# (0, 2) Quantum Cohomology

#### Sheldon Katz

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2 (0, 2) [Gauged Linear Sigma Model](#page-22-0)

3 (0, 2) [Quantum Cohomology](#page-65-0)

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# Joint work with Ron Donagi, Josh Guffin, and Eric Sharpe

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Sheldon Katz (0, 2) [Quantum Cohomology](#page-0-0)

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## Physical Model.

Spacetime  $M = \mathbf{R}^{3,1} \times X$ , X 6 dimensional and compact

$$
S = \int d^{10}x \sqrt{-g} \left( \partial_i x^\mu \partial^i x_\mu + \bar{\psi}_\mu \Gamma^\nu \partial_\nu \psi^\mu + \bar{\lambda}_a \Gamma^\nu \partial_\nu \lambda^a + \right. \\
\left. + R_{\mu \bar{\nu} a \bar{b}} \psi^\mu \bar{\psi}^\nu \lambda^a \bar{\lambda}^b + \dots \right)
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- $\psi^{\mu}$  right-handed fermions in *TX*;  $\lambda^{a}$  left-handed fermions living in gauge bundle  $E$ ; a gauge index,  $\psi^{\mu}$  superpartner of  $x^{\mu}$ ,  $(0, 2)$  SUSY
- $\bullet$  In an  $E_6$  heterotic, matter fits into 27, 27, singlet multiplets
- Yukawa couplings 27<sup>3</sup>, 27<sup>3</sup>
- <span id="page-4-0"></span>If  $E = TX$  (i.e. (2, 2) SUSY), 27<sup>3</sup> and  $27^3$  Yukawa couplings can be computed *exactly* by techniques of algebraic geometry (only tree level co[up](#page-3-0)l[in](#page-5-0)[g](#page-3-0)[s](#page-4-0)[\)](#page-8-0)

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#### Quantum cohomology

- What makes this work so well: (2, 2) theory can be twisted to become topological (A model)
- Fermions take values in simpler bundles (trivial and canonical)
- In topological sector, worldsheet vertex operators are identified with cohomology *H* ∗ (*X*)
- Operator products computed in terms of three point couplings, define quantum cohomology, and the algebra *H*<sup>∗</sup>(*X*) still closes after quantum corrections

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## Half-twisted (0, 2) theories

#### However, we can *half-twist* (right handed fermions only)

- Not topological, but have a finite-dimensional *quasi-topological* sector
- Three point couplings in half-twisted theory coincide with Yukawa couplings in the physical theory
- These theories are closely related to *gauged linear sigma models* (and sometimes coincide)
- Many other motivations from mathematics, especially algebraic geometry
- The rest of the talk will focus on  $(0, 2)$  GLSMs

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#### **Outline**



#### 2 (0, 2) [Gauged Linear Sigma Model](#page-22-0)

3 (0, 2) [Quantum Cohomology](#page-65-0)

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- The (0, 2) GLSM is a 2-dimensional gauge theory with (0, 2) supersymmetry *Witten*
	- Gauge group  $G = U(1)$ <sup>*r*</sup>
	- Chiral fields  $\Phi^1, \ldots, \Phi^n, \bar{\mathcal{D}} \Phi^i = 0$ , lowest component scalar
	- Fermi superfields Υ*<sup>a</sup>* , lowest component left-handed fermion
	- $\bar{D}\Upsilon^a = E^a(\Phi)$ ,  $E^a$  holomorphic

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#### Prior work.

- First studied in this context by *Adams, Basu, and Sethi*, who computed by mirror symmetry
- Direct computation of quantum cohomology ring done for a fixed (0, 2) deformation by Guffin-K-Sharpe
- All  $(0, 2)$  deformations of  $\mathsf{P}^1 \times \mathsf{P}^1$  model computed by Guffin-K
- General form for linear deformations conjectured by McOrist-Melnikov
- We verify their conjecture and show that it holds verbatim for nonlinear deformations as well

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#### Vacuum Moduli Space

#### • The moduli space of vacua is a toric variety

- $Q^i_\alpha$  charge of  $\Phi^i$  under  $\alpha^{\text{th}}$  *U*(1)
- FI terms  $\sum Q_\alpha^i |\Phi^i|^2 r_\alpha = 0$
- Gauge equivalence classes give moduli space

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# $G = U(1)$

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\bullet \ \sum |\Phi^i|^2 - r = 0
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M_{\text{vac}} = {\phi |\sum |\phi^i|^2 = r}/(\phi^i \sim e^{i\theta} \phi^i)
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This is complex projective *n* space **P** *n*

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#### Language of Algebraic Geometry

#### Complex projective space **CP***<sup>n</sup>* = **P** *n* :

Described by homogeneous coordinates:

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(x_0,\ldots x_n)\sim(\lambda x_0,\ldots,\lambda x_n)
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Have holomorphic line bundles  $O(k)$  on  $\mathbf{P}^n$  whose holomorphic sections are homogeneous degree *k* polynomials  $f(x_0, \ldots, x_n)$ 

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## Identification of Vacuum Moduli Space by Algebraic **Geometry**

 $M_{\text{vac}} \simeq \mathbf{P}^n$  $(\phi^0, \ldots, \phi^n) \mapsto (\phi^0, \ldots, \phi^n)$ 

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 $(\phi^0,\ldots,\phi^n)\mapsto \frac{1}{\sqrt{2}}$ *r*

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## Identification of Vacuum Moduli Space by Algebraic **Geometry**

 $M_{\text{vac}} \simeq \mathbf{P}^n$  $\bullet$  $(\phi^0, \ldots, \phi^n) \mapsto (\phi^0, \ldots, \phi^n)$  $\bullet$  $(\phi^0,\ldots,\phi^n)\mapsto \frac{1}{\sqrt{2}}$  $(\phi^0,\ldots,\phi^n)$ *r*

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#### Instanton Contributions

- Fix the worldsheet as  $S^2 = P^1$ . Fix a gauge instanton background with  $c_1 = d$
- $\overline{\mathcal{D}}\Phi^i=0$  implies  $\bar{\partial}\phi^i=0$
- $\phi^i$  is a holomorphic section of  $\mathcal{O}_{\mathbf{P}^1} (d),$  a degree  $a$ polynomial
- These are identified with degree *d* holomorphic maps (worldsheet instantons)

$$
\phi : \mathbf{P}^1 \to \mathbf{P}^n, \phi(x_0, x_1) = (\phi^0(x_0, x_1), \dots, \phi^n(x_0, x_1))
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This gives the gauge theory/string theory dictionary: gauge  $instantons \leftrightarrow worldsheet instantons$ 

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 $\mathsf{P}^1\times\mathsf{P}^1$ 

Now consider  $G=U(1)^2,$  charged chirals  $\Phi^1,\ldots,\Phi^4$ • Charge matrix

$$
Q=\left(\begin{array}{rrrr}1&1&0&0\\0&0&1&1\end{array}\right)
$$

$$
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$$

$$
|\Phi^1|^2+|\Phi^2|^2=r_1,\qquad |\Phi^3|^2+|\Phi^4|^2=r_2
$$

- After gauge equivalence, this is  $\mathsf{P}^1 \times \mathsf{P}^1$
- **In general,**  $M_{\text{vac}}$  **is a toric variety**

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#### The (0, 2) Deformation

- For the rest of this talk, we specialize to the case where the Υ*<sup>i</sup>* are in one to one correspondence with the Φ *i* , same charges, i.e. *E* is a deformation of the tangent bundle
- Convenient to let *W* be 2-dimensional vector space containing the lattice of charges (in  $P<sup>1</sup> \times P<sup>1</sup>$  case)
- *W* also generates the ring of operators (arising in the gauge sector)
- Since <sup>γ*i*</sup> and *E<sup>i</sup>*(Φ) have the same charges, we must have for  $a_{ii}$ ,  $\tilde{a}_{ii} \in W$  $E^1(\Phi) = a_{11}\Phi^1 + a_{12}\Phi^2$ ,  $E^2(\Phi) = a_{21}\Phi^1 + a_{22}\Phi^2$ ,  $E^3(\Phi) = \tilde{a}_{11} \Phi^3 + \tilde{a}_{12} \Phi^4$ ,  $E^4(\Phi) = \tilde{a}_{21} \Phi^1 + \tilde{a}_{22} \Phi^4$

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#### Bundle *E*.

- The bundle  $E$  on  $\mathsf{P}^1 \times \mathsf{P}^1$  determined by the  $\Upsilon^i$  is the quotient of  $\mathcal{O}_{\mathbf{P}^1\times\mathbf{P}^1}(1,0)^2\oplus\mathcal{O}_{\mathbf{P}^1\times\mathbf{P}^1}(0,1)^2$  by the subbundle spanned by the two-dimensional space of sections  $(E^1(\phi),\ldots,E^4(\phi))$  of  $\mathcal{O}_{\mathbf{P}^1\times\mathbf{P}^1}(1,0)^2\oplus\mathcal{O}_{\mathbf{P}^1\times\mathbf{P}^1}(0,1)^2$
- In the language of algebraic geometry, this corresponds to the exact sequence of vector bundles on  $\mathsf{P}^1\times\mathsf{P}^1$

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0 \to \mathcal{O}^{2} \overset{(E^1, \ldots, E^4)}{\to} \mathcal{O}(1,0)^2 \oplus \mathcal{O}(0,1)^2 \to E \to 0
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## Relation to Heterotic String Theory.

#### • Can rewrite by dualizing as

$$
0 \to E^* \to \mathcal{O}(-1,0)^2 \oplus \mathcal{O}(0,-1)^2 \to W \otimes \mathcal{O} \to 0
$$

- Since  $H^1({\cal O}(-1,0))=H^1({\cal O}(0,-1))=0$ , we conclude  $W \simeq H^1(E^*)$
- $H^1(E^*)$  parametrize the fermion vertex operators in the heterotic string
- This completes the GLSM heterotic dictionary

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[Background](#page-2-0)

(0, 2) [Gauged Linear Sigma Model](#page-22-0)

(0, 2) [Quantum Cohomology](#page-65-0)

#### **Outline**



#### 2 (0, 2) [Gauged Linear Sigma Model](#page-22-0)



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 $\bullet$ 

#### • Let *W* be the 2-dimensional space of the  $a_{ii}$ ,  $\tilde{a}_{ii}$

# $\mathcal{A} = \left( \begin{array}{cc} a_{11} & a_{21} \ a_{21} & a_{22} \end{array} \right), \qquad \tilde{\mathcal{A}} = \left( \begin{array}{cc} \tilde{a}_{11} & \tilde{a}_{21} \ \tilde{a}_{21} & \tilde{a}_{22} \end{array} \right),$

Sheldon Katz (0, 2) [Quantum Cohomology](#page-0-0)

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Sheldon Katz (0, 2) [Quantum Cohomology](#page-0-0)

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# **Couplings**

#### • Let *W* be generated by  $\psi, \tilde{\psi}$

- $P(\psi, \tilde{\psi})$  any operator
- $\langle P \rangle = \sum_{d, \tilde{d}} \langle P \rangle_{d, \tilde{d}} q^d \tilde{q}^{\tilde{d}}$  where  $q, \tilde{q}$  depend on FI parameters
- Analogous to instanton sum in string theory
- $\bullet$  Let *Q* = det(*A*),  $\tilde{Q}$  = det( $\tilde{A}$ ) ∈ Sym<sup>2</sup>*W*
- $\bullet$  Then the  $(0, 2)$  quantum cohomology algebra is

$$
Q^2 = q, \qquad \tilde{Q}^2 = \tilde{q}
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## Explanation.

 $\bullet$ 

Can compute  $\left\langle P\right\rangle _{d,\widetilde{d}}$  by algebraic geometry

$$
\otimes^k H^1(E^*) \to H^k(\Lambda^k E^*) \simeq H^k(\Omega^k_{M_{d,\tilde{d}}}) \simeq \mathbf{C}
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$$
k = 2d + 2\tilde{d} + 2, M_{d,\tilde{d}} = \mathbf{P}^{2d+1} \times \mathbf{P}^{2\tilde{d}+1}
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- Normalization is nontrivial, but much easier to prove identities between correlation functions directly
- Quantum cohomology relations follow from identities

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\left\langle PQ \right\rangle_{d+1, \tilde{d}} = \left\langle P \right\rangle_{d, \tilde{d}} = \left\langle P \tilde{Q} \right\rangle_{d, \tilde{d}+1}
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\left\langle PQ \right\rangle_{d+1, \tilde{d}} = \left\langle P \right\rangle_{d, \tilde{d}} = \left\langle P \tilde{Q} \right\rangle_{d, \tilde{d}+1}
$$

• Proven directly by geometry

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## Explanation.

 $\bullet$ 

Can compute  $\left\langle P\right\rangle _{d,\widetilde{d}}$  by algebraic geometry

$$
\otimes^k H^1(E^*) \to H^k(\Lambda^k E^*) \simeq H^k(\Omega^k_{M_{d,\tilde{d}}}) \simeq \mathbf{C}
$$

$$
k = 2d + 2\tilde{d} + 2, M_{d,\tilde{d}} = \mathbf{P}^{2d+1} \times \mathbf{P}^{2\tilde{d}+1}
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- Normalization is nontrivial, but much easier to prove identities between correlation functions directly
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## Relations.

#### *Generalized Koszul complex* of

$$
0 \to E^* \to Z = \mathcal{O}(-1,0)^2 \oplus \mathcal{O}(0,-1)^2 \to W \otimes \mathcal{O} \to 0
$$
 is

$$
0\to\Lambda^2 E^*\to\Lambda^2 Z\to W\otimes Z\to\text{Sym}^2 W\otimes\mathcal{O}\to 0
$$

$$
\bullet \ \mathcal{O}(-2,0) \subset \Lambda^2 Z \text{ and } H^1(\mathcal{O}(-2,0)) \neq 0
$$

- This cohomology class produces a classical cohomology relation  $\langle Q \rangle_{0.0} = 0$
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## Gauge Instanton Moduli Space.

- In the  $(d, \tilde{d})$  instanton background, the zero modes of  $\Phi^1$ and Φ <sup>2</sup> are polynomials *f* <sup>1</sup>,<sup>2</sup> of degree *d* in the homogeneous coordinates  $(x_0, x_1)$  on the worldsheet
- Similarly, the zero modes of  $\Phi^3$  and  $\Phi^4$  are polynomials  $f^{3,4}$ of degree  $\tilde{q}$  in the homogeneous coordinates  $(x_0, x_1)$  on the worldsheet
- The FI constraints prevent either  $(f<sup>1</sup>, f<sup>2</sup>)$  or  $(f<sup>3</sup>, f<sup>4</sup>)$  from being identically zero.
- Since  $(f<sup>1</sup>, f<sup>2</sup>)$  has 2( $d + 1$ ) parameters and  $(f<sup>3</sup>, f<sup>4</sup>)$  has  $2(\tilde{d} + 1)$  parameters, after imposing the FI constraint and gauge equivalence, the moduli space is  $\mathbf{P}^{2d+1}\times\mathbf{P}^{2\widetilde{d}+1}$
- In general, this is also a toric variety, so the same techniques apply as in the classical (from the viewpoint of geometry) case! 4 ロ ) (何 ) (日 ) (日 )

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• In an instanton background, we get  $Q^{d+1} = 0$  (from  $H^{2d+1}(\mathbf{P}^{2d+1} \times \mathbf{P}^{2d+1}, \mathcal{O}(-2d-2, 0)) = 0$  and  $\tilde{Q}^{d+1} = 0$ .

**•** This is consistent with

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#### General Case.

#### The general case relies on toric geometry rather than the geometry of projective space

- One new ingredient in general: if a field lives in a bundle with negative chern class in an instanton background, there are no zero modes. To compensate, new terms enter into the calculation from the four-fermi terms in the action
- These turn out to be powers of the analogues of the same *Q*, *Q* in the general case.

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### A little toric geometry

 $\bullet$ 

#### $\bullet$  In general,  $M_{\text{vac}}$  is a toric variety.

- To each field  $\Phi^i,$  we associate the divisor  $D_i\subset M_{\text{vac}}$  defined by  $\phi^i=0$
- $\bullet$  Have line bundles  $\mathcal{O}(D_i)$  associated with these divisors by general algebraic geometry

 $0 \to E^* \otimes \mathcal{O} \to \oplus_i \mathcal{O}(-D_i) \stackrel{E^a}{\to} \mathcal{W} \otimes \mathcal{O} \to 0$ 

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- Now group together all Φ *i* (hence Υ*<sup>i</sup>* ) whose associated bundles  $O(D_i)$  are isomorphic (or in physical terms, whose charges are equal)
- These *E <sup>i</sup>* can be expressed as *W*-valued linear expressions in these Φ, plus higher degree polynomials which we show can be ignored
- Labelling each collection by an index *c*, these expressions for the *E <sup>i</sup>* can be arranged into a *W*-valued square matrix *Ac*
- Put  $Q_c = det(A_c)$

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### The Result.

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$$
\prod_{c\in [K]}Q_c=q_K^\beta\prod_{c\in [K^-]}Q_c^{-d_c^{\beta_K}}
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#### where the notation will not be explained in a 30 minute talk!

- This form is precisely the form conjectured by McOrist and  $\bullet$ Melnikov
- We prove that the result is true in general, independent of any nonlinear deformations

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### Four-Fermi Terms

- Suppose Φ *i* lives in a bundle with negative chern class *d<sup>i</sup>* in an instanton background  $\beta$
- Then any Φ *j* in the same class *c* lives in the same bundle
- The four-fermi terms are  $Q_c^{-d_c-1}$
- In GLSM, four-fermi terms are generated by Yukawa couplings

$$
\sigma^a \psi^i \bar{\psi}_i, \qquad \sigma^a \lambda^i \bar{\lambda}_i
$$

- $\dim H^1(O_{\mathbf{P}^1}(d_c)) = -d_c 1$
- This is the analogue of the virtual fundamental class of Gromov-Witten theory

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[Background](#page-2-0) (0, 2) [Gauged Linear Sigma Model](#page-22-0) (0, 2) [Quantum Cohomology](#page-65-0)

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