Constraints on String Cosmology

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with Stephen Green, Emil Martinec and Savdeep Sethi

Accelerated Expansion and Supergravity

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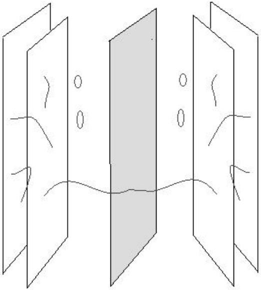
- No-Go Thm. $\Rightarrow$ SEC is inherited upon (warped) compactification.
- Does this rule out inflation/dS in string theory?

Accelerated Expansion and String Theory

- String theory is not supergravity
- Have objects like O-planes, with negative tension



- These can violate SEC

SEC Violation in Heterotic String Theory

In heterotic *all* O-planes effects are encoded in $\alpha' R^2$ corrections.

Several reasons to do study this problem in heterotic:

- Heterotic is "complete" (all couplings known to this order - Bergshoeff & de Roo '89).

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- Heterotic supergravity is reliable (no localized sources)
- Same qualitative features should carry over to other frames
- Simple calculation, can extract robust statements

Outline

1. **Introduction & Motivations**
2. **Review**
   - Strong Energy Condition
   - No-Go Theorem
   - Higher Order Couplings
3. **Calculations**
   - Set Up
   - Method
   - Results
4. **Summary**
   - Results

Strong Energy Condition

- Roughly speaking, SEC requires $R_{00} \geq 0$.
- More precisely, take any forward pointing time-like vector u, then

$$R_{\mu\nu} u^{\mu} u^{\nu} \geq 0.$$

- Physically, SEC says that gravity is locally attractive.
- Main point for us, in FRW universe

$$R_{00} = -3 \left(\frac{\ddot{a}}{a} \right)$$

- So $\ddot{a} \geq 0 \Leftrightarrow$ SEC is violated.

SEC in $D = 10$ $\mathcal{N} = 1$ supergravity

Trace-reversed Einstein equations (in Einstein frame):

$$R_{MN} = \frac{1}{2}\nabla_M\phi\nabla_N\phi + \frac{1}{4}e^{-\phi}H_{MPQ}H_N{}^{PQ} - \frac{1}{8}e^{-\phi}g_{MN}|H|^2 + \frac{\alpha'}{4}e^{-\phi/2}\left[\mathrm{tr}\,F_{MP}F_N{}^P - \frac{1}{8}g_{MN}\mathrm{tr}\,|F|^2\right]$$

In particular,

$$R_{00} = \frac{1}{2}(\dot{\phi})^2 + \frac{1}{8}e^{-\phi}\left(H_{0IJ}H_0{}^{IJ} + H_{IJK}H^{IJK}\right) + \frac{\alpha'}{32}e^{-\phi/2}\left(7F_{0I}F_0{}^I + \frac{1}{2}F_{IJ}F^{IJ}\right) \geq 0$$

So, $D = 10$ sugra satisfies SEC.

Proof of the No-Go Theorem

Extremely simple and elegant (Gibbons '84, see also Maldecena-Nunez)

1. Take warped product metric

$$ds^2 = W^2(y)\left(g_{\mu\nu}(x)dx^\mu dx^\nu + g_{ij}(y)dy^i dy^j\right)$$

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$$R_{00}^{(D)} = R_{00}^{(d)} + \frac{1}{(D-2)W^{D-2}}\nabla^2 W^{D-2} \geq 0$$

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3. Multiply by W^{D-2}, and integrate over Y.

As long as $W(y) \neq 0$, guaranteed that $R_{00}^{(d)} \geq 0$.

Heterotic Supergravity

$D = 10$ $\mathcal{N} = 1$ supergravity with specific $O(\alpha')$ correction, and field definitions

$$S = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g}\, e^{-2\phi} \Big[R + 4(\partial\phi)^2 - \frac{1}{2}|H|^2 - \frac{\alpha'}{4} \left(\mathrm{tr}\,|F|^2 - \mathrm{tr}\,|R_+|^2 \right) + O(\alpha'^2) \Big],$$

where

$$\mathrm{tr}\,|R_+|^2 = \frac{1}{2} R_{MNAB}(\Omega_+) R^{MNAB}(\Omega_+)$$

$$\Omega_{\pm}^{AB}{}_M = \Omega^{AB}{}_M \pm \frac{1}{2} H^{AB}{}_M + O(\alpha')$$

$$H = dB + \frac{\alpha'}{4} \left[\mathrm{CS}(\Omega_+) - \mathrm{CS}(A) \right],$$

Modified Equations of Motion

These R_+^2 terms modify the Ricci tensor. In particular

$$\tilde{R}_{00} = \ldots + \frac{\alpha'}{16} e^{-\phi/2} \left[\left(7 R_{+\,0I0J} R_{+\,0}{}^{I}{}_{0}{}^{J} + \frac{1}{2} R_{+\,IJ0K} R_{+}{}^{IJ}{}_{0}{}^{K} \right) - \frac{1}{2} \left(7 R_{+\,0IJK} R_{+\,0}{}^{IJK} + \frac{1}{2} R_{+\,IJKL} R_{+}{}^{IJKL} \right) \right]$$

- No longer positive definite.

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- No longer positive definite.
- How large can negative contributions be? Can no-go be evaded?
- Cannot dial up arbitrarily large
  must satisfy dilaton e.o.m. (and Bianchi)
- These constraints can be relaxed in local/non-compact models. In global/compact models, these gives bounds to how much $|R_+|^2$ is allowed.

Caveats

- We assume α' expansion is valid
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- We assume α' expansion is valid
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- Ignore perturbative g_s corrections
    - One string-loop comes in at $O(\alpha'^3)$
- Ignore non-perturbative g_s corrections
    - $NS5$-branes are just point-like instantons (no SEC violation)
    - Other spacetime non-perturbative effects, like gaugino condensation, could alter this picture

Ansatz

- We'll take an extremely simple ansatz for our spacetime fields:

$$\begin{aligned} ds^2 &= e^{2\omega(y)}\left(\hat{g}_{\mu\nu}(x)dx^\mu dx^\nu + \hat{g}_{mn}(y)dy^m dy^n\right) \\ H &= H_{mnp}(y)dy^m dy^n dy^p \\ F &= F_{mn}(y)dy^m dy^n \\ \phi &= \phi(y) \end{aligned}$$

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\end{aligned}
$$

- In particular $\dot{\phi} = 0$.
- Ignore axion h, $dh = *H$.
- Ignore all other light scalars from compactifying.

  $\Rightarrow$ No dynamical scalars (moduli or otherwise).

Two Cases

We consider two simple choices for spacetime metrics

1. Maximally symmetric spacetime

$$\hat{R}_{\mu\nu} = \Lambda \hat{g}_{\mu\nu}$$

2. FLRW spacetime

$$ds_X^2 = -dt^2 + a(t)^2 h_{ij} dx^i dx^j$$

$\Rightarrow$ focus on effects of $|R_+|^2$ on Λ and $a(t)$, respectively.

Method

- We don't know form of internal fields $R_{mnpq}(\Omega_+)$, H_{mnp}, etc.
- Use the dilaton equation

$$\tilde{\nabla}^M \tilde{\nabla}_M \phi + \frac{1}{2} e^{-\phi} |H|^2 + \frac{\alpha'}{8} e^{-\phi/2} \left(\mathrm{tr}\,|F|^2 - \mathrm{tr}\,|R_+|^2 \right) = O(\alpha'^2)$$

to eliminate these from the Einstein equations

$$\begin{aligned} \tilde{R}_{MN} \quad = \quad & \frac{1}{4} \tilde{g}_{MN} \tilde{\nabla}^P \tilde{\nabla}_P \phi + \frac{1}{2} \tilde{\nabla}_M \phi \tilde{\nabla}_N \phi + \frac{1}{4} e^{-\phi} H_{MPQ} H_N{}^{PQ} \\ & + \frac{\alpha'}{4} e^{-\phi/2} \left[\mathrm{tr}\, F_{MP} F_N{}^P - R_{+\,MPAB} R_{+\,N}{}^{PAB} \right]. \end{aligned}$$

Method

Write in product frame, look at spacetime components:

$$\hat{R}_{\mu\nu} = \hat{g}_{\mu\nu} W^{-8} \hat{g}^{mn} \hat{\nabla}_m \left(W^8 \hat{\nabla}_n \omega \right) - \frac{\alpha'}{4} e^{-2\omega} \left[\hat{R}_{\mu\rho\lambda}{}^{\sigma} \hat{R}_{\nu}{}^{\rho\lambda}{}_{\sigma} - 4 \hat{R}_{\mu\nu} |\hat{\nabla}_m \omega|^2 + 2\hat{g}_{\mu\nu} \left(3 \left(|\hat{\nabla}_m \omega|^2 \right)^2 + 2|X_{mn}|^2 + \frac{1}{2} e^{-4\omega} |H_{mn}{}^p \hat{\nabla}_p \omega|^2 \right) \right]$$

where

$$X_{mn}(\omega) = \hat{\nabla}_m \omega \hat{\nabla}_n \omega - \hat{\nabla}_m \hat{\nabla}_n \omega - \hat{g}_{mn} |\hat{\nabla}_p \omega|^2$$

Maximally symmetric spacetimes

- For $\hat{R}_{\mu\nu} = \Lambda \hat{g}_{\mu\nu}$, this reduces to

$$W^{-8}\hat{\nabla}^m \left(W^8 \hat{\nabla}_m \omega \right) = \Lambda$$
$$+ \frac{\alpha'}{2} e^{-2\omega} \left[\frac{1}{3} \left(\Lambda - 3|\hat{\nabla}_m \omega|^2 \right)^2 + 2|X_{mn}|^2 + \frac{1}{2} e^{-4\omega} |H_{mn}{}^p \hat{\nabla}_p \omega|^2 \right]$$

- Note: R^2 terms and $|\hat{\nabla}\omega|^2$ combine into perfect square!
- As in no-go thm, multiply by W^8 and integrate over Y

Result:

$$\Lambda = -\frac{\alpha'}{2V'}\int_{\mathcal{M}} d^6y\sqrt{\hat{g}}W^8 e^{-2\omega}\Big[3|\hat{\nabla}_m\omega|^2 + 2|X_{mn}|^2 + \frac{1}{2}e^{-4\omega}|H_{mn}{}^p\hat{\nabla}_p\omega|^2\Big] + O(\alpha'^2)$$

where $V' = \int_{\mathcal{M}} d^6y\sqrt{\hat{g}}W^8$

- In particular, $\Lambda \leq 0$.
- No de Sitter solutions in this setup

Minkowski:

- No warping in string frame (consistent with Strominger '86)
- All supersymmetric solutions are of this type (Strominger)
- $\exists$? non-SUSY Minkowski vacua (probably not, finely-tuned)

AdS:

- Should exist! Not possible in supergravity
- $\Lambda \sim \mathcal{O}(\alpha'/R^4)$, so weakly curved
- Non-SUSY
- Seem to be generic solutions
- AdS/CFT? Worldsheet description?

dS:

- not possible by $O(\alpha')$ corrections
- these effects are dual to O-planes in other duality frames
- this indicates that O-planes are probably not sufficient to violate SEC in fully global models

Summary & Future Directions

- Looked for SEC violation from $O(\alpha')$ effects in heterotic supergravity
- Dilaton plays important role in constraining size of SEC violation
- In the absence of dynamical scalars, solutions are maximally symmetric with $\Lambda \leq 0$
- Only relevant missing piece is spacetime non-perturbative effects
- Could these generate $V > 0$ for, say, dilaton ($\sim$ racetrack)?
- Begun to include model-independent scalars ϕ, h and volume
- *Brief* periods of inflation are possible, requires further analysis

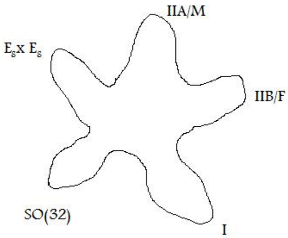
THANK YOU!

Future Directions

- Begun to include model-independent scalars ϕ and h
  Periods of inflation *may* be possible, require further analysis
- Could extend to model-dependent scalars
- Only relevant missing piece is spacetime non-perturbative effects
- Could these generate $V > 0$ for, say, dilaton ($\sim$ racetrack)?

SEC Violation in String Theory

- These effects are present in every corner of string landscape



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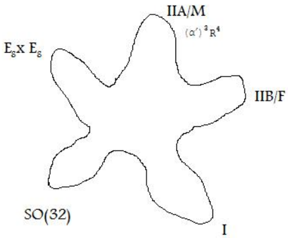

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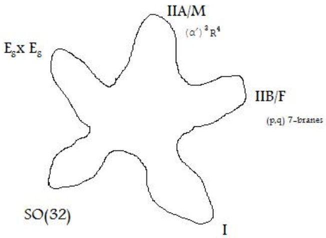

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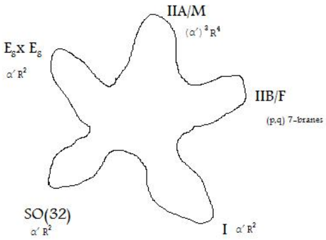

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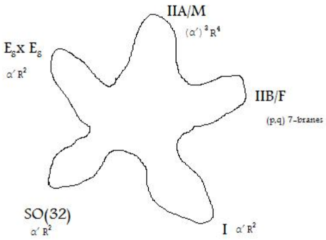

- These couplings make flux compactifications possible
  - Even though higher order, cannot neglect
  - Produce background charges, act as sinks
  - Allow solutions to Gauss Law with non-trivial flux

Comment on different frames

- Action written with string frame metric, g
- EoM/SEC best studied in Einstein frame,

$$g = e^{\phi/2}\tilde{g}$$

- In practice, useful to use product frame metric

$$g = e^{\phi/2}W^2\hat{g}$$

- Define

$$\omega = \log W + \frac{1}{4}\phi$$

conformal factor between string and product frame

Modified Equations of Motion

In Einstein frame,

- dilaton EoM

$$\tilde{\nabla}^M \tilde{\nabla}_M \phi + \frac{1}{2} e^{-\phi} |H|^2 + \frac{\alpha'}{8} e^{-\phi/2} \left(\mathrm{tr}\,|F|^2 - \mathrm{tr}\,|R_+|^2 \right) = O(\alpha'^2)$$

- Einstein equation (trace-reversed)

$$\tilde{R}_{MN} = \ldots + \frac{\alpha'}{4} e^{-\phi/2} \left[\frac{1}{8} \tilde{g}_{MN} \mathrm{tr}\,|R_+|^2 - R_{+\,MPAB} R_{+\,N}{}^{PAB} \right] + O(\alpha'^2)$$

- won't need others

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- All warping (in Einstein frame) comes from ϕ.
- String frame metric is (unwarped) direct product.
- Consistent with analysis by Strominger ('86):
  Max'ly symm. SUSY spacetimes are Minkowski, with $\nabla\omega = 0$.
- Are there other (non-SUSY) Minkowski solutions?

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Generic solution will have $\Lambda < 0$.

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- Surprising, since no known AdS_4 solutions in heterotic
  Forbidden in pure supergravity
- $|\Lambda| \sim O(\alpha')$, so weakly curved
- Non-SUSY, since all SUSY sol'n have $\Lambda = 0$
- Only shown solutions are possible, need to construct examples
  Possible dual: IIB on $K3 \times T^2$ with $G^{(3,0)} \neq 0$

Case 2: FLRW spacetimes

Performing similar analysis, find the Friedmann equations:

$$\begin{aligned} & W^{-8}\hat{\nabla}^m\left(W^8\hat{\nabla}_m\omega\right) \\ = \quad & 3\left(\frac{\ddot{a}}{a}\right)+\frac{\alpha'}{2}e^{-2\omega}\left[3\left(\frac{\ddot{a}}{a}-|\hat{\nabla}_m\omega|^2\right)^2\right. \\ & \qquad \left. +2|X_{mn}|^2+\frac{1}{2}e^{-4\omega}|H_{mn}{}^p\hat{\nabla}_p\omega|^2\right] \\ = \quad & -3\left(\frac{\dot{a}^2+k}{a^2}\right)-\frac{\alpha'}{2}e^{-2\omega}\left[3\left(\frac{\dot{a}^2+k}{a^2}-|\hat{\nabla}_m\omega|^2\right)^2\right. \\ & \qquad \left. +2|X_{mn}|^2+\frac{1}{2}e^{-4\omega}|H_{mn}{}^p\hat{\nabla}_p\omega|^2\right] \end{aligned}$$

Case 2: FLRW spacetimes

Integrating over Y

$$\frac{\ddot{a}}{a} = \frac{\dot{a}^2 + k}{a^2} = \frac{\Lambda_{eff}}{3}$$

where

$$\Lambda_{eff} = -\frac{\alpha'}{2V'} \int_{\mathcal{M}} d^6y \sqrt{\hat{g}}\, W^8 e^{-2\omega} \Big[3|\hat{\nabla}_m \omega|^2 + 2|X_{mn}|^2 + \frac{1}{2} e^{-4\omega} |H_{mn}{}^p \hat{\nabla}_p \omega|^2 \Big] + O(\alpha'^2)$$

Once again $\Lambda_{eff} \leq 0$

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This is Milne Universe, equivalent to a patch of Minkowski

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This is Milne Universe, equivalent to a patch of Minkowski

- $\Lambda_{eff} < 0$

$$\Rightarrow \begin{cases} a(t) \sim \sin\left(\sqrt{|\Lambda_{eff}|/3}t\right) \\ k = \Lambda_{eff}/3 \end{cases}$$

This "bounce" solution is actually just a patch of AdS

Summary of FRW Results:

- No new FRW solutions
- In particular, no acceleration $\ddot{a} > 0$
- Not surprising in retrospect;
  no dynamical fields to source non-trivial behaviour or $a(t)$

Extending the Results?

- What if we relax our assumptions?
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- Consider $0m$ (time-internal) components of Einstein equations

$$0 = -3\alpha' e^{-6\omega} h \left(\frac{\dot{a}}{a}\right) \hat{\nabla}_m \omega + O(\dot{\phi})$$

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  - First option we have studied already
  - Second option is uninteresting
  - Third is too constraining (reduces to $\ddot{a} = 0$)

Including $\dot{\phi} \neq 0$

- Simplest extension will require time-dependent dilaton
- This vastly complicates the e.o.m.
  e.g. The first Friedmann equations become

$$
\begin{aligned}
3\left(\frac{\ddot{a}}{a}\right) &= -\frac{1}{V'}\int d^6y\sqrt{g_6}W^8\left[-\hat{\nabla}^{\mu}\hat{\nabla}_{\mu}\omega + 8\dot{\omega}^2\right. \\
&+\frac{\alpha'}{2}e^{-2\omega}\left\{3\left(\frac{\ddot{a}}{a}+\ddot{\omega}+\dot{\omega}\left(\frac{\dot{a}}{a}\right)-|\hat{\nabla}_m\omega|^2\right)^2\right. \\
&+2\left|\hat{g}_{mn}\ddot{\omega}+X_{mn}\right|^2 - 8\left|\hat{\nabla}_m\dot{\omega}-\dot{\omega}\hat{\nabla}_m\omega\right|^2 \\
&\left.\left.+3e^{-4\omega}\left(\frac{1}{6}\left|H_{mn}{}^{p}\hat{\nabla}_p\omega\right|^2 - h^2\left(\dot{\omega}+\frac{\dot{a}}{a}\right)^2 - \dot{\omega}^2\left|H_{mnp}\right|^2\right)\right\}\right].
\end{aligned}
$$

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- Other field equations, too. And Bianchi.
- Sufficiently complicated that we cannot yet rule out *periods* of inflation
- Although, a pure dS phase looks improbable/impossible.