Non-extremality and the IR limit of large N QCD

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March. 2 2012, Great Lakes Strings Conference
Purdue University, IN, USA.

Based On

- 'Non-extremality, chemical potential and the infrared limit of large N QCD', Mohammed Mia, Fang Chen, Keshav Dasgupta, Paul Franche and Sachideo Vaidya, arXiv:1202.5321 [hep-th].
- 'Phase transitions in holographic QCD', Mohammed Mia, Fang Chen and Miklos Gyulassy, *To Appear*.
- 'A holographic model for large N thermal QCD', Mohammed Mia, Keshav Dasgupta, Charles Gale and Sangyong Jeon, J.Phys.G.Nucl.Part.Phys.39 (2012), arXiv:1108.0684 [hep-th].

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- However QCD is non-conformal in the temperature regime explored by the heavy ion collisions.

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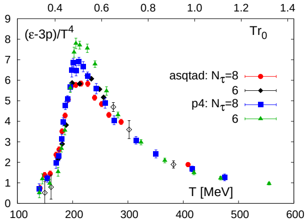

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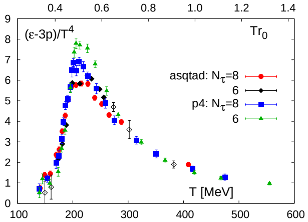

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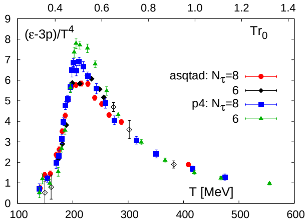

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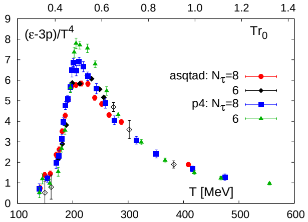

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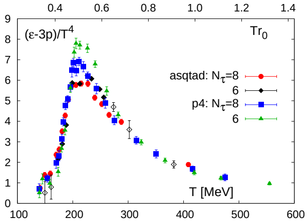

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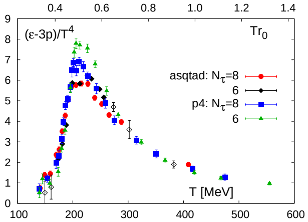

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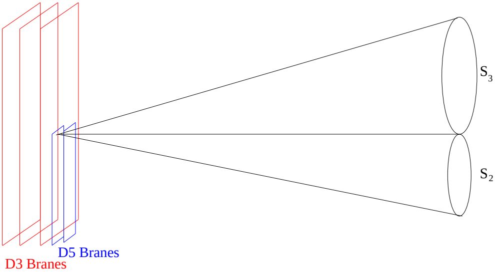
- We find new non-extremal solutions of modified KS model derived directly from 10d type IIB action with fluxes.
- Using this black hole geometry, we explore the thermodynamics of the gauge theory and find qualitative consistency with thermal QCD.

Non-Conformal Field Theory:The Brane Setup



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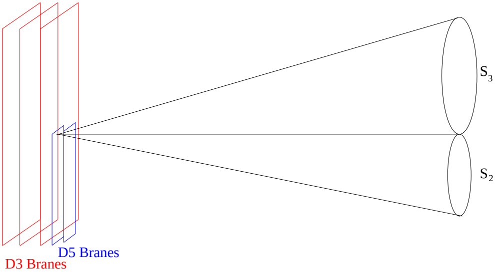

- The gauge group is $SU(N+M) \times SU(N)$, *logarithmic running of gauge coupling* but $N_{\rm eff}$ diverges in the UV and the theory has Landau Poles.

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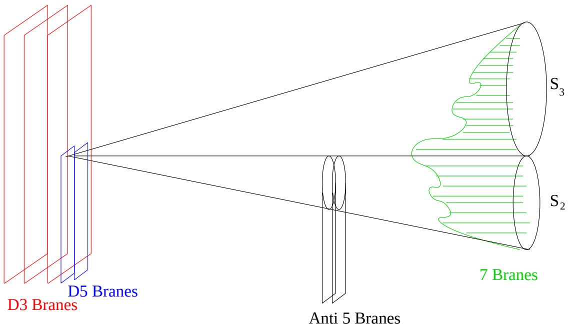

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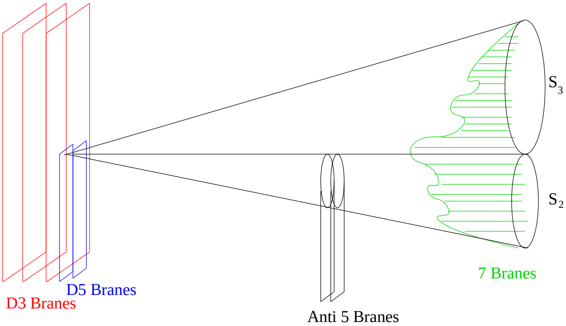

- Now we have $SU(N+M) \times SU(N)$ gauge group in the IR , $SU(N+M) \times SU(N+M)$ in the UV and $SU(N_f) \times SU(N_f)$ flavor symmetry with *fundamental matter and logarithmic running of gauge coupling*. N_{eff} large but finite in the far UV and gauge group cascades to $SU(\bar{M})$ in the far IR.

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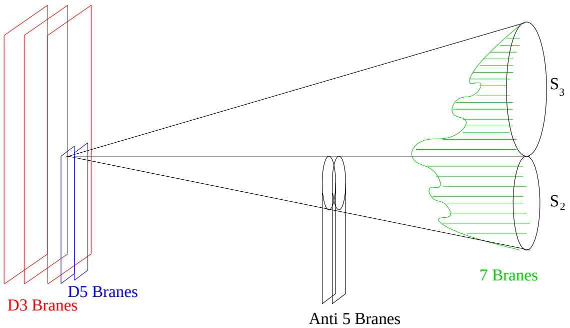

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- How to analyze the strongly coupled gauge theory?

Dual Geometry

- For large M, 'tHooft coupling $\Lambda_1 = g_1(k+1)M, \Lambda_2 = g_2 kM) \gg 1$, and the gauge theory can be described by type IIB supergravity

$$S_{\rm IIB} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\,\sqrt{-g}\left[R - \frac{\partial_a\tau\partial^a\tau}{2|{\rm Im}\tau|^2} - \frac{G_3\cdot\bar{G}_3}{12{\rm Im}\tau} - \frac{\widetilde{F}_5^2}{4\cdot 5!}\right]$$
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- The metric takes the form

$$ds^2 = \frac{1}{\sqrt{h}}\left[-g_1(r)dt^2 + dx^2 + dy^2 + dz^2\right] + \sqrt{h}\left[g_2(r)^{-1}dr^2 + d\mathcal{M}_5^2\right] \equiv -e^{2A+2B}dt^2 + e^{2A}\delta_{ij}dx^i dx^j + e^{-2A-2B}\widetilde{g}_{mn}dx^m dx^n$$

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- Warped four dimensional Minkowski space and warped six dimensional cone with five dimensional compact space $\mathcal{S}_5$ as it's base.

Dual Geometry

- With fluxes and warp factors A, B only depending on cone coordinates x^m, we have the Einstein equations

$$
\begin{aligned}
R_{\mu\nu} &= -g_{\mu\nu}\left[\frac{G_3\cdot\bar{G}_3}{48\,\mathrm{Im}\tau}+\frac{\widetilde{F}_5^2}{8\cdot 5!}\right]+\frac{\widetilde{F}_{\mu abcd}\widetilde{F}_{\nu}{}^{abcd}}{4\cdot 4!}+\kappa_{10}^2\left(T^{\mathrm{loc}}_{\mu\nu}-\frac{1}{8}g_{\mu\nu}T^{\mathrm{loc}}\right)\\
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- We consider closed three form fluxes H_3, F_3 (sourced by M number of five branes) and self dual five form flux

$$\widetilde{F}_5 = (1+\star)d\alpha\wedge dx^0\wedge dx^1\wedge dx^2\wedge dx^3 \tag{2}$$

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- Solve the Einstein equations and Bianchi identity to obtain A, B and $\tilde{g}_{mn}$.

Dual Geometry

- We have *four* equations and unknown functions A, B and $\tilde{g}_{mn}$. We use the following ansatz

$$\widetilde{g}_{mn}dx^m dx^n = dr^2 + r^2 e^{2B}\Bigg[\frac{1}{9}(d\psi + \cos\,\theta_1 d\phi_1 + \cos\,\theta_2 d\phi_2)^2 + \frac{1}{6}(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) + \frac{1}{6}(1+F)(1+\mathcal{G})\left(\frac{d\theta_2^2}{1+\mathcal{G}} + \sin^2\theta_2 d\phi_2^2\right)\Bigg]$$

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- In the limit $M = 0$ and $\partial\tau \sim \mathcal{O}(N_f) = 0$, an exact solution exist

$$e^{-4A} = \alpha^{-1} = L^4/r^4, \quad e^{2B} = 1 - r_h^4/r^4, \quad F = \mathcal{G} = 0$$

  the well known non-extremal limit of Klebanov-Witten geometry - $AdS_5 \times T^{1,1}$ with a black hole with $L^4 = g_s N\alpha'^2$

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- When $N_f, M \neq 0$, but $B = 0$, we have the extremal Klebanov-Tseytlin geometry with Ouyang D7 embedding. Again exact solution exists with

$$\begin{aligned}e^{-4A} &= \alpha^{-1} = h^0 = \frac{L^4}{r^4}\left\{1 + \frac{3g_sM^2}{2\pi N}\,\mathrm{log}r\left[1 + \frac{3g_sN_f}{2\pi}\left(\mathrm{log}r + \frac{1}{2}\right)\right]\right.\\ &+ \left.\frac{3g_s^2M^2N_f}{8\pi^2N}\,\mathrm{log}r\,\log\left(\sin\frac{\theta_1}{2}\sin\frac{\theta_2}{2}\right)\right\}\end{aligned}$$

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- Hence in the non-extremal limit, $F, \mathcal{G} \sim \mathcal{O}(M, r_h)$. With $g_s M^2/N \ll 1$, ignoring $\mathcal{O}(g_s M^2/N)\mathcal{O}(F, \mathcal{G})$ and considering upto linear order in $F, \mathcal{G}$ we get the Bianchi identity

$$\left[\partial_r \partial_r h^1 + \frac{1}{g}\partial_{\theta_i}\left(\bar{g}_0^{\theta_i\theta_i}\partial_{\theta_i} h^1\right) + \frac{r_h^4/r^4}{g}\partial_{\theta_i}\left(\bar{g}_0^{\theta_i\theta_i}\partial_{\theta_i} h^0\right)\right] r^5 + 5r^4\partial_r h^1$$
$$= 4L^4\partial_r\left(F + \mathcal{G}/2\right)$$

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$$F, \mathcal{G} \sim \mathcal{O}(M/N) + \mathcal{O}(g_s^2 M^2 N_f/N)$$

  which give $F, \mathcal{G} \ll 1$ for $N \gg M$. This also justifies ignoring $\mathcal{O}(g_s M^2/N)\mathcal{O}(F, \mathcal{G})$ and the linear approximation.

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  which give $F, \mathcal{G} \ll 1$ for $N \gg M$. This also justifies ignoring $\mathcal{O}(g_s M^2/N)\mathcal{O}(F, \mathcal{G})$ and the linear approximation.

- Writing $e^{2B} = 1 - \frac{r_h^4}{r^4} + G$, a similar analysis gives

$$G \sim \mathcal{O}(M/N) + \mathcal{O}(g_s^2 M^2 N_f/N)$$

  Also writing $h^1 = \frac{L^4}{r^4}A^1$ gives $A^1 \sim \mathcal{O}(M/N)$.

Dual Geometry

- Thus in the limit $N \gg M$ and $g_s M^2/N \ll 1$, we can consider only linear terms in $F, G, \mathcal{G}$ and A^1. However, there are only three non-trivial equations upto linear order and hence we can set $\mathcal{G} = 0$. In fact, G_3 do not enter explicitly in the equations for F, G and A^1 and we find an exact solution to all equations with

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\begin{aligned}
h^1 &= \frac{L^4}{r^4}\left(A_0 + A_1 \log r + A_2 \log^2 r\right) \\
e^{2B} &\equiv g = 1 - \frac{\bar{r}_h^4}{r^4} + G \equiv 1 - \frac{\bar{r}_h^4}{r^4} + g_0 + g_1 \log r + g_2 \log^2 r \\
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- The *background* warp factor h^0 couples directly to G_3 which is non-ISD. Our solution for the field strength is

$$\alpha = e^{4A} + \mathcal{O}(F^2), \quad F_3, H_3 \sim M(1 + \mathcal{O}(r_h, F))$$

where the non-ISD part G_3 is of $\mathcal{O}(F, r_h)$. In the limit $F = r_h = 0$, we recover ISD G_3.

Dual Geometry

- In the limit $N_f = 0$ but $M \neq 0$, the equations drastically simplify and we get

$$A_0(r) = \sum_{k=1}^{\infty} \bar{a}_k^0 \left(\frac{r_h}{r}\right)^k, \qquad F_0(r) = \sum_{k=1}^{\infty} \bar{f}_k^0 \left(\frac{r_h}{r}\right)^k, \quad g_0(r) = \sum_{k=1}^{\infty} \bar{\zeta}_k^0 \left(\frac{r_h}{r}\right)^k$$

with $A_i = F_i = g_i = 0, i = 2, 3.$ These forms also satisfy the boundary conditions

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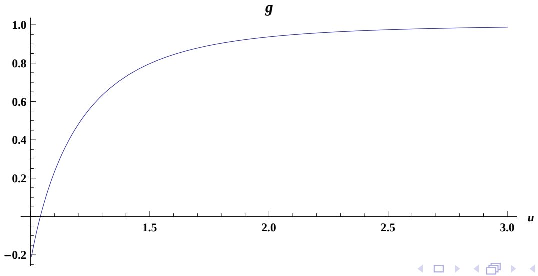

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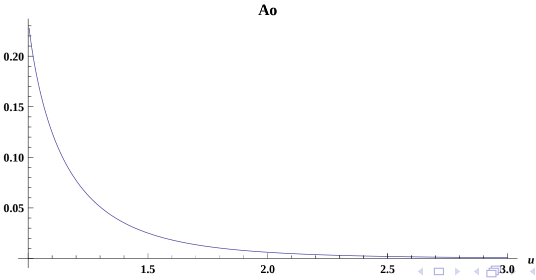

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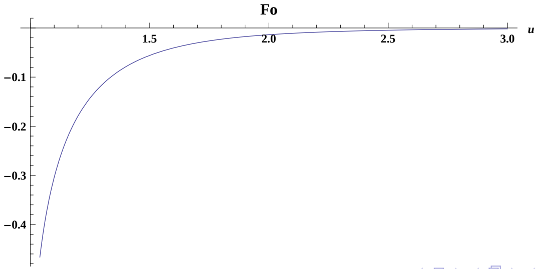

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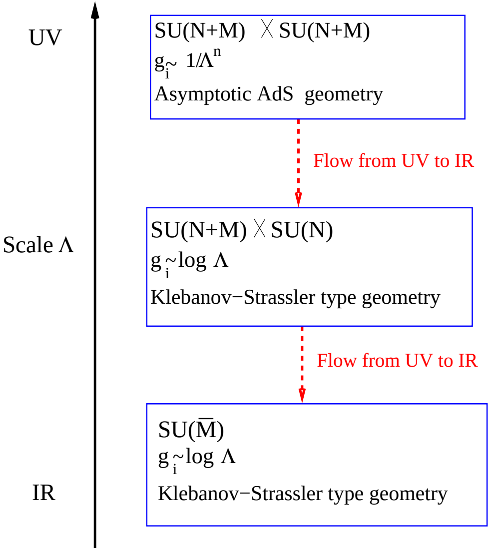
Dual Geometry

- The analysis can easily be generalized for KS type geometries with modifications at asymptotically large r.
- Three form fluxes $\sim \bar{M}(r)/r^A$ with $\bar{M}(r) = M\left(1 - \frac{exp[\alpha(r-r_0-b)]}{1+exp[\alpha(r-r_0-b)]}\right)$. Near $r \sim r_0$ we have fluxes sourced by anti five branes.
- $\lim_{r\to b} M(r) \to M$ and $\lim_{r\to\infty} \bar{M}(r) \to 0$. Hence $B_2 \sim M\ln(r)$ for $r \ll r_0$ and $B_2 \sim 0$ for $r \gg r_0$.
- 7 branes source τ, we can arrange them in such a way that for $r \gg r_0$, $\tau \sim 1/r^n$. On the other hand for $r < r_0$, $\tau \sim ln(r)$.
- We expect the squashing function F and warp factors e^A, e^B to again be described by our ansatz with inverse power law behavior.

The Story so far



Entropy From Dual Gravity

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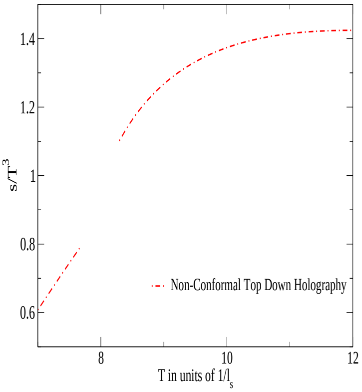

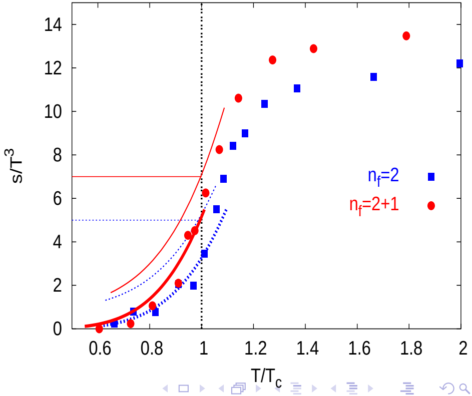

Conformal anomaly from non-AdS Dual Gravity

- We computed free energy of the thermal gauge theory from the ten dimensional type IIB supergravity action, $I_{\rm gravity} = I_{\rm gauge} = \beta F$.
- We obtain pressure $p = -\frac{\partial F}{\partial V} = -f$ and then using the black hole entropy, we compute internal energy $e = f + Ts$.

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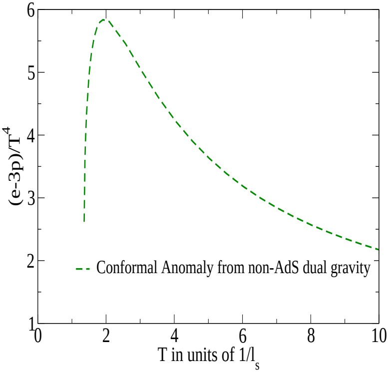
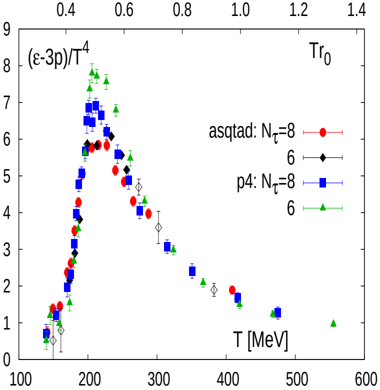
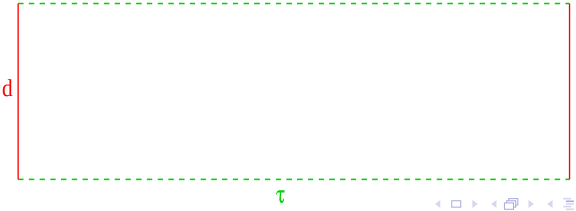




Linear confinement from holography

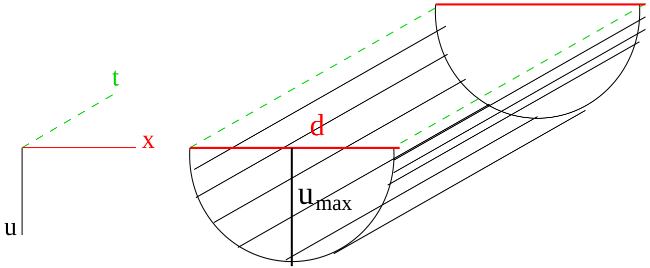
- We want to analyze the confinement mechanism for the fundamental matter arising from string theory. While lattice QCD gives linear confinement of quarks at low temperatures, do our quark strings confine?
- We will study $Q\bar{Q}$ free energy as a function of inter quark separation and temperature. Free energy can be obtained from the Wilson loop

$$< W_C > \sim \exp\left(-F(d,T)/T\right)$$

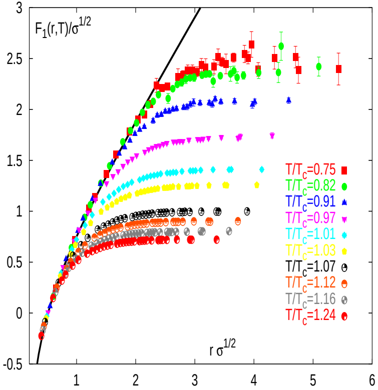


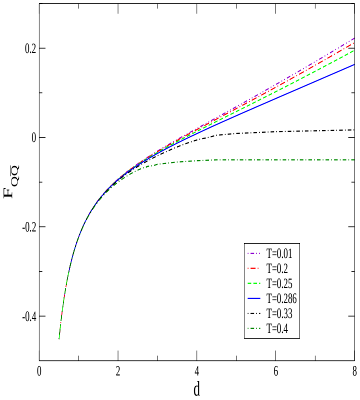
- Holography gives

$$< W_C >\sim \exp\left(-S_{\rm NG}\right)$$



Lattice VS Dual Gravity





[Olaf Kaczmarek et al]

Summary

- We have constructed non-extremal generalizations of warped resolved deformed conifold geometry.
- It is possible to construct the dual gravity of gauge theory which has logarithmic running of coupling in IR but behaves almost conformal in the UV.
- IR of the field theory we analyzed mimics large N QCD.
- Entropy of black hole is qualitatively similar to that of strongly coupled QCD while the conformal anomaly of the dual gauge theory is in agreement with QCD.
- The dual geometry realizes linear confinement of both heavy and light quarks.
- We expect the IR of the gauge theory to be thermodynamically equivalent to strongly coupled large N QCD.

Future Directions

- What are the phases and order of phase transitions?
- What about chemical potential and critical point?
- We are now studying various phases of nuclear matter at strong coupling by varying chemical potential and temperature coming from ten dimensional black hole geometries in string theory. We expect Hawking-Page phase transition ...
- For a trivial D7 embedding the chemical potential scales as $\mu \sim T(1 + \log(r_h))$.
- Using this non-extremal geometries we are also studying trailing and falling strings giving rise to heavy and light quark energy loss [*Andrej Ficnar, Miklos Gyulassy*]... Stay tuned!