Dynamical Supersymmetry Breaking for Field Theories in 2+1 dimensions

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[1] Based on 0807:1500, 0906:2390, 1004.0903, 1104.3517, 1105.3687 with Aharony, Hirano, Ouyang

Starting point: ABJ(M) theory

$$U(N)_k \times U(N+l)_{-k} \leftrightarrow AdS_4 \times S_7/Z_k$$

“Entropy”

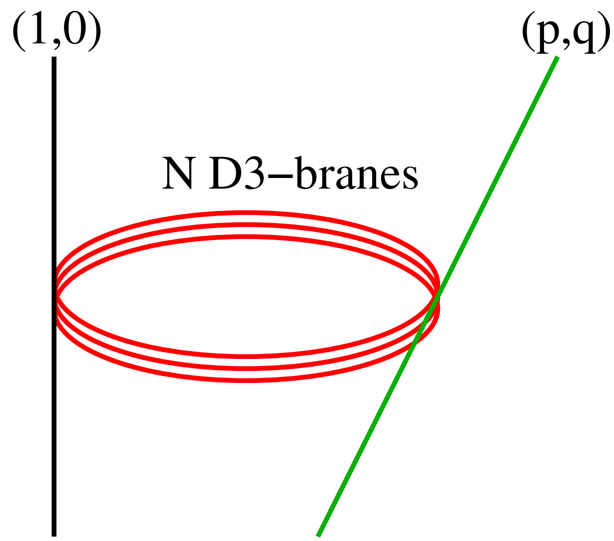
$$S \sim Q^{3/2}, \qquad Q = N - \frac{l(l-k)}{2k}$$

Computed on SUGRA and using localization techniques.

What happens when $Q < 0$?

One way to frame this question: RG flow

After all, ABJM was originally conceived as the IR of



We can decouple in sequence:

IIB String Theory $\to$ Defect Field Theory in 3+1d $\to$
Chern-Simons-Yang-Mills-Matter Theory in 2+1 $\to$ ABJM SCFT

The question: what is the universality class of CSYMM theory when

$$Q = N - \frac{l(l-k)}{2k} < 0$$

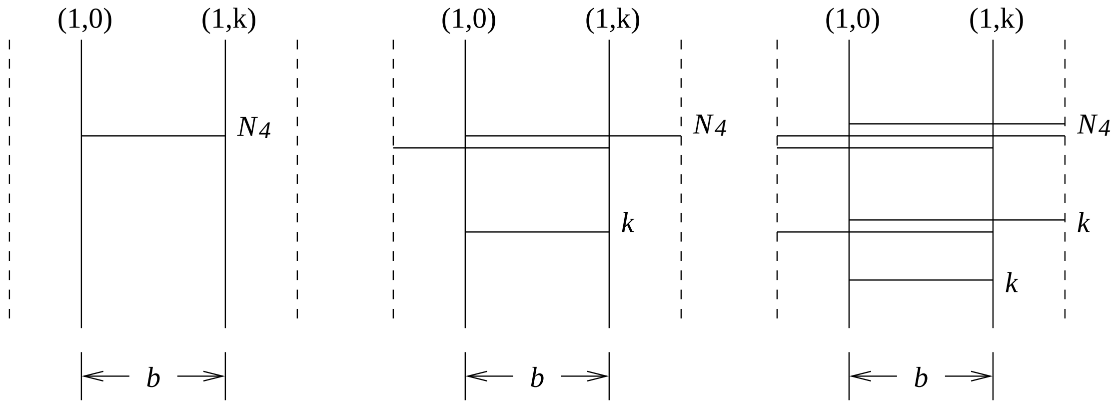
- UV field theory well defined
- $\mathcal{N} = 3$ SUSY in the UV, enhanced to $\mathcal{N} = 6, 8$ in the IR
- Needs fractional brane l
- Causes cascade
- Can be addressed in SUGRA

k: Start with M-theory 8d $sp(2)$ manifold $\mathcal{M}_8$ (LWY)

l: Turn on self-dual 4-form flux on $\mathcal{M}_8$

N: Add M2-brane sources and compute the back reaction

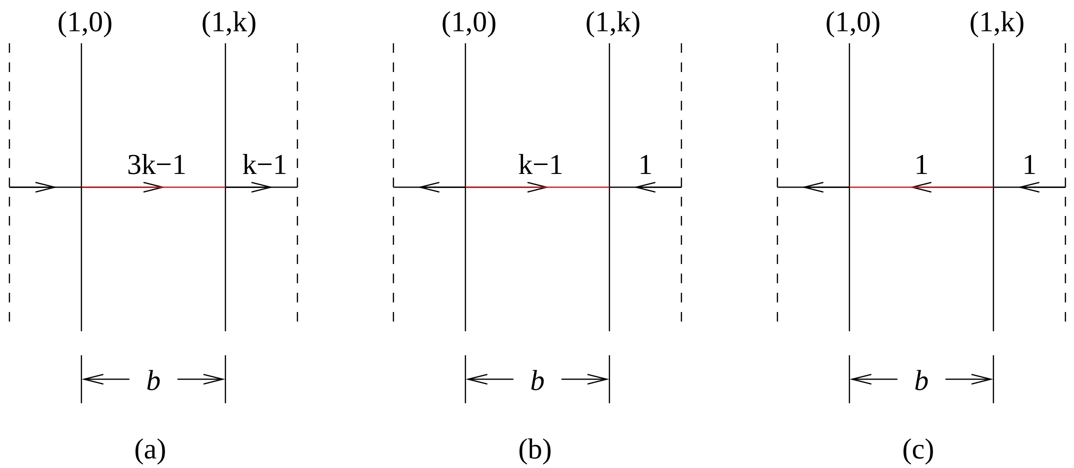
Duality Cascade:



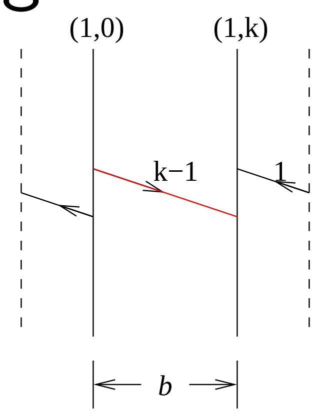
$$
\begin{aligned}
N &\longrightarrow N + l \\
l &\longrightarrow l + k \\
k &\longrightarrow k
\end{aligned}
$$

$$
Q = N - \frac{l(l-k)}{2k} \;\longrightarrow\; Q = N - \frac{l(l-k)}{2k}
$$

$Q < 0$ if N is too small so that an anti D3 appear in the cascade chain



Anti-branes in this configuration repel[2]



[2]Mukhi and Suryanarayana hep-th/0003219

Questions. Given N, l, k, and b_∞ (relative g^2_{YM} coupling)

- How does one compute the mass gap

- What is the low energy effective field thoery in the deep IR

- How does one characterize the phase

- What are the values of the relevant order parameters

LYW Metric:

$$ds^2 = V_{ij} d\vec{y}_i d\vec{y}_j + (V^{-1})^{ij} R_i R_j (d\varphi_i + A_i)(d\varphi_j + A_j)$$
$$V_{ij} = \delta_{ij} + \frac{1}{2} \frac{R_i p_i R_j p_j}{|R_1 p_1 \vec{y}_1 + R_2 p_2 \vec{y}_2|} + \frac{1}{2} \frac{R_i \tilde{p}_i R_j \tilde{p}_j}{|R_1 \tilde{p}_1 \vec{y}_1 + R_2 \tilde{p}_2 \vec{y}_2|} \ ,$$

- Originally constructed by Lee, Weinberg, and Yi for the moduli-space of monopoles and dyons

- Lots of structure

- Self-dual 4-forms are conjectured by Sen to exist on this space based on S-duality

- No explicit construction as of yet

- Greens Function

Alternative for $\mathcal{M}_8$?

- $(TN \times TN)/Z_k$
- $spin(7)$ holonomy manifold A_8/Z_k
- $spin(7)$ holonomy mainfold B_8/Z_k
- Stenzel geometry
- Generalized Taub-NUT
- $G_2 \times S_1$
- LLM geometry
- ...

Many of these examples involve deformation parameter that blows up a non-trivial cycle in the IR very much like the deformed conifold.

$spin(7)$ manifold A_8/Z_k

- R^8/Z_k near core, $R^7 \times (S_1/Z_k)$ at infinity
- Metric known explicitly $(r \geq \ell)$

$$
\begin{aligned}
ds^2_{A_8} &= h(r)^2 dr^2 + a(r)^2 (D\mu^i)^2 + b(r)^2 \sigma^2 + c(r)^2 d\Omega_4 \\
& h(r)^2 = \frac{(r+\ell)^2}{(r+3\ell)(r-\ell)}, \qquad a(r)^2 = \frac{1}{4}(r+3\ell)(r-\ell) \\
& b(r)^2 = \frac{\ell^2 (r+3\ell)(r-\ell)}{(r+\ell)^2}, \qquad c(r)^2 = \frac{1}{2}(r^2-\ell^2)
\end{aligned}
$$

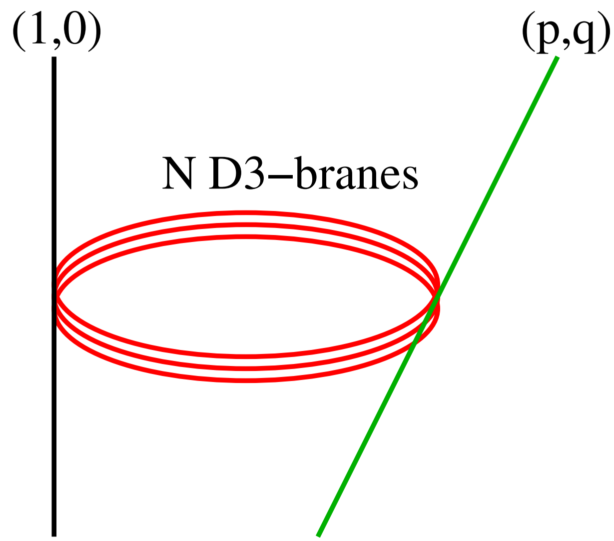
- self-dual 4-form and the Greens function also known
- geometry regular at $r = \ell$

One can therefore construct a warped solution of the form

$$\begin{aligned} ds^2 &= H^{-2/3}(-dt^2 + dx_1^2 + dx_2^2) + H^{1/3} ds_8^2 \\ F_4 &= dt \wedge dx_1 \wedge dx_2 \wedge dH^{-1} + mG_4 \end{aligned}$$

should be dual to some RG flow which in the IR is ABJM, and the UV is something resembling a Yang-Mills theory (because of the S_1)

Description in terms of brane constructions[3]



Configuration	Angles	Condition	SUSY	second 5-brane
1	θ_4	$\theta_4 = 0$	$\mathcal{N} = 4$	NS5 (12345)
2(i)	$\theta_2,\ \theta_3$	$\theta_2 = \theta_3$	$\mathcal{N} = 2$	NS5 $(123[48]_{\theta_2}[59]_{\theta_3})$
2(ii)	$\theta_3,\ \theta_4$	$\theta_3 = \theta_4$	$\mathcal{N} = 2$	$(p,q)5\ (1234[59]_{\theta_3})$
3(i)	$\theta_1,\ \theta_2,\ \theta_3$	$\theta_3 = \theta_1 + \theta_2$	$\mathcal{N} = 1$	NS5 $(12[37]_{\theta_1}[48]_{\theta_2}[59]_{\theta_3})$
3(ii)	$\theta_2,\ \theta_3,\ \theta_4$	$\theta_3 = \theta_2 + \theta_4$	$\mathcal{N} = 1$	$(p,q)5\ (123[48]_{\theta_2}[59]_{\theta_3})$
4(i)	$\theta_1,\ \theta_2,\ \theta_3,\ \theta_4$	$\theta_4 = \theta_1 + \theta_2 + \theta_3$	$\mathcal{N} = 1$	$(p,q)5\ (12[37]_{\theta_1}[48]_{\theta_2}[59]_{\theta_3})$
4(ii)	$\theta_1,\ \theta_2,\ \theta_3,\ \theta_4$	$\theta_1 = -\theta_2,\ \theta_3 = \theta_4$	$\mathcal{N} = 2$	$(p,q)5\ (12[37]_{\theta_1}[48]_{\theta_2}[59]_{\theta_3})$
4(iii)	$\theta_1,\ \theta_2,\ \theta_3,\ \theta_4$	$\theta_1 = \theta_2 = \theta_3 = \theta_4$	$\mathcal{N} = 3$	$(p,q)5\ (12[37]_{\theta_1}[48]_{\theta_2}[59]_{\theta_3})$

[3] Kitao, Ohta, Ohta

Gravity solution can be constructed by starting with $Q = 0$ where all the warp factor is sourced by the flux term

$$d * F = \frac{1}{2} F \wedge F$$

which is normalizable.

- IR geometry is regular
- UV geometry is warped by D2 charge
- $Q > 0$ then means adding D2-branes:
  $\rightarrow$ locally $AdS_4 \times S_7/Z_k$ with flux Q

$Q < 0$ should then correspond to adding anti D2-branes

• Very similar to the Saclay group program of adding anti D3-branes to deformed conifold

• Non-BPS

In the case of deformed conifold, anti D3-branes is a candidate for $metastable$ states

• Decay to SUSY vacum via KPV instanton

• Similar mechanism at work in 2+1 examples involving IR deformation e.g. B_8, Stenzel, etc

The case of A_8 is different because there are no deformed cycle in the IR for the KPV instanton to wrap

- Expectation is that $Q < 0$ is non-SUSY globally stable vacum, i.e. spontaneously broken SUSY
- UV is simpler in 2+1 (cascade terminates)
- Great subject to explore the leading non-BPS deformation
- Technical and hard

Case of $Q = 0$

- SUGRA solution is explicitly known
- Mass-gap

$$E_{gap} = g_{YM2}^2 N \left(\frac{N}{k}\right)^{3/2}$$

- Would be interesting to have similar results for $Q < 0$

More probe of the vacua: Confinement scale

- Fundamental string at $r = \ell$ is $tensionless$
- This is clear because the M-theory cycle is collapsing at $r = \ell$
- Seemingly screening

More probe of the vacua: 't Hooft operator

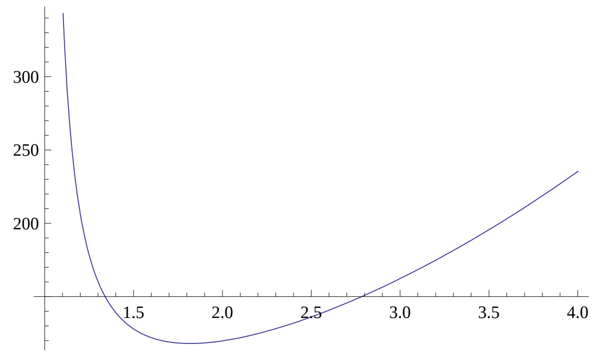
- The *area law* in 3+1 is the *length law* in 2+1
- Dynamics of D0-probe in $Q = 0$ background:

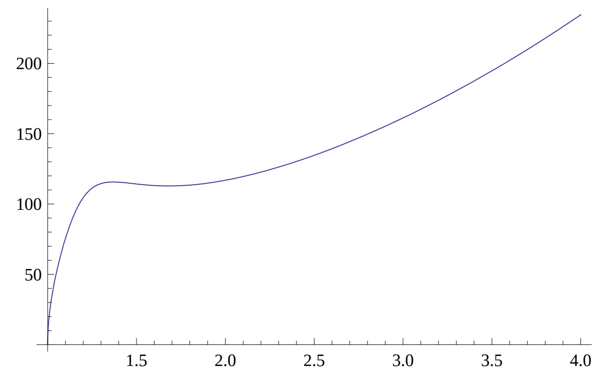
- The D2 charge pulls D0 at large r toward small r

- The D6 charge pushes the D0 at small r toward large r

- Equilibrate at r of order ℓ

$$S = e^{-\phi}\sqrt{-g_{00}} = H^{-1/2}(r)b^{-1}(r)$$





- D0 is a gravity mode in the M-theory lift and is related to the mass gap

- Appears to support *screening*

Some issues remain:

- Which gauge group does the Wilson line (dual to F1) source?

- Which gauge group does the 't Hooft operator (dual to D0) source?

- D1 in deformed conifold was the first sign of baryon branch of the moduli space.