Symmetries of Holographic Super-Minimal Models

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(Based on work-in-progress with Kentaro Hanaki)

Motivation

Recent progress

Higher spin gravity in $AdS_3 \Leftrightarrow$ 2d W_N-minimal models

(Gaberdiel, Gopakumar 2010, Gaberdiel, Hartman, 2011 ...)

- Why consider higher spin?
  - In AdS spacetime, interaction of higher spin modes is possible.
  - More operators in the duality, richer study of holography.

Motivation

- **Can we supersymmetrize it?**
  - More symmetries, even richer content.
  - Connection to superstring theories

$$\Downarrow$$

- **Proposal:**

Higher spin supergravity in $\mathbf{AdS}_3 \Leftrightarrow$ 2d $\mathcal{N}=2$ $\mathbb{C}\mathbf{P}^n$ minimal models

(Creutzig, Hikida, Ronne, 2011)

Outline

In the forthcoming work, we find further evidence of the duality "Higher spin supergravity in $\text{AdS}_3 \Leftrightarrow$ 2d $\mathcal{N}=2$ $\mathbb{C}\text{P}^n$ minimal model":

- We compute the asymptotic symmetry $\mathcal{SW}_\infty[\lambda]$ of higher spin supergravity in AdS_3
- We propose that it matches with the chiral algebra super-W_n of the $\mathbb{C}\text{P}^n$ model in the 't Hooft limit.
- We also provide two non-trival checks to be described.

Higher spin supergravity as a Chern-Simons theory

Classically, higher spin supergravity can be formulated as $shs[\lambda]_L \times shs[\lambda]_R$ Chern-Simons theory in AdS$_3$ spacetime.

- $shs[\lambda]$: super higher spin algebra, admits $\mathcal{N} = 2$ SUSY.

  (Bergshoeff, de Wit, Vasiliev 1991)

- Connection: $\Gamma = \sum_s (A^{(s)} L^{(s)} + \psi^{(s)} G^{(s)}), \quad s \in \mathbb{Z}/2$

$\mathcal{SW}_\infty[\lambda]$ algebra as an asymptotic symmetry

- **Asymptotic symmetry** is the gauge transformation that leaves the field invariant asymtotically: $(\Gamma - \Gamma_{AdS_3})|_{\partial\mathcal{M}} = \mathcal{O}(1)$.
- Get the asymptotic algebra from $shs[\lambda]$ by the classical Drinfeld-Sokolov (CDS) reduction.
- We can compute the variation under this algebra: e.g.

$$\delta^B_{\frac{5}{2}} A^{(\frac{5}{2})} = \frac{1-4\lambda}{3}\left(2A^{(\frac{5}{2})}\eta' + (A^{(\frac{5}{2})})'\eta\right) - N^B_{5/2}\left(2A^{(2)}\eta + (A^{(2)})'\eta\right) + \frac{k_{CS}N^B_{5/2}}{18\pi}\eta'''$$

Upshot

The asymptotic symmetry includes generators with arbitrary spin, has central charge $c = \frac{3l}{2G}$, is called $\mathcal{SW}_\infty[\lambda]$ algebra.

The dual CFT: $\mathcal{N}=2$ $\mathbb{C}\mathrm{P}^n$ model in the 't Hooft limit

- Facts about $\mathcal{N}=2$ $\mathbb{C}\mathrm{P}^n$ model in the 't Hooft limit:
  - $\mathcal{N}=2$ $\mathbb{C}\mathrm{P}^n$ model: $\frac{\widehat{SU(n+1)}_k \widehat{SO(2n)}_1}{\widehat{SU(n)}_{k+1} \widehat{U(1)}_{n(n+1)(k+n+1)}}$
  - 't Hooft limit: $n, k \to \infty$ with $\lim_{n\to\infty} \frac{n}{k+n}$ =const
  - central charge: $c = \frac{3kn}{k+n+1}$
  - chiral algebra: super-W_n algebra (Ito,91,93)

  (Quantum Drinfeld-Solokov (QDS) reduction of $sl(n+1,n)$)
- Trivial checks of the duality:
  - large central charge
  - Match of higher spin currents

Check of the duality I: The Algebra

- *Goal*: Chiral algebra in the 't Hooft limit $\frac{2}{\div}$ AdS asymptotic algebra.
- *How*: Take OPE at n=2,3,4,5, then extrapolate to $n \to \infty$ limit.
- *Result*: For example, the OPE $:W_{\frac{5}{2}} W_{\frac{5}{2}}:$ in CFT side matches with the variation $\delta^B_{\frac{5}{2}} A^{(\frac{5}{2})}$ in AdS side and defines $\lim_{n\to\infty} \frac{n}{2(n+k+1)} = \lambda$, which agrees with the proposal in CHR paper.

Key fact

This check is made available due to supersymmetry.

Check of the duality II: The Degenerate Representations

- *Goal*: Check if degenerate representations match.
- *How*:

$$\begin{array}{ccc} sl(n+1,n) & \xrightarrow{n \to -\lambda} & shs[\lambda] \\ \downarrow \text{QDS} & & \downarrow \text{CDS} \\ \text{super-}W_n & \xrightarrow[\substack{n \to -\lambda \\ k_{DS} \to \infty}]{\text{classical limit}} & \mathcal{SW}_\infty(\lambda) \end{array}$$

AdS$_3$/CFT$_2$ Proposal

$$\text{super-}W_n \xrightarrow[?]{\text{'t Hooft limit}} \mathcal{SW}_\infty(\lambda)$$

Comparing the conformal weight and the $U(1)$ charge of any degenerate representation of super-W_n in the two limits and see if they match.

Result

Chiral primaries match in a simple way.

Summary

In this work, we find further evidence of the duality:
"Higher spin supergravity in $\mathrm{AdS}_3 \Leftrightarrow$ 2d $\mathcal{N}=2$ $\mathbb{C}\mathrm{P}^n$ minimal model":

- We compute the asymptotic symmetry $\mathcal{SW}_\infty[\lambda]$ of higher spin supergravity in AdS_3
- We propose that it matches with the chiral algebra of the $\mathbb{C}\mathrm{P}^n$ model in the 't Hooft limit.
- We provide two checks:
  1. compute the OPE between currents with low conformal weight at finite n and extrapolate to $n \to \infty$. The result matches with the variation of the asymptotic algebra on gravity side.
  2. match the degenerate representations of the algebras on both sides.

Thank You