Critical current via gauge/gravity duality*

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March 3, 2012

*arXiv:1201.5103

Motivation

- Computation of properties of SN transition at a critical (depairing) current is a classic (and surprisingly difficult) problem in theory of superconductivity.
- 1D superconductor is supposed to be the simplest: the current is uniform over the wire's cross section. The wire is still multichannel, though:

  $$N_{\mathrm{ch}} \sim k_F^2 A \gg 1$$

  It is important to know if the transition is 1st or 2nd order (i.e., if superconductivity can be "softened" by application of current).
- The difficulty lies in incorporating inter-channel scattering necessary to define a current-carrying *normal* state. Gauge/gravity duality to the rescue.

Application of gauge/gravity duality

- Consider a large N $SU(N)$ gauge theory and identify N with $N_{\rm ch}$. Consider N species of electrons in (1+1) dimensions. For each, define a Dirac spinor ($\gamma_0 = \sigma^1$)

$$\psi = \left[\begin{array}{c} \sum_{k>0}(a_{Rk}e^{ikx} + b^\dagger_{Rk}e^{-ikx}) \\ \sum_{k<0}(a_{Lk}e^{ikx} + b^\dagger_{Lk}e^{-ikx}) \end{array} \right]$$

  The superconducting channel is $\sum_{A=1}^{N} \bar{\psi}_A \psi_A$.

- Gravitational description: N D3 branes + single (probe) D5 intersecting over a line:

	0	1	2	3	4	5	6	7	8	9
$D3$:	$\times$	$\times$	$\times$	$\times$						
$D5$:	$\times$	$\times$			$\times$	$\times$	$\times$	$\times$		

  Note: electrons are (1+1) but the gauge field is (3+1). Compare: D3/D7 intersecting over a plane (graphene/QHE) [Rey 07; Davis, Kraus and Shah 08].

D5 action

- Need the full D3 metric ("throat"), including the AF region:

$$ds^2 = \frac{1}{\sqrt{f}} dx^\mu dx_\mu + \sqrt{f}\left(d\Delta^2 + \Delta^2 d\phi^2 + d\rho^2 + \rho^2 d\Omega_3^2\right)$$

with $f = 1 + \frac{R^4}{(\Delta^2+\rho^2)^2}$. Set $R = 1$. "Order parameter": $\Delta e^{i\phi}$.
- First look at these static embeddings: $\Delta = \Delta(\rho)$, $\phi = 0$, $A_t = A_t(\rho)$. The DBI action is $S_{\rm DBI} = -\int dt \mathcal{F}$,

$$\mathcal{F} = 2\pi^2 T_5 \int dx d\rho \rho^3 \sqrt{f} (1 + \Delta_{,\rho}^2 - F_{t\rho}^2)^{1/2} \,,$$

where $F_{t\rho} = -\partial_\rho A_t$. Note: A_t couples to $\psi^\dagger \psi$, i.e., to the electric current

$$J = \frac{1}{2\pi^2 T_5} \frac{\delta \mathcal{F}}{\delta F_{t\rho}} \,.$$

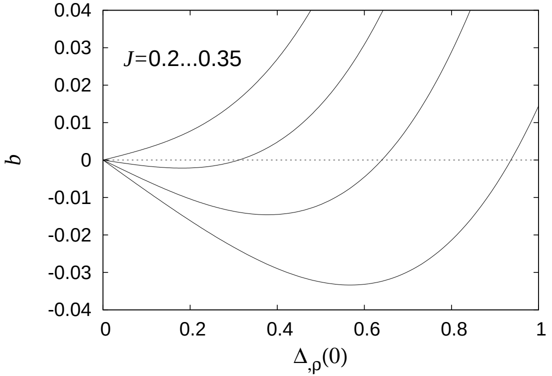
- Legendre transform:

$$\widehat{\mathcal{F}} = 2\pi^2 T_5 \int dx d\rho (\rho^6 f + J^2)^{1/2} (1 + \Delta_{,\rho}^2)^{1/2}$$

Describes physics at fixed J.

Properties of solutions

- $F_{t\rho}$ has nowhere on the brane to end, hence $\Delta(\rho \to 0) = 0$: the superconductor is gapless.
- $\Delta(\rho)$ goes to a constant at large ρ: $\Delta(\rho \to \infty) = b$.
- Equation-of-state curve: b as a function of $\Delta_{,\rho}(0)$ (one for each J). A nontrivial zero of such a curve = spontaneous breaking of $\phi \to \phi +$ const symmetry. Merging of zeroes = continuous phase transition (here at $J = J_c = 0.3197$).



Linear stability analysis

- Linearize near $\Delta = 0$ but include t and x dependence: $\Delta \sim e^{-i\omega t + ikx}\Delta^{(n)}(\rho)$. Frequency of the unstable mode:

$$\mathrm{Im}\omega = \Omega_0(k, J) \approx a(J_c - J) - a' k^2 \,,$$

  for J near J_c and small k^2. $a, a' > 0$.
- All the scaling exponents here are Gaussian, but that's the ultraviolet behavior, reflecting the suppression of interactions among the collective modes by the large N (i.e., the thickness of the wire). To find the infrared scaling, we construct an effective field theory near $J = J_c$.

Effective field theory near $J = J_c$

- Substitute the single-mode approximation

$$\Delta e^{i\phi}(x, \rho; J) = \int dk e^{ikx} \Psi(k) \Delta^{(0)}(\rho; k, J)$$

into the DBI action and expand to the fourth order in $\Psi(x)$ (but second in $\Psi_{,x}$). Obtain the GL-type free energy:

$$\mathcal{F}_{\mathrm{GL}} = \pi T_5 \left[-\int dk \Omega_0(k, J) |\Psi(k)|^2 + c \int dx |\Psi(x)|^4 \right]$$

Note: $c > 0$.

- Add the dissipative term ($\Omega=$ Euclidean frequency)

$$S_{\mathrm{dissip}} = \pi T_5 \int dx d\Omega |\Omega| |\Psi(x, \Omega)|^2$$

- The total

$$S_E = \int d\tau \mathcal{F}_{\mathrm{GL}} + S_{\mathrm{dissip}}$$

is the dissipative XY model, known to have a nontrivial infrared fixed point [e.g., Sachdev, Werner and Troyer 04].

Comparison to experiments

- Since in the UV the $|\Psi|^4$ coupling is small, even a small deviation of J from J_c may lock the system in the domain controlled by the Gaussian fixed point. Then, e.g., the free-energy barrier for thermal phase slips scales as

$$\delta\mathcal{F} \sim (\text{free-energy density}) \times \xi \sim (J_c - J)^{3/2}$$

- Experimentally, the scaling exponent for $\delta\mathcal{F}$ has been deduced from statistics of the switching current [Sahu et al. 08, Li et al. 10, Aref et al.] and found to be close to 3/2 for Al and amorphous MoGe wires, albeit closer to 5/4 for crystalline Mo_3Ge.

Conclusions

- As an approach to the depairing transition, the gauge/gravity duality allows one to include effects (such as the electron-electron scattering) that may be difficult to include by other means.
- The transition at fixed current is found to be second-order, in the dissipative XY universality class. The second order is consistent with the experiments on the switching currents.
- The Gaussian UV fixed point controls scaling up to large distances (a consequence of the initial $1/N$ suppression). Materials with large coherence lengths (crystalline?) will probably be needed to observe deviations from the Gaussian scaling.