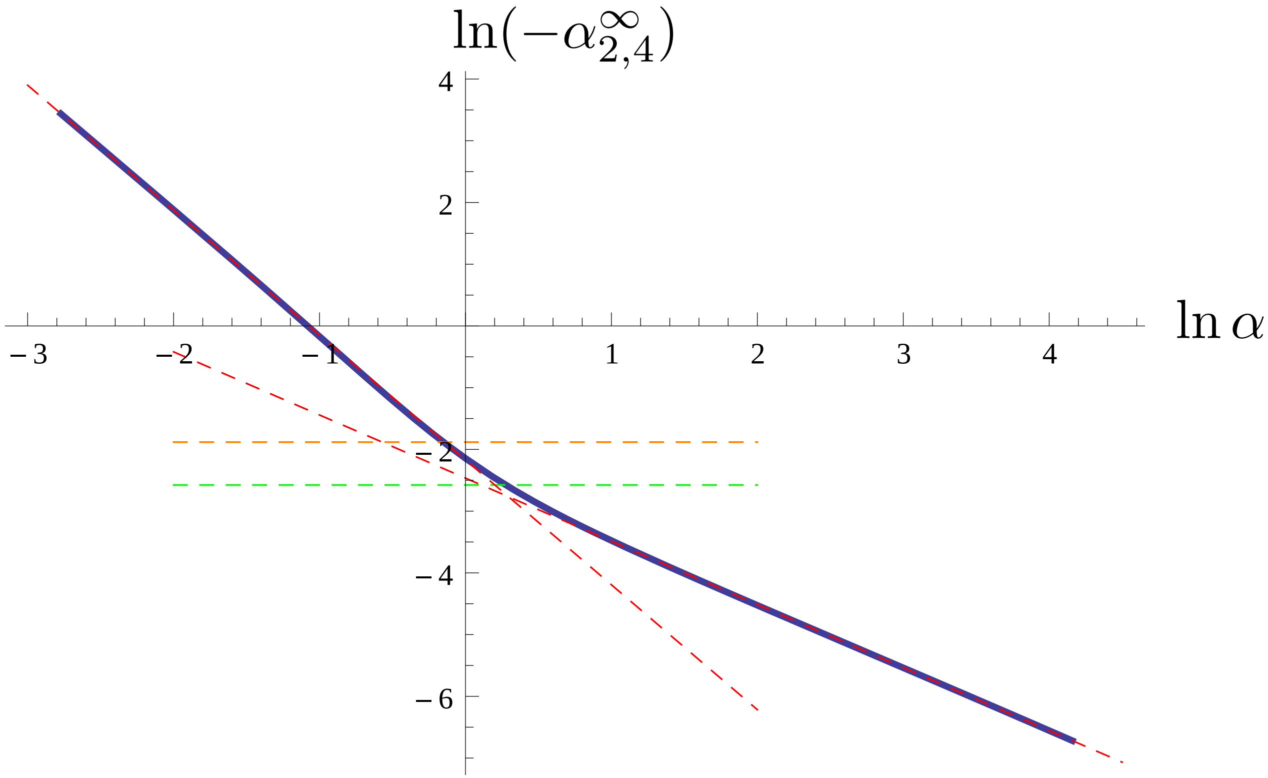






Recall:

$$\frac{T_f}{T_i} = \left(1 + \left(\pm\frac{\Gamma\left(\frac{3}{4}\right)^4}{3\pi^2} - \frac{1}{2}a_4^\infty\right)\frac{(m_f^0)^2}{\pi^2 T_i^2} + \mathcal{O}\left(\frac{(m_f^0)^4}{T_i^4}\right)\right)$$



$\Rightarrow$ Note that quenches *always* results in pumping energy into the system

$$\text{slow}: \qquad \ln(-a^{\infty}_{2,4})\Big|^{fit}_{red,dashed} = -2.46(5) - 1.0(2)\ \ln\alpha\,, \qquad \alpha \gg 1$$

$$\text{fast}: \qquad \ln(-a^{\infty}_{2,4})\Big|^{fit}_{red,dashed} = -2.17(0) - 2.0(2)\ \ln\alpha\,, \qquad \alpha \ll 1$$

Above asymptotic behaviour translates into

$$\frac{|\Delta T|}{T_i} \equiv \frac{|T_f - T_i|}{T_i} = \begin{cases} \propto \frac{1}{\alpha}\ \frac{(m_f^0)^2}{T_i^2},\ \alpha \gg 1 \\ \propto \frac{1}{\alpha^2}\ \frac{(m_f^0)^2}{T_i^2},\ \alpha \ll 1 \end{cases}$$

and similarly for the relative change in the energy density $\mathcal{E}$ and the pressure $\mathcal{P}$.

$\Rightarrow$ Note that infinitely sharp quenches

$$\alpha \to 0$$

are not allowed

In general, quenching the coupling λ_Δ of $\mathcal{O}_\Delta$ as

$$\lambda_\Delta(t) = \lambda_\Delta^0 \left(\frac{1}{2} + \frac{1}{2} \tanh \frac{T_i\, t}{\alpha} \right)$$

results in the following scaling of physical observables for fast $\alpha \ll 1$ quenches:

$$\propto \begin{cases} \left(\frac{1}{\alpha}\right)^{|2\Delta - 4|}, & |2\Delta - 4| = \text{integer} \neq 0 \\ (-\ln \alpha)\,, & \text{otherwise} \end{cases}$$

Comment on scheme-independent observables:

- while the following observables are scheme-dependent,

$$\mathcal{E} = \frac{3}{8}\pi^2 N^2 T_i^4 \left(1 - \left(2a_4 + \frac{1}{3}(p_0')^2\ \ln\frac{\pi T_i}{\Lambda_2} + \frac{1}{9}(p_0')^2 + \frac{2}{3}p_0 p_2\right)\frac{(m_f^0)^2}{\pi^2 T_i^2} + \mathcal{O}\left(\frac{(m_f^0)^4}{T_i^4}\right)\right)$$

$$\mathcal{O}_3 = -\frac{\sqrt{2}}{2} N^2 T_i^2 m_f^0 \left(p_2 - \frac{1}{4}p_0'' + \frac{1}{2}\ln\frac{\pi T_i}{\Lambda_2}\ p_0'' + \mathcal{O}\left(\frac{(m_f^0)^2}{T_i^2}\right)\right)$$

- the following combination is renormalization-scheme independent:

$$\left(\mathcal{E}(\tau) - \frac{m_f^0}{\sqrt{2}}\ \int_{-\infty}^{\tau} ds\, p_0'(s)\mathcal{O}_3(s)\right)$$

Open questions:

- Fully-nonlinear quenches, not necessarily of thermal states
- Sound waves in quenches
- Quenches in various dimensions
- Non-local observables during quenches
- Quenches of SUSY couplings
- Quenches accross the phase transitions
- $\cdots$