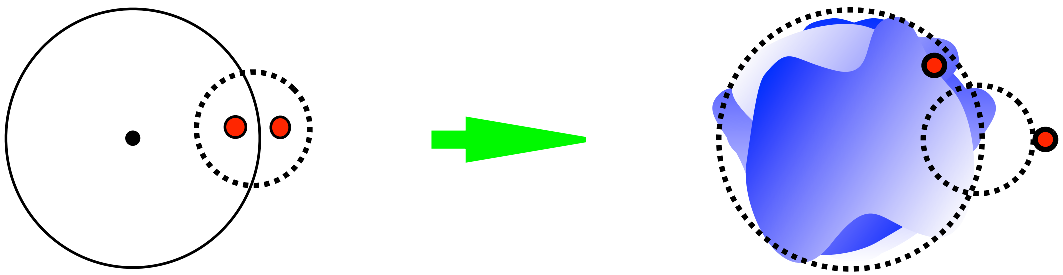
Cosmic acceleration from fuzzball evolution

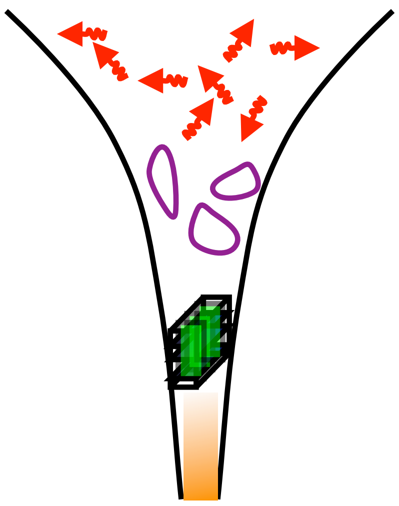
Great Lakes 2012

Outline

(A) Black hole information paradox tells us
something new about quantum gravity

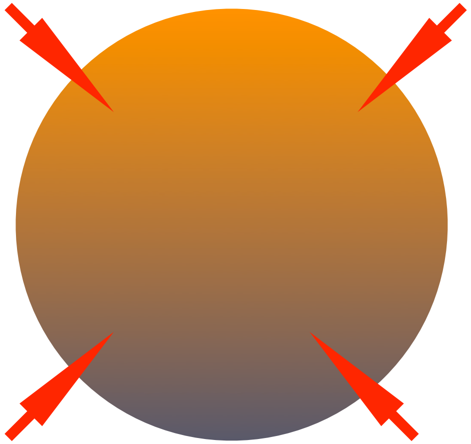


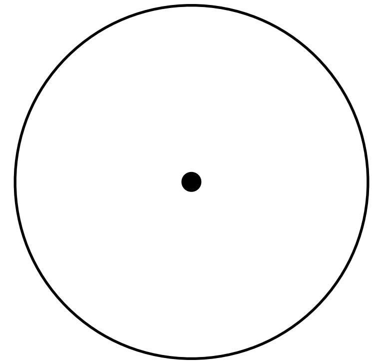
(B) Early Universe had a high density, so these new
quantum gravity effects may be relevant

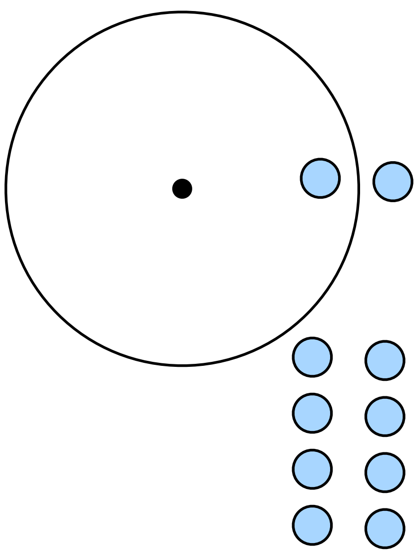


We take each thing we have learnt about black holes
and use it to speculate on what might happen in the
Early Universe

Information paradox: Schwinger process for gravity







$$\Psi_M$$

$$\otimes \quad |0\rangle_1|0\rangle_{1'} + |1\rangle_1|1\rangle_{1'}$$

$$\otimes \quad |0\rangle_2|0\rangle_{2'} + |1\rangle_2|1\rangle_{2'}$$

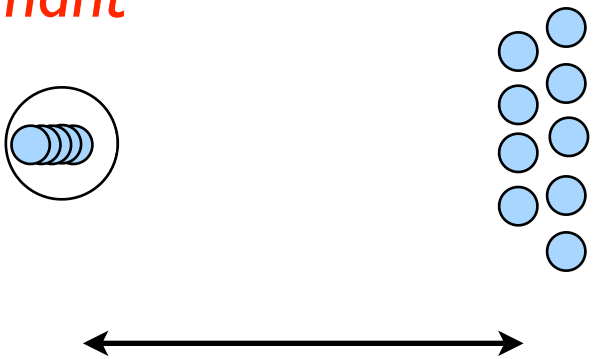
$$\cdots$$

$$\otimes \quad |0\rangle_n|0\rangle_{n'} + |1\rangle_n|1\rangle_{n'}$$

Schwinger process in the gravitational field

Possibilities

Planck mass remnant

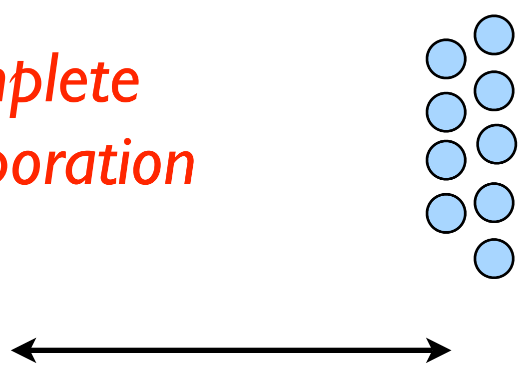


$$S_{ent} = N \ln 2$$

To have this entanglement, the remnant should have at least 2^N internal states

But how can we have an unbounded degeneracy for objects with a given mass ?

Complete evaporation



$$S_{ent} = N \ln 2$$

The radiated quanta are in an entangled state, but there is nothing that they are entangled with !

They cannot be described by any wavefunction, but only by a density matrix

$\longrightarrow$ *failure of quantum mechanics*

Black hole evaporation leads to information loss or remnants

GR person

We cannot image that this is a serious problem

There must be small corrections to Hawking's computation that make the information come out

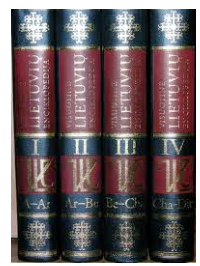
String theorist

So who is right ?

In 2005, Stephen Hawking surrendered his bet to John Preskill, based on such an argument of ‘small corrections’ ...

(Subleading saddle points in a Euclidean path integral give exponentially small corrections to the leading order evaporation process)

Stephen Hawking

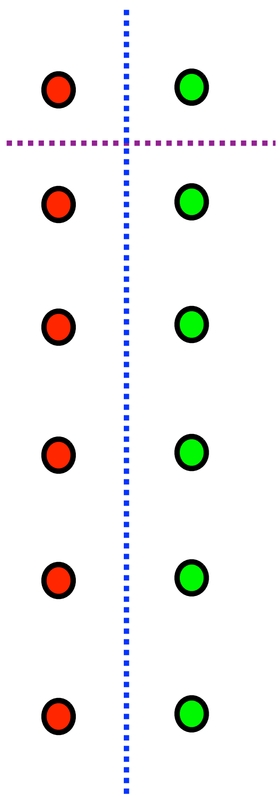


John Preskill

Kip Thorne

But Kip Thorne did not agree to surrender the bet ...

Theorem: Small corrections to Hawking's leading order computation do NOT remove the entanglement

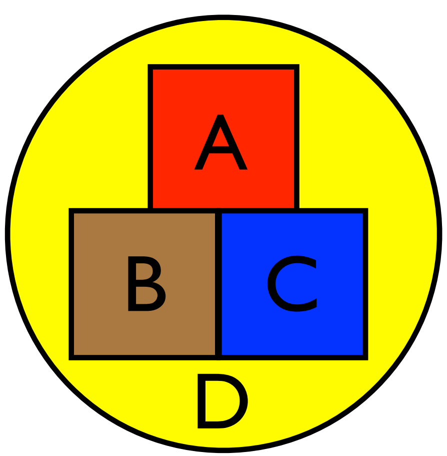


$$\frac{\delta S_{ent}}{S_{ent}} < 2\epsilon$$

(SDM 09)

Bound does not depend on the number of pairs N

Basic tool : Strong Subadditivity (Lieb + Ruskai '73)



$$S(A) = -Tr[\rho_A \ln \rho_A]$$ etc.

$$S(A+B) + S(B+C) \geq S(A) + S(C)$$

So, what is the resolution?

(Avery, Balasubramanian, Bena, Chowdhury, de Boer, Gimon, Giusto, Keski-Vakkuri, Levi, Lunin, Maldacena, Maoz, Park, Peet, Potvin, Ross, Ruef, Saxena, Skenderis, Srivastava, Taylor, Turton, Warner ...)

The ‘no-hair’ theorem tells us that the black hole metric is unique:

$$ds^2 = -(1-\frac{2M}{r})dt^2 + \frac{dr^2}{(1-\frac{2M}{r})} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

But how did we get this metric ?

We take an ansatz where the metric coefficients had no dependence on angular variables or on the compact directions

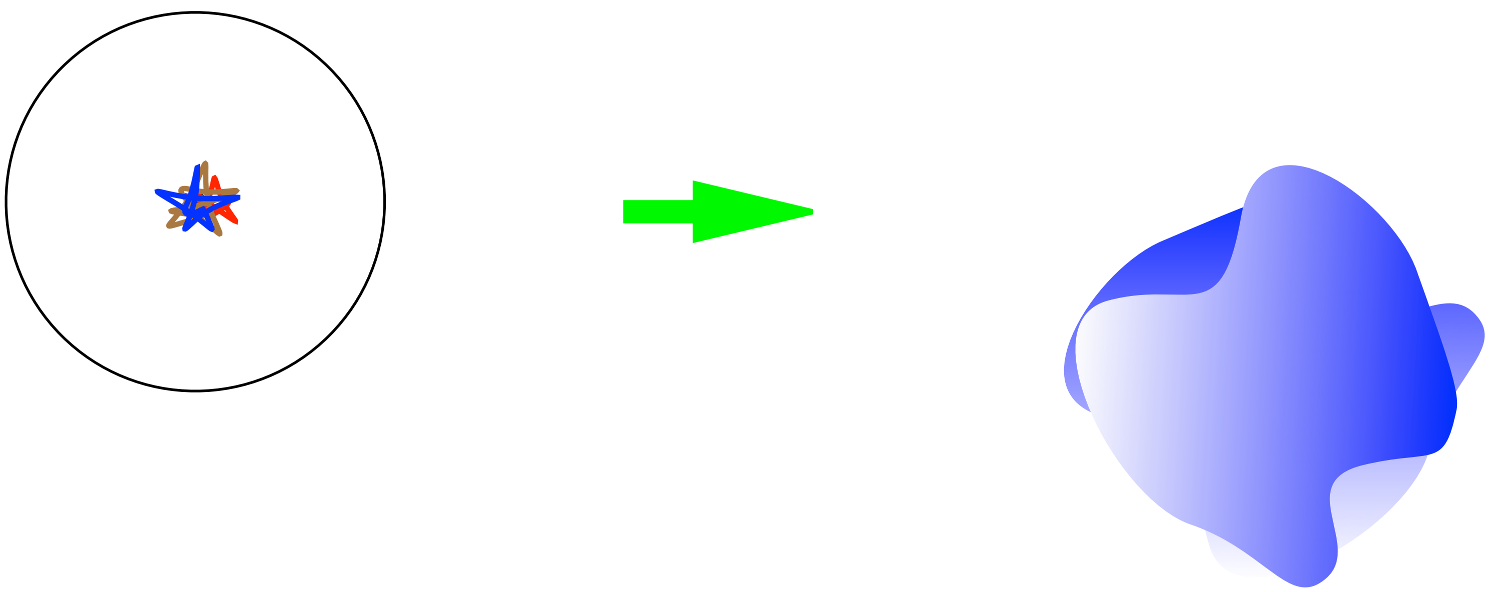
$$ds^2 = -f(r)dt^2 + g(r)dr^2 + r^2 d\Omega_2^2 + dz_i dz_i$$

The solution we get is singular, however, at the origin, so we cannot be sure it is a solution of the full quantum gravity theory

Now let us look for solutions that have no spherical symmetry and the compact directions are also not trivially tensored

Then there are a large number of regular solutions - no horizon and no singularity - with the same M, Q, J as the black hole

→ 'Fuzzballs'

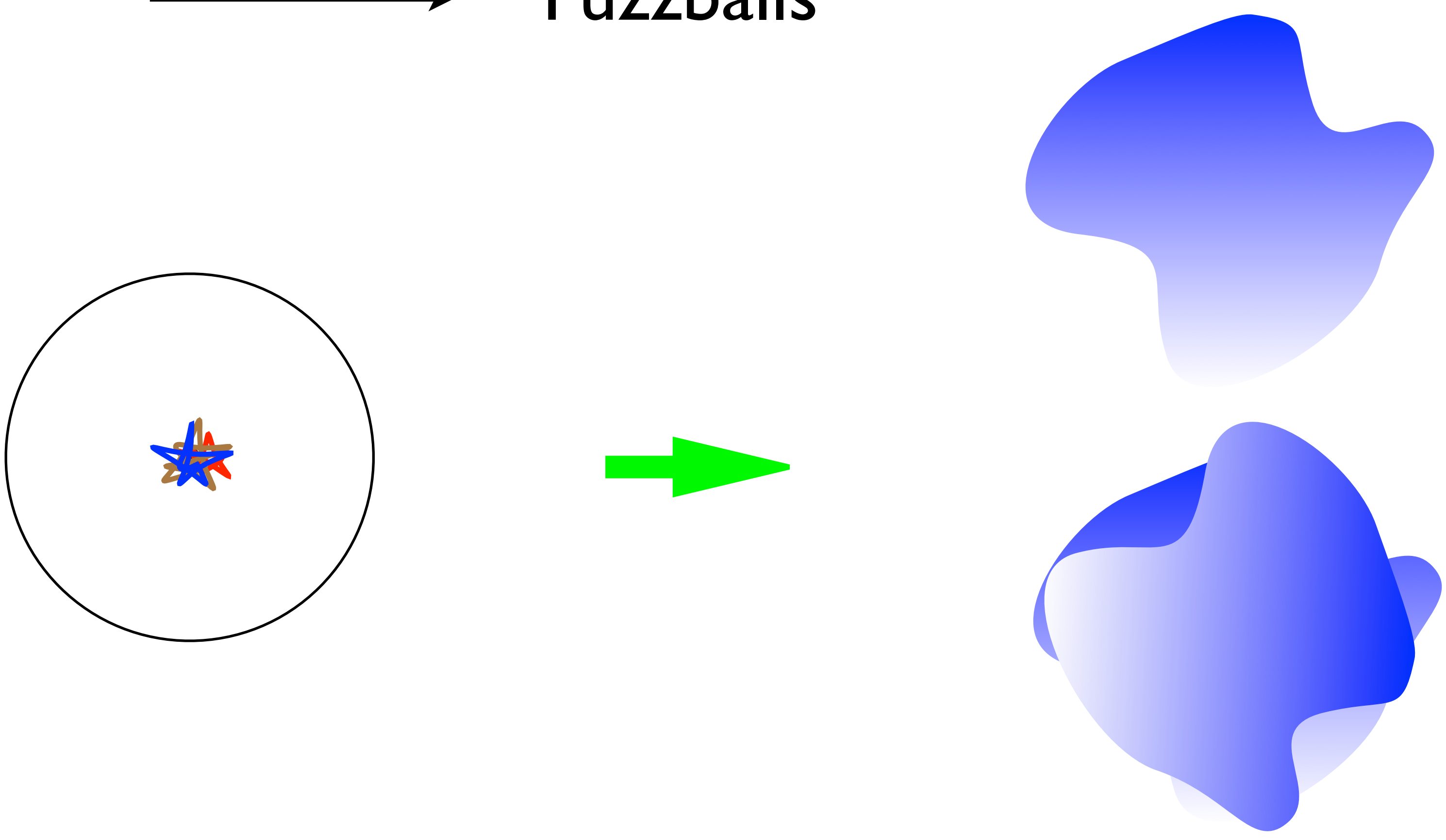


....

Now let us look for solutions that have no spherical symmetry and the compact directions are also not trivially tensored

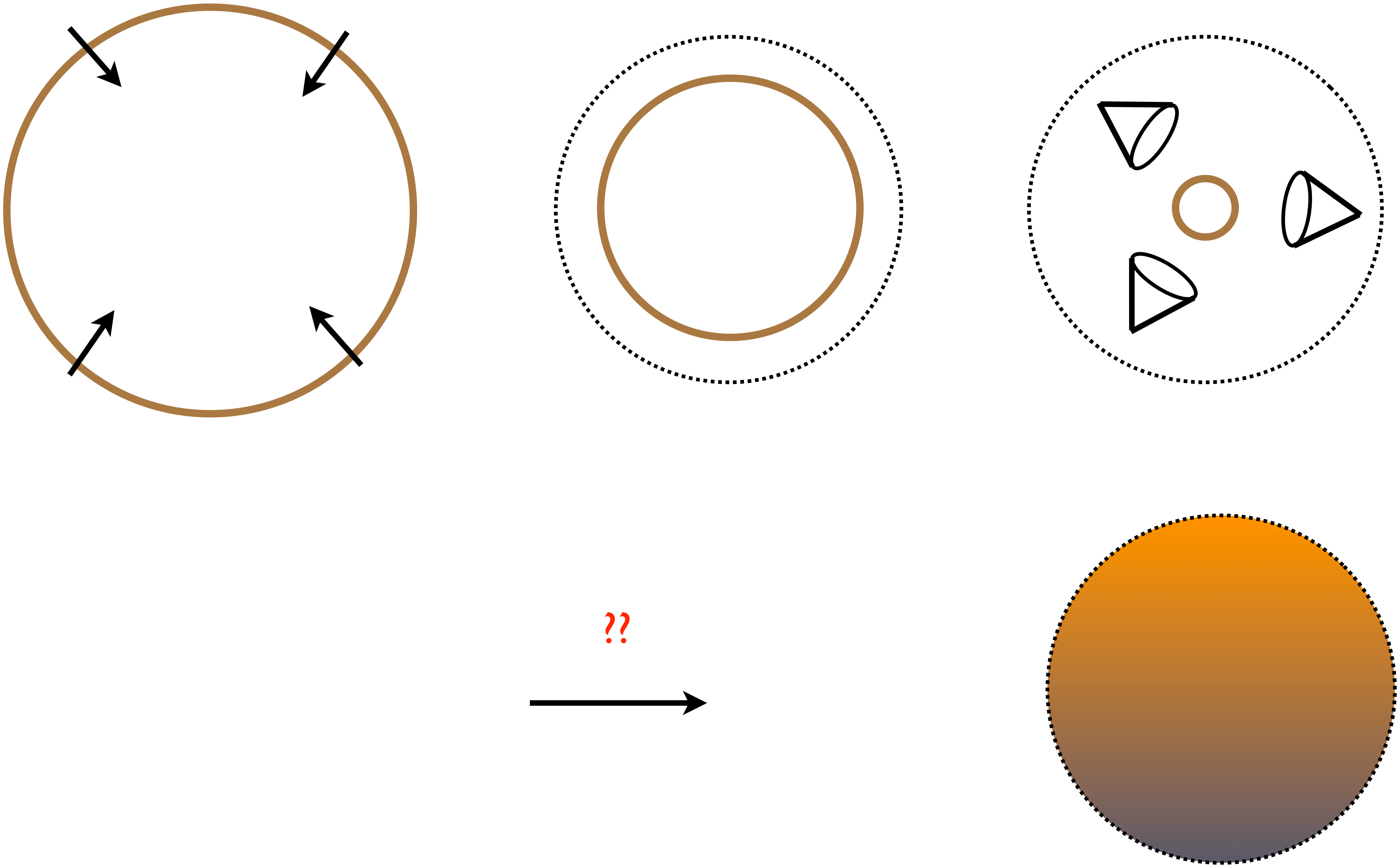
Then there are a large number of regular solutions - no horizon and no singularity - with the same M, Q, J as the black hole

⟶ ‘Fuzzballs’

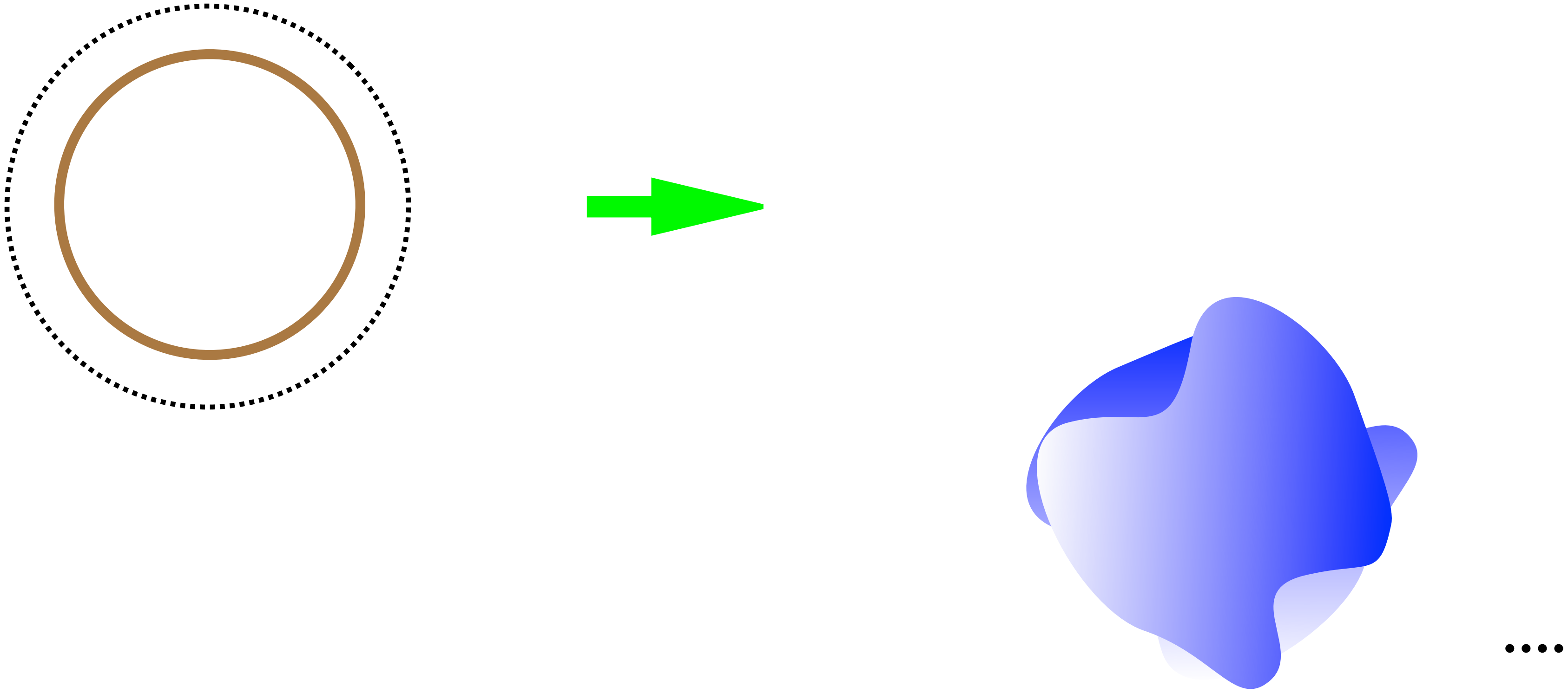


....

How does a collapsing shell become fuzzballs ?

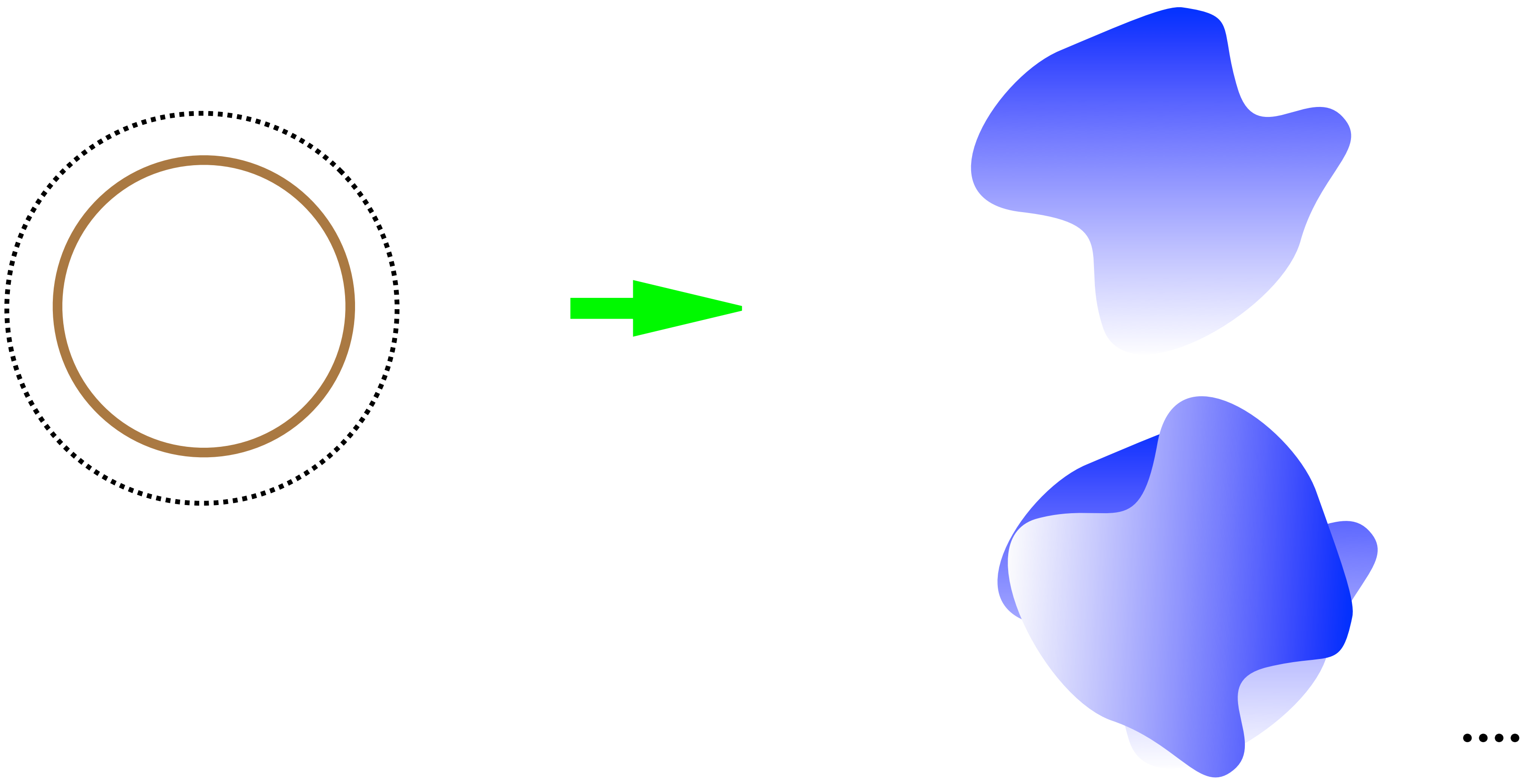


There is a small amplitude for the shell to tunnel into one of the fuzzball solutions ...



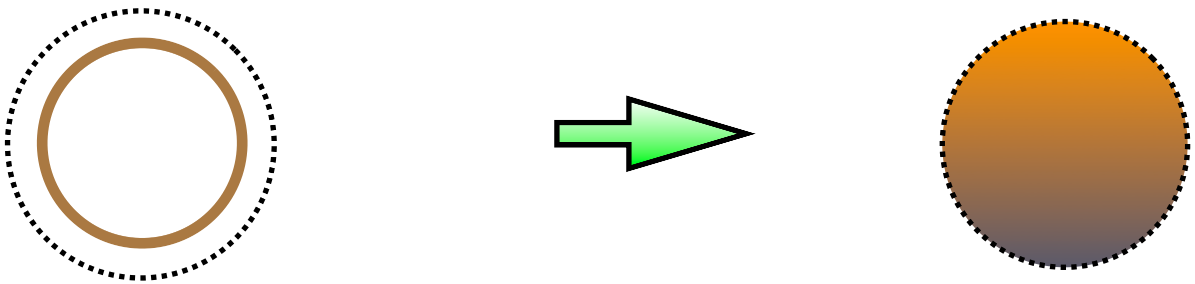
But we must multiply the tunneling probability by the number of solutions we can tunnel to ....

There is a small amplitude for the shell to tunnel into one of the fuzzball solutions ...



But we must multiply the tunneling probability by the number of solutions we can tunnel to ....

Consider the amplitude for the shell to tunnel to a fuzzball state



$$S_{tunnel} \sim \frac{1}{G}\int R d^4x \sim \frac{1}{G}\frac{1}{(GM)^2}(GM)^4 \sim GM^2$$

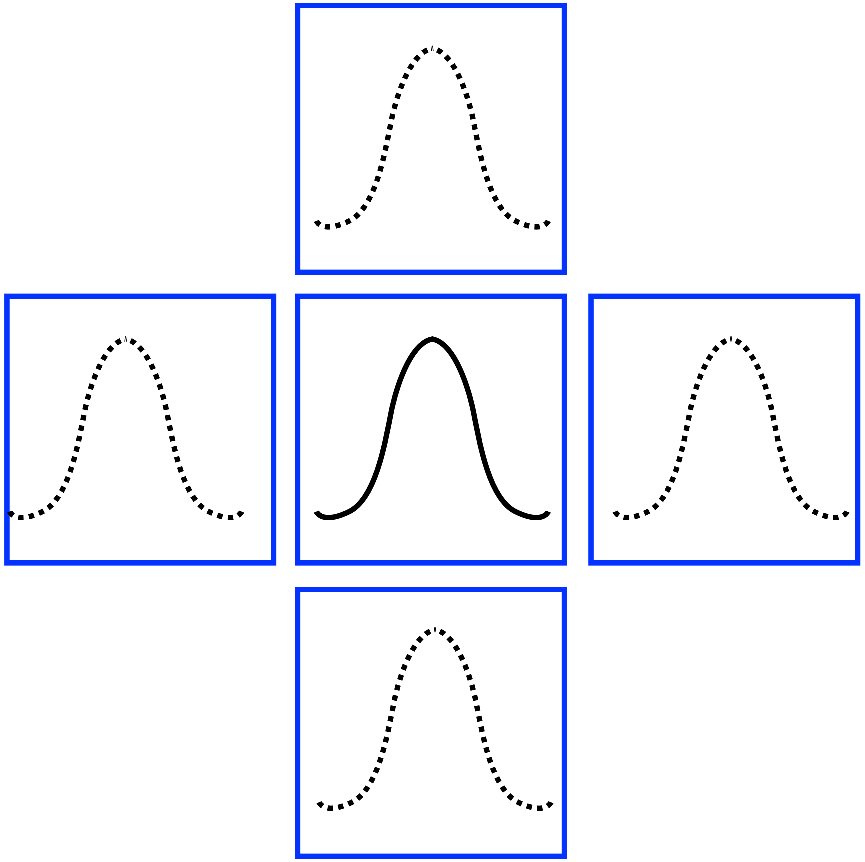
$$\mathcal{A} \sim e^{-S_{tunnel}}$$

Amplitude to tunnel is very small

$$\mathcal{N} \sim e^{S_{bek}} \sim e^{GM^2}$$

But the number of states that one can tunnel to is very large !

Toy model: Small amplitude to tunnel to a neighboring well, but there are a correspondingly large number of adjacent wells



In a time of order unity, the wavefunction in the central well becomes a linear combination of states in all wells (SDM 07)

Summary:

$$Z = \int D[g] e^{-\frac{1}{\hbar} S[g]}$$

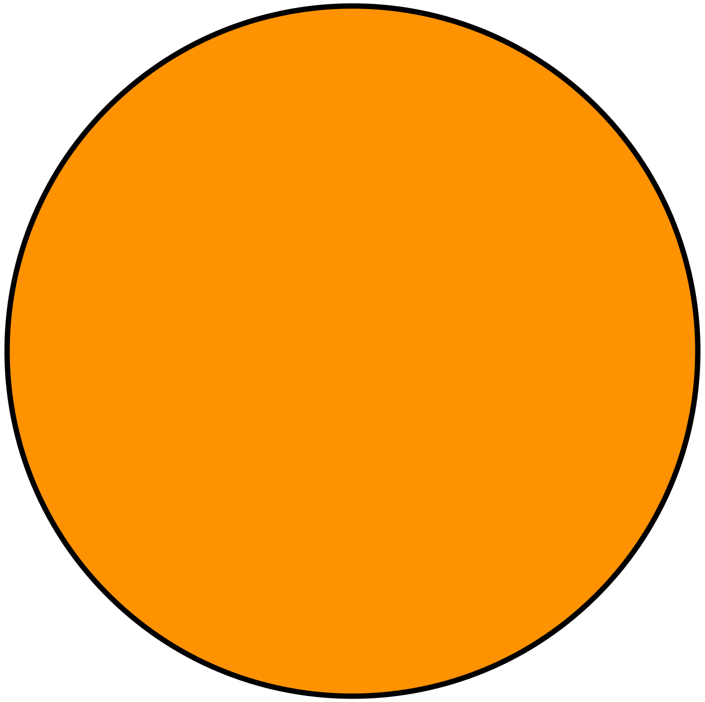
Path integral

Measure has degeneracy of states

Action determines classical trajectory

For traditional macroscopic objects the measure is order $\hbar$ while the action is order unity

But for black holes the entropy is so large that the two are comparable ...



Summary:

$$Z = \int D[g] e^{-\frac{1}{\hbar} S[g]}$$

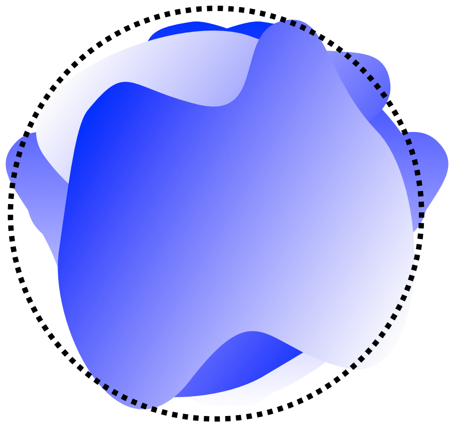
Path integral

Measure has degeneracy of states

Action determines classical trajectory

For traditional macroscopic objects the measure is order $\hbar$ while the action is order unity

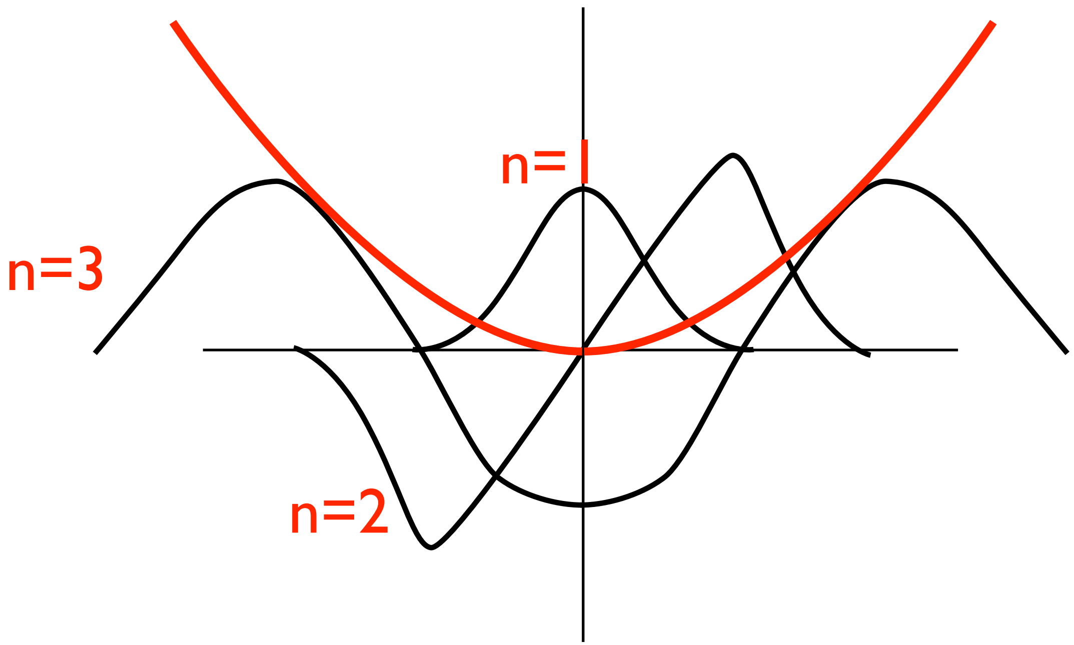
But for black holes the entropy is so large that the two are comparable ...



Why do black hole states become fuzzballs ?

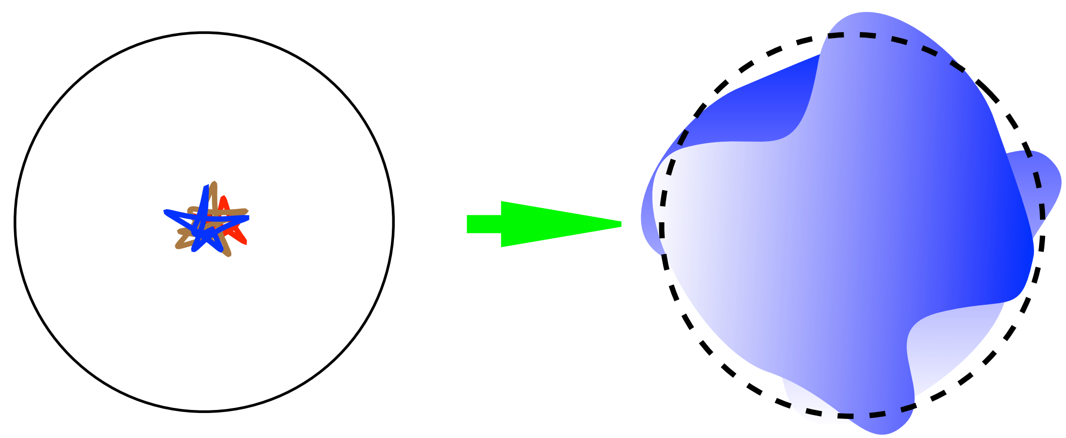
There are Exp[S] states

Their wavefunctions must be orthogonal to each other



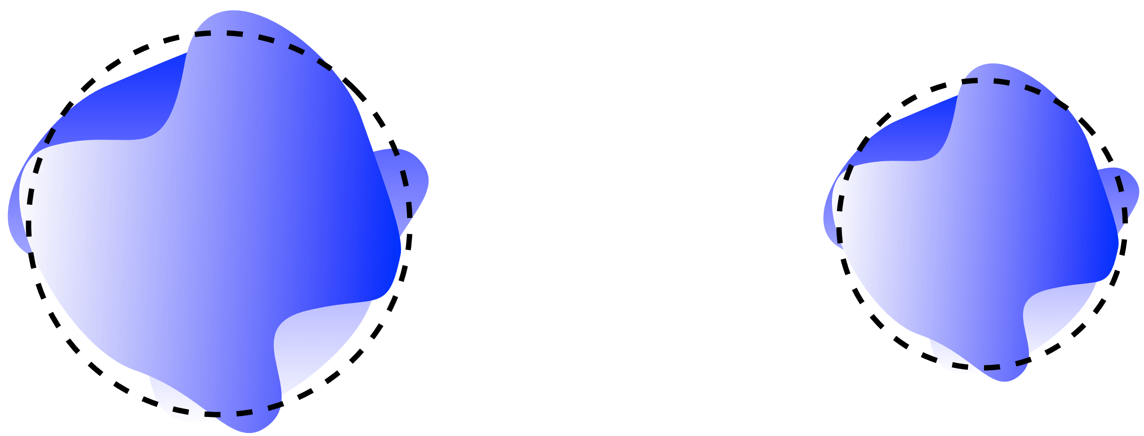
Harmonic oscillator wavefunctions

It needs a sizeble region to hold Exp[S] orthogonal states → horizon radius



A further result for 2-charge states

If we take a region smaller than the black hole size, less states fit in



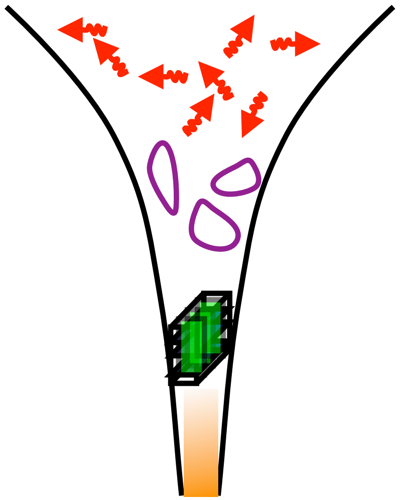
$$S \sim \frac{A}{G} \qquad\qquad S' \sim \frac{A'}{G}$$

Smaller volume, less orthogonal states, entropy still satisfies Bekenstein law

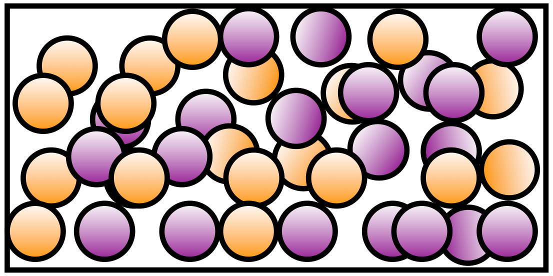
(SDM 07)

Early Universe

(a) Lots of matter was crushed into a small volume, so physics looks similar to black hole physics



(b) Put energy E in volume V

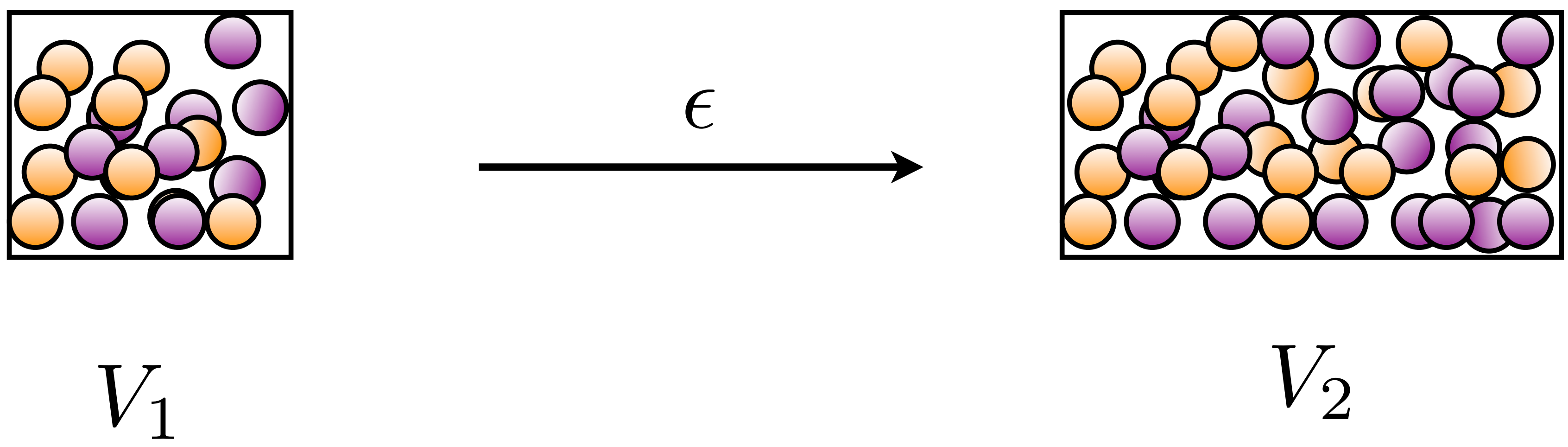


Question: How many states can fit in ?

V small, less states

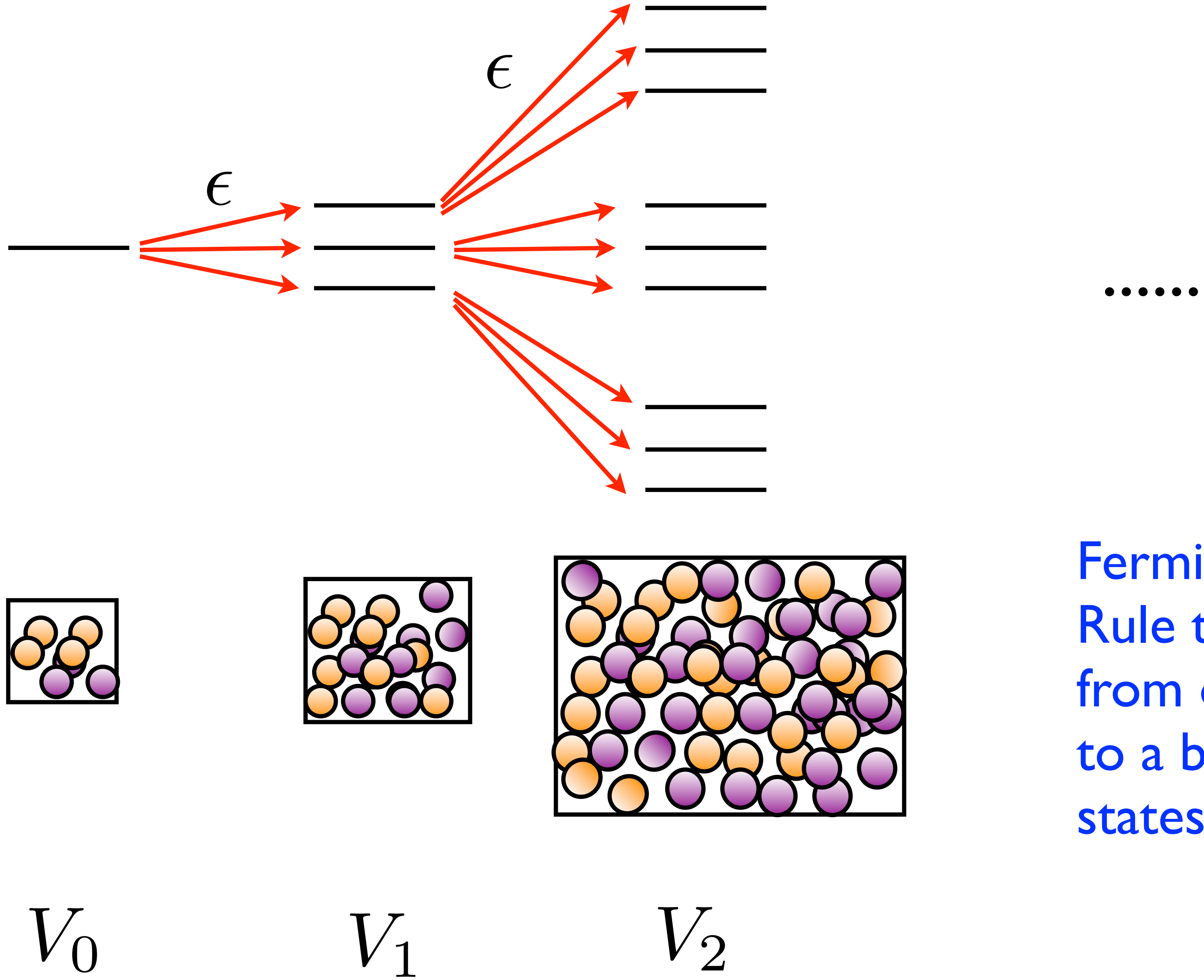
V large, more states

Suppose we start with small volume



Let there be a small amplitude for transition to a state in the larger volume

Here is the nature of the quantum mechanical problem ....



Fermi Golden Rule transitions from one state to a band of states

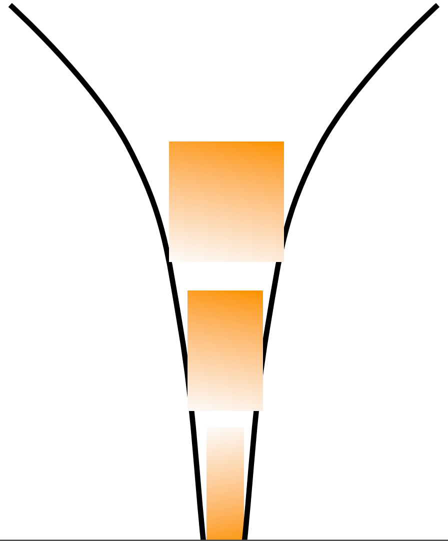
Larger phase space at larger volumes makes the system drift towards larger volumes

This would happen in any system, but to be significant the phase space has to be large

At high densities, we expect

Entropy = Log (phase space volume) ~ Bekenstein law

With such large phase space volumes, the phase space effect competes with the classical Einstein action ...



We get an extra ‘push’ towards larger volumes

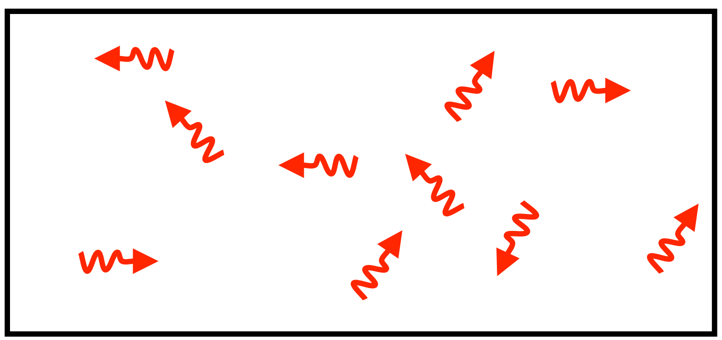
→ Inflation in the Early Universe ?

Cosmology

Start with a box of volume V

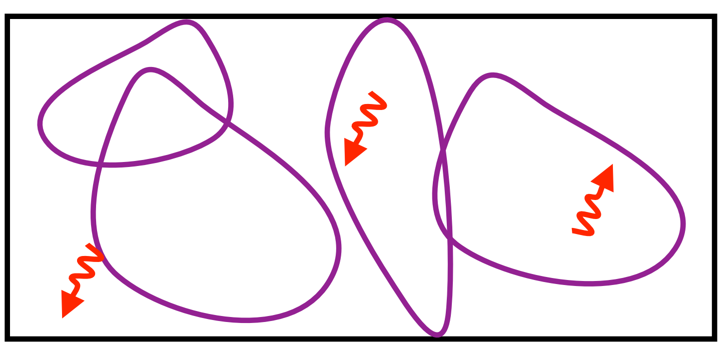
In the box put energy E

Question: What is the state of maximal entropy S, and how much is S(E) ?



Radiation

$$S \sim E^{\frac{D-1}{D}}$$



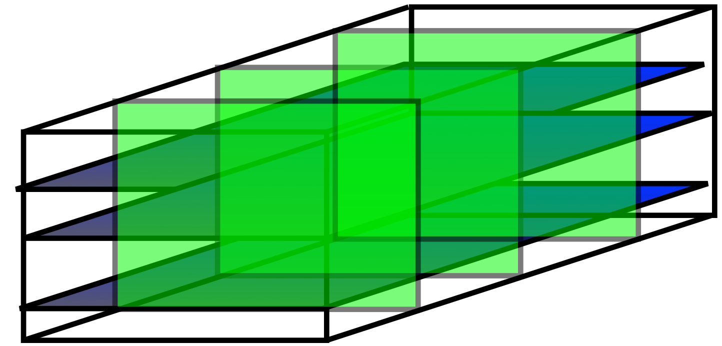
String gas 'Hagedorn phase'

$$S \sim E \sim \sqrt{E}\sqrt{E}$$

(Brandenberger+Vafa)

Black holes:

3-charges: $S \sim \sqrt{n_1 n_2 n_3} \sim E^{\frac{3}{2}}$



Entropy comes from different ways to group the

$n_1 n_2 n_3$ intersection points

The 4-charge extremal case works the same way (3+1 d black holes)

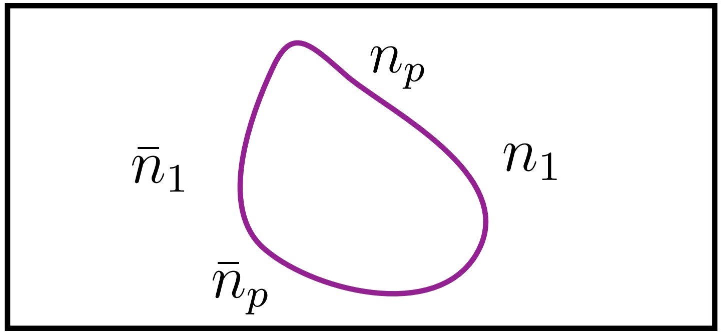
$$S_{micro} = 2\pi\sqrt{n_1 n_2 n_3 n_4} = S_{bek}$$

Charges can be taken as
D1 D5 P KK, or D3 D3 D3 D3

Note that $S \sim \sqrt{n_1 n_2 n_3 n_4} \sim E^2$

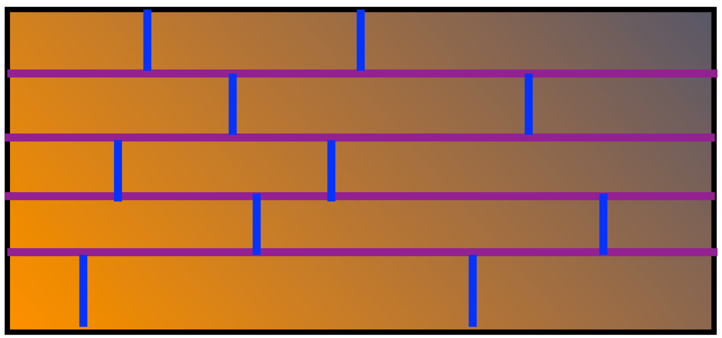
(Johnson,Khuri,Myers 96,
Horowitz,Lowe,Maldacena 96)

Neutral holes ⟶ Cosmology



Two charges

$$S = 2\pi\sqrt{2}(\sqrt{n_1} + \sqrt{\bar{n}_1})(\sqrt{n_p} + \sqrt{\bar{n}_p}) \sim \sqrt{E}\sqrt{E} \sim E$$



Three charges

$$S = 2\pi(\sqrt{n_1} + \sqrt{\bar{n}_1})(\sqrt{n_5} + \sqrt{\bar{n}_5})(\sqrt{n_p} + \sqrt{\bar{n}_p}) \sim E^{\frac{3}{2}}$$

(Horowitz+Maldacena+Strominger 96)

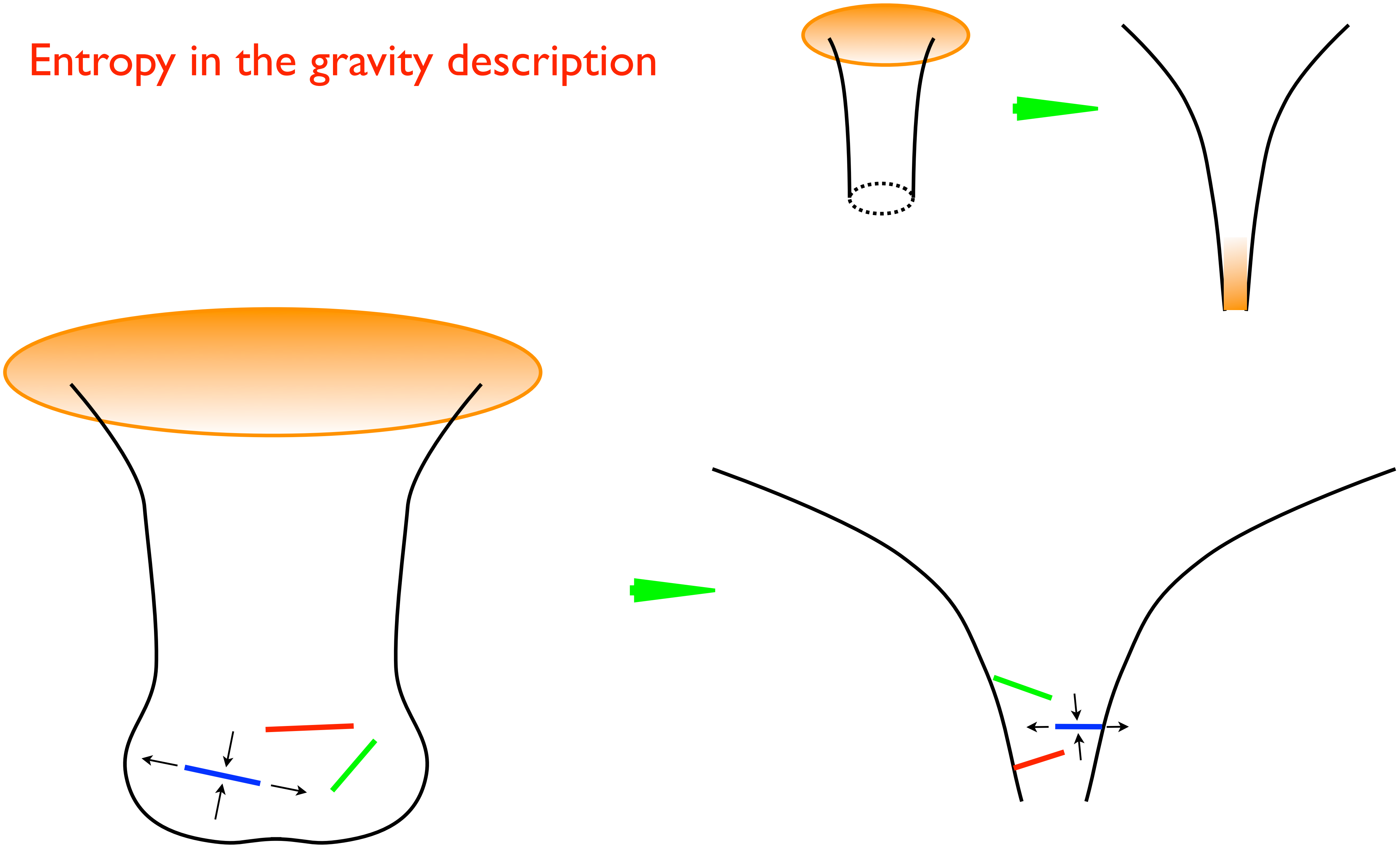
Four charges $S = 2\pi(\sqrt{n_1} + \sqrt{\bar{n}_1})(\sqrt{n_2} + \sqrt{\bar{n}_2})(\sqrt{n_3} + \sqrt{\bar{n}_3})(\sqrt{n_4} + \sqrt{\bar{n}_4}) \sim E^2$

(Horowitz+Lowe+Maldacena 96)

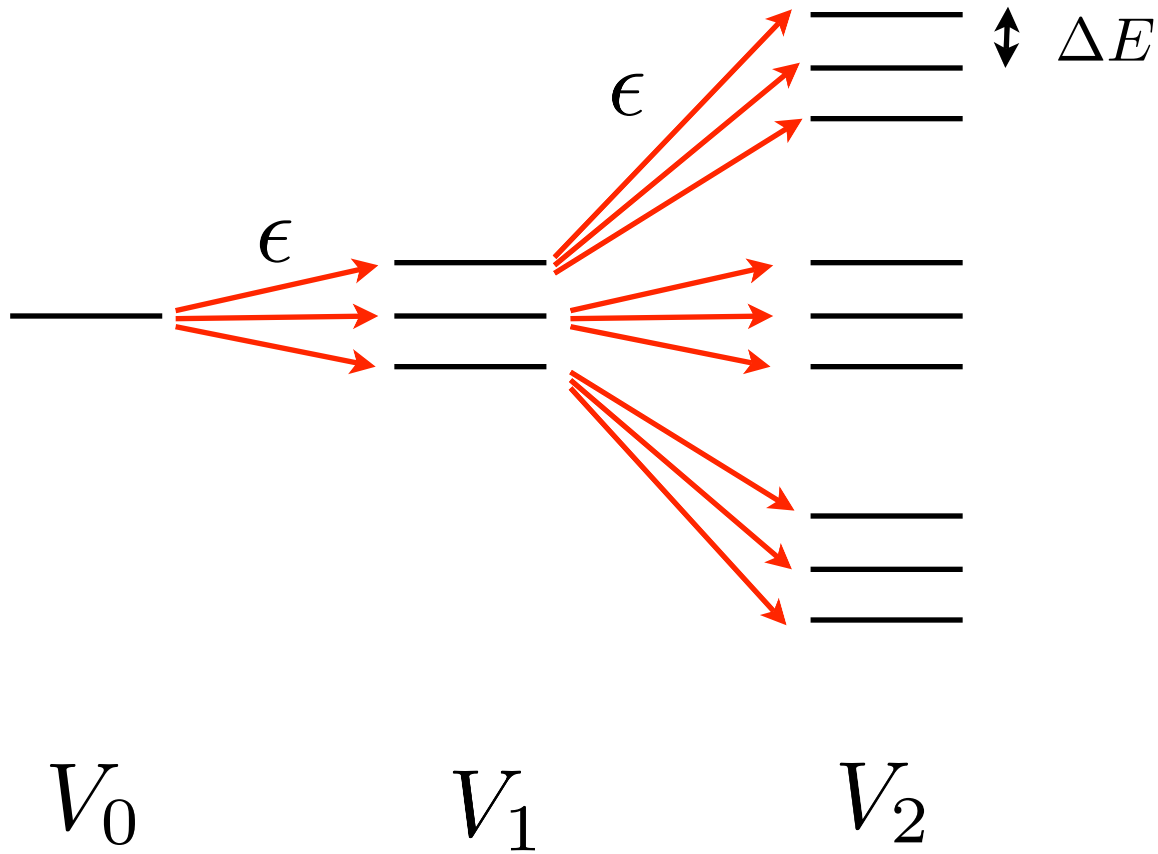
N charges, postulate .....

$$S = A_N \prod_{i=1}^{N} (\sqrt{n_i} + \sqrt{\bar{n}_i}) \sim E^{\frac{N}{2}}$$

Entropy in the gravity description



Fuzzball solutions are similar to the BKL singularity



Expansion due to phase space effects

Probability spreads over all volumes, peaks at

$$V_k : \qquad k_{peak} = \frac{2\pi\epsilon^2}{\Delta E} t$$

Push towards larger volumes by effects of measure, not classical action

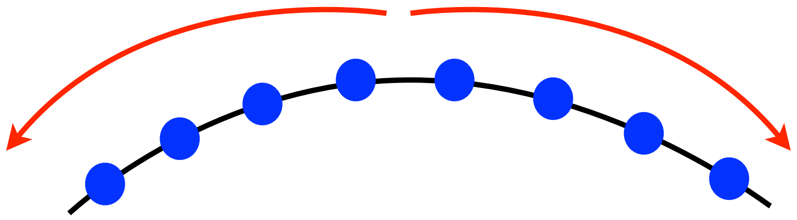
Can this be relevant today ?

Mass inside one Cosmological horizon is just of the order to make a black hole with radius equal to horizon radius

$$H^2 \sim G\rho$$ (Einstein equation)

$$M = \rho R^3 = \rho H^{-3} \sim \frac{H^{-1}}{G} \sim \frac{R}{G}$$

$R \sim H^{-1}$

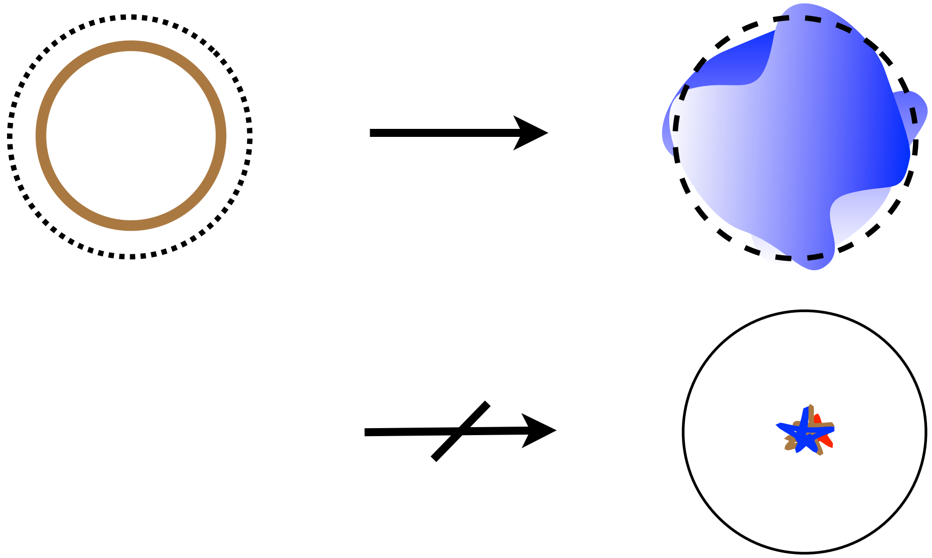


(This is the black hole condition)

Extra 'push' expected to be order unity ... could be related to dark energy ?

Summary

(A) Black holes

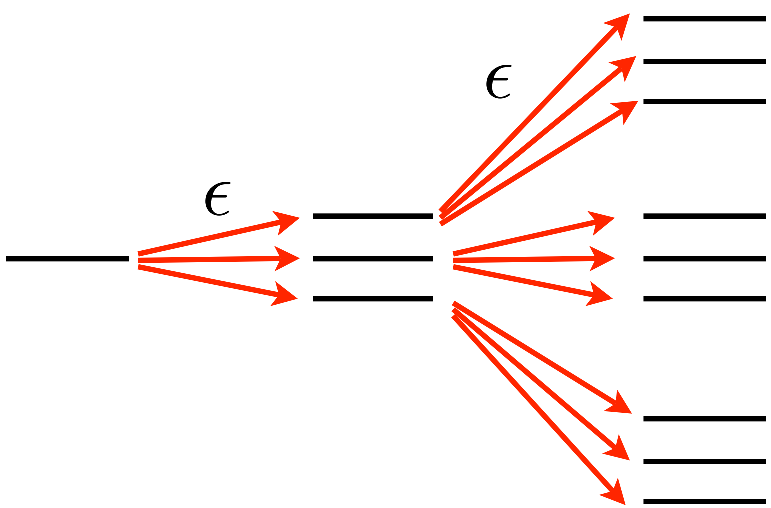


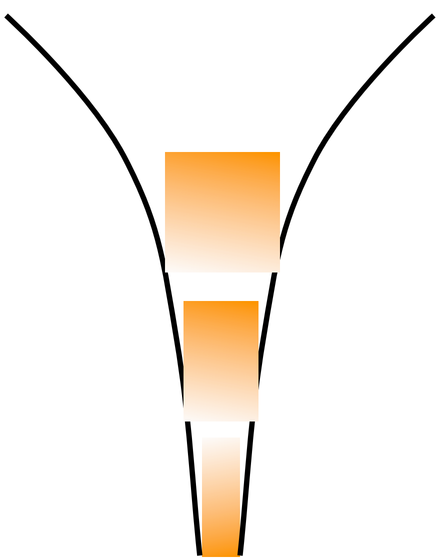
Large entropy

Large phase space

Wavefunction spreads all over phase space

(B) Cosmology





More phase space at larger volumes gives a push towards larger volumes

Inflation ??

Dark Energy ??