Quantum Compactifications

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Great Lakes Strings, Purdue
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Based on “Linear Sigma Models with Torsion”
by Callum Quigley and S.S.

arXiv: 1107.0714

“New Branches of (0,2) Theories”
by Callum Quigley, S.S. and Mark Stern

to appear

Outline:

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- Knowns and desires

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- Non-compact examples

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- Compact examples
- Classical and quantum geometries

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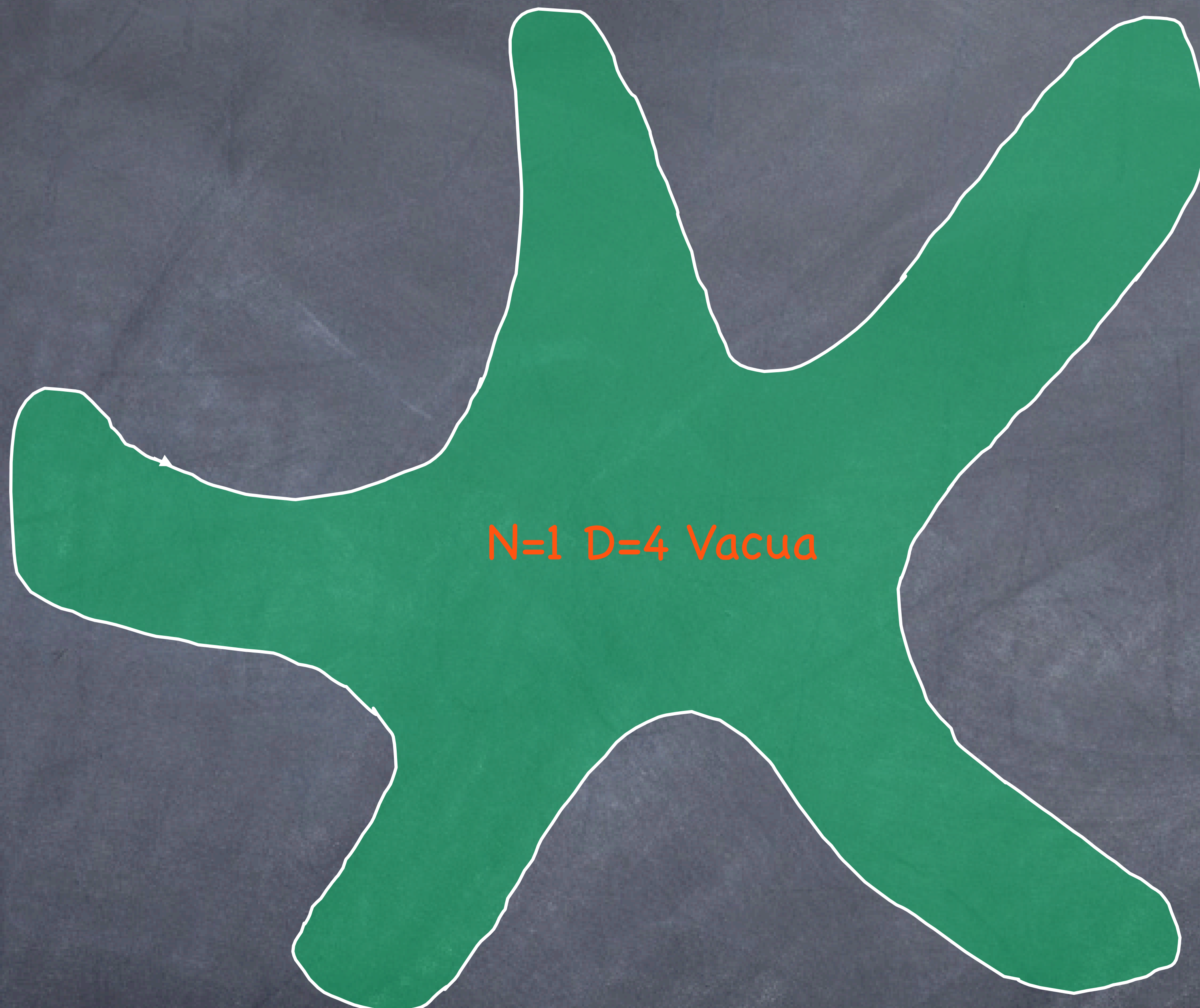
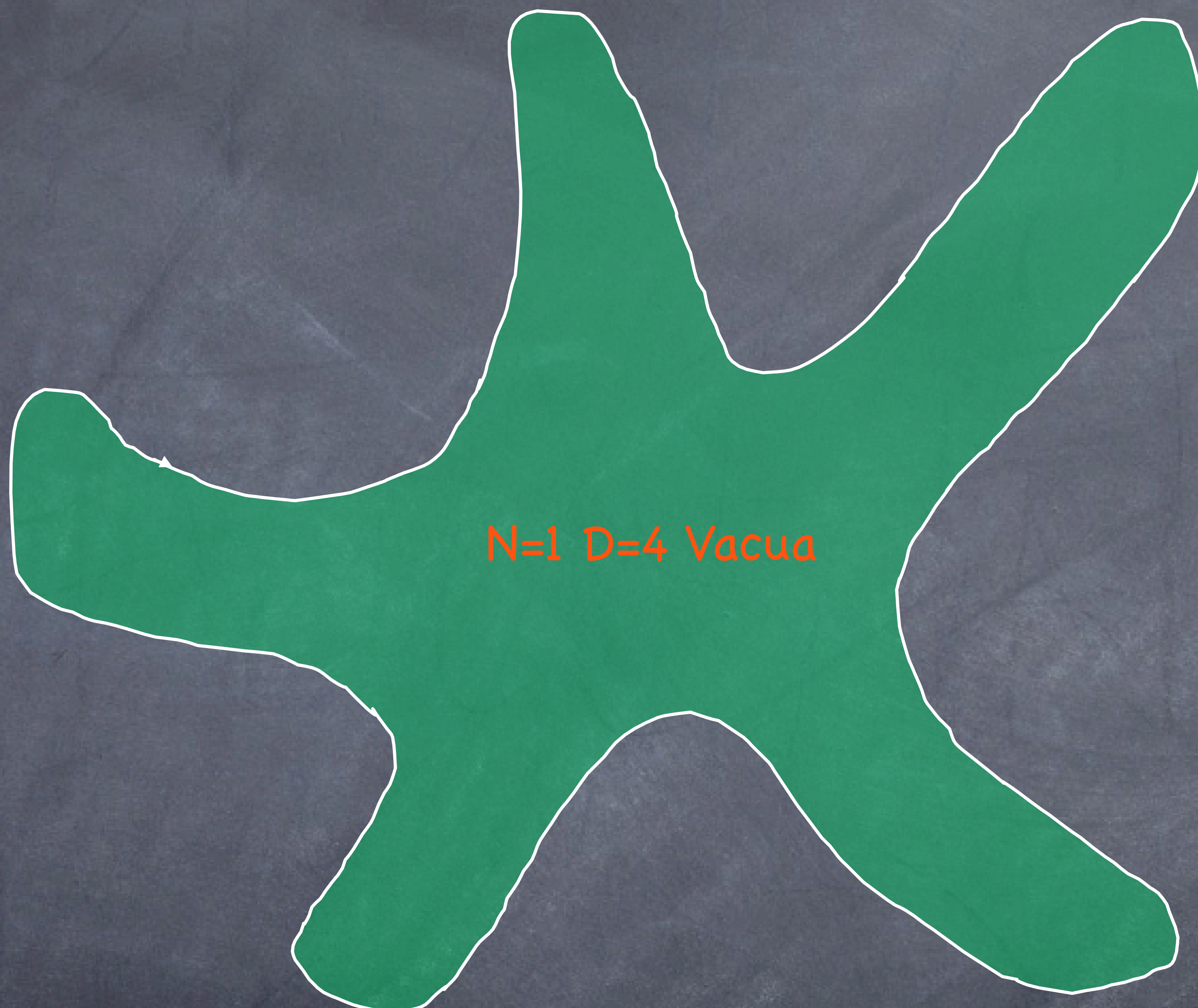
- The space of string compactifications is still largely mysterious.
- We need more powerful approaches to understand the interplay between cosmology, particle physics and Planck scale SUSY.


N=1 D=4 Vacua

Type I



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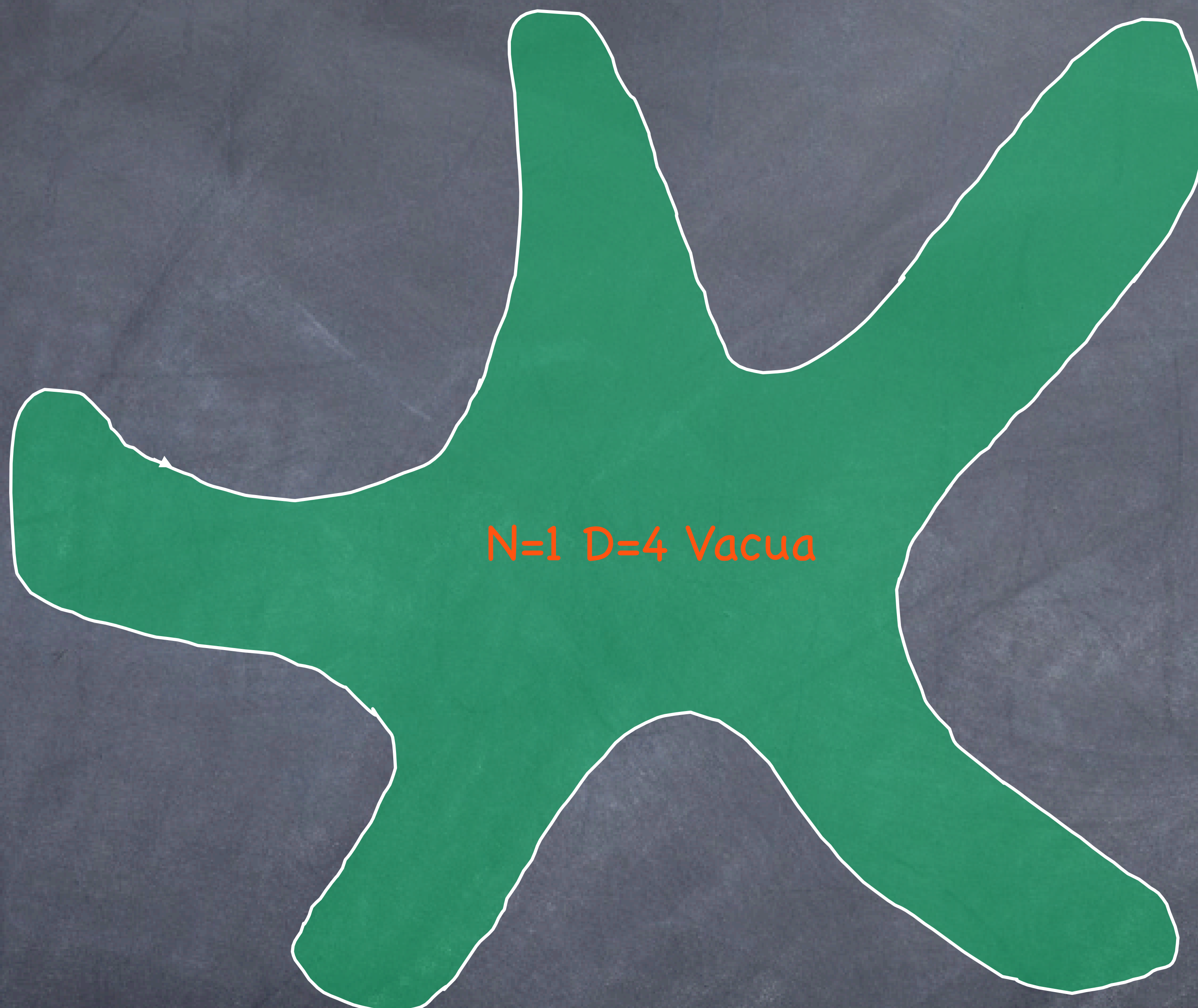

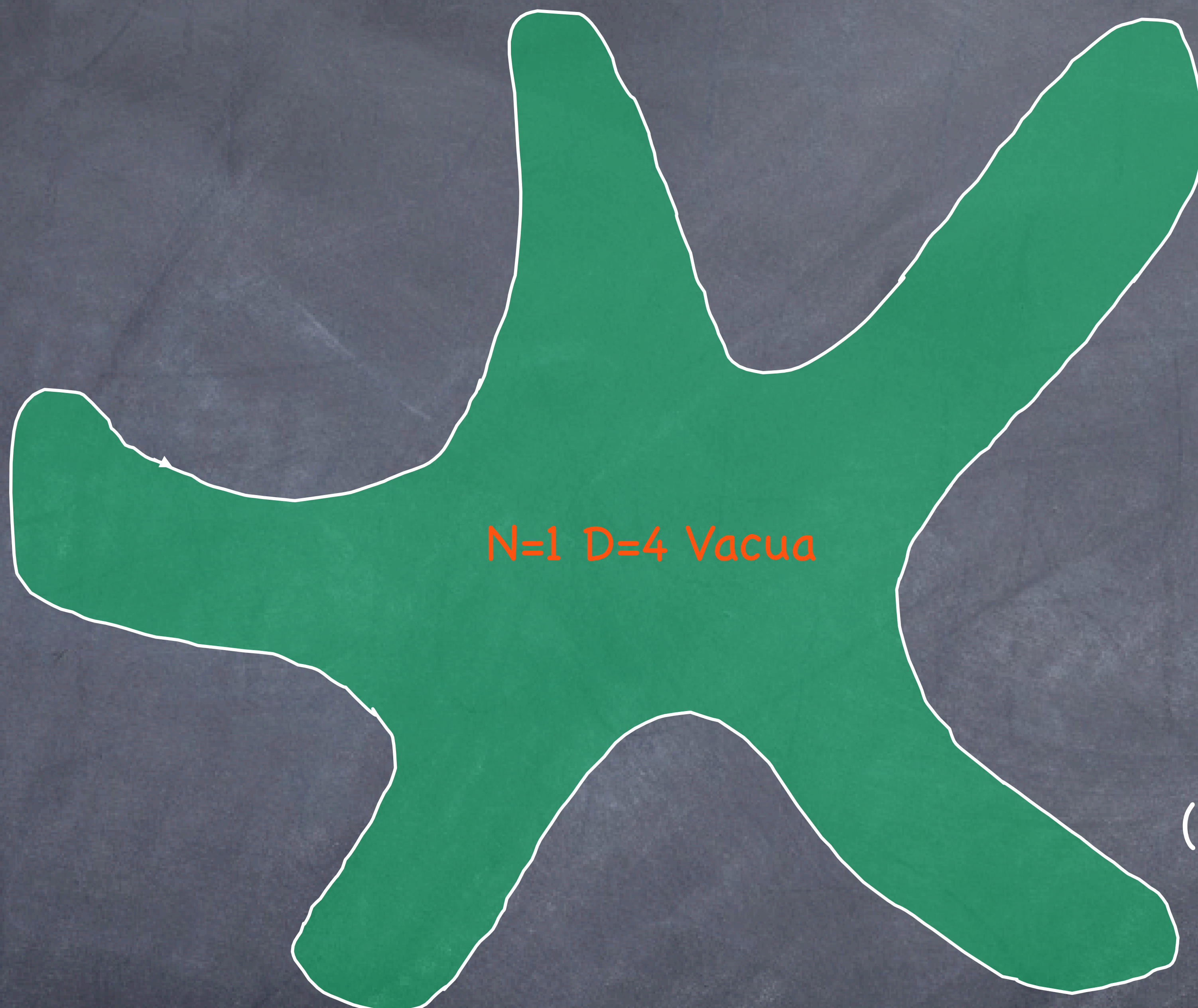

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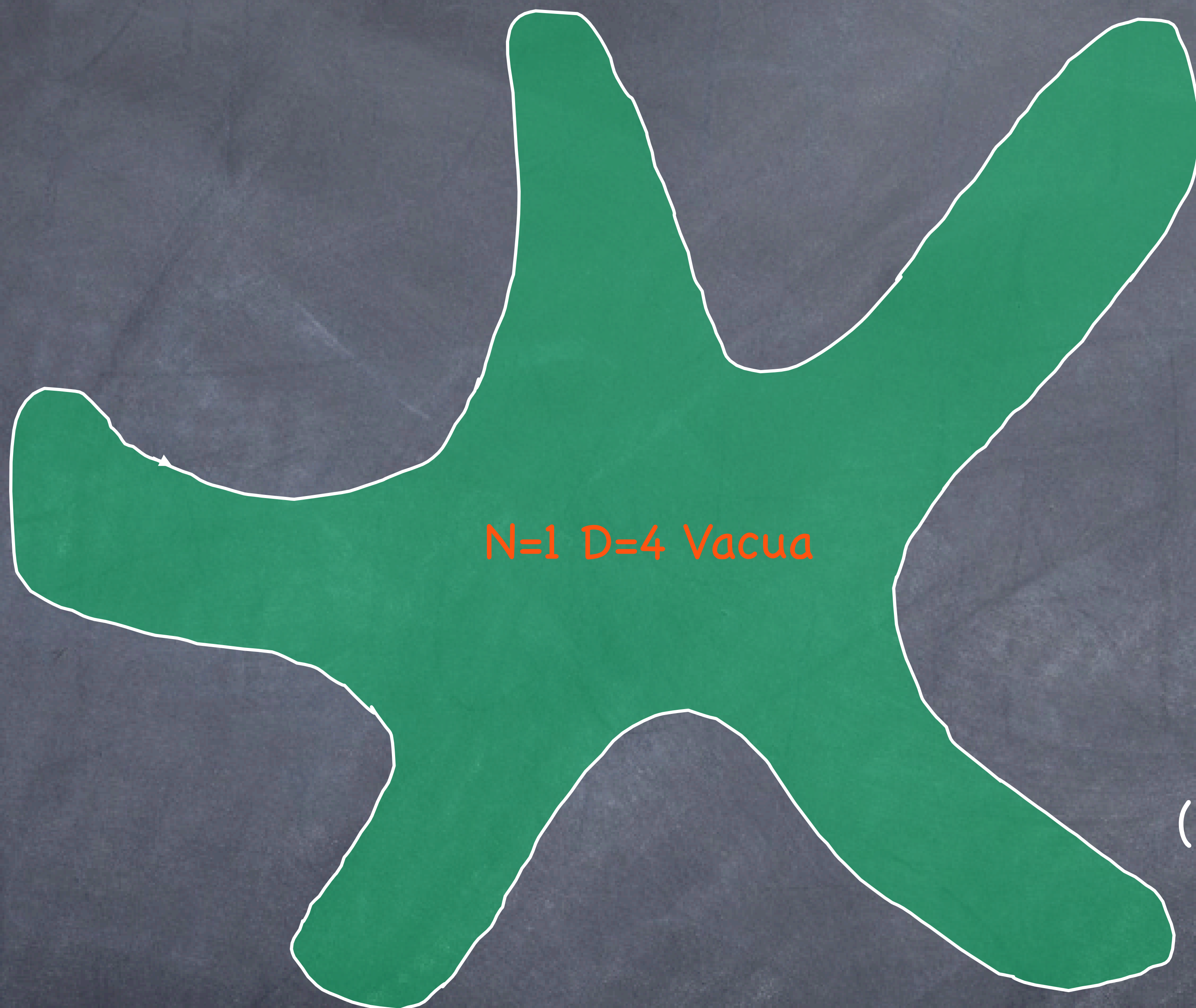
(IIB orientifolds)



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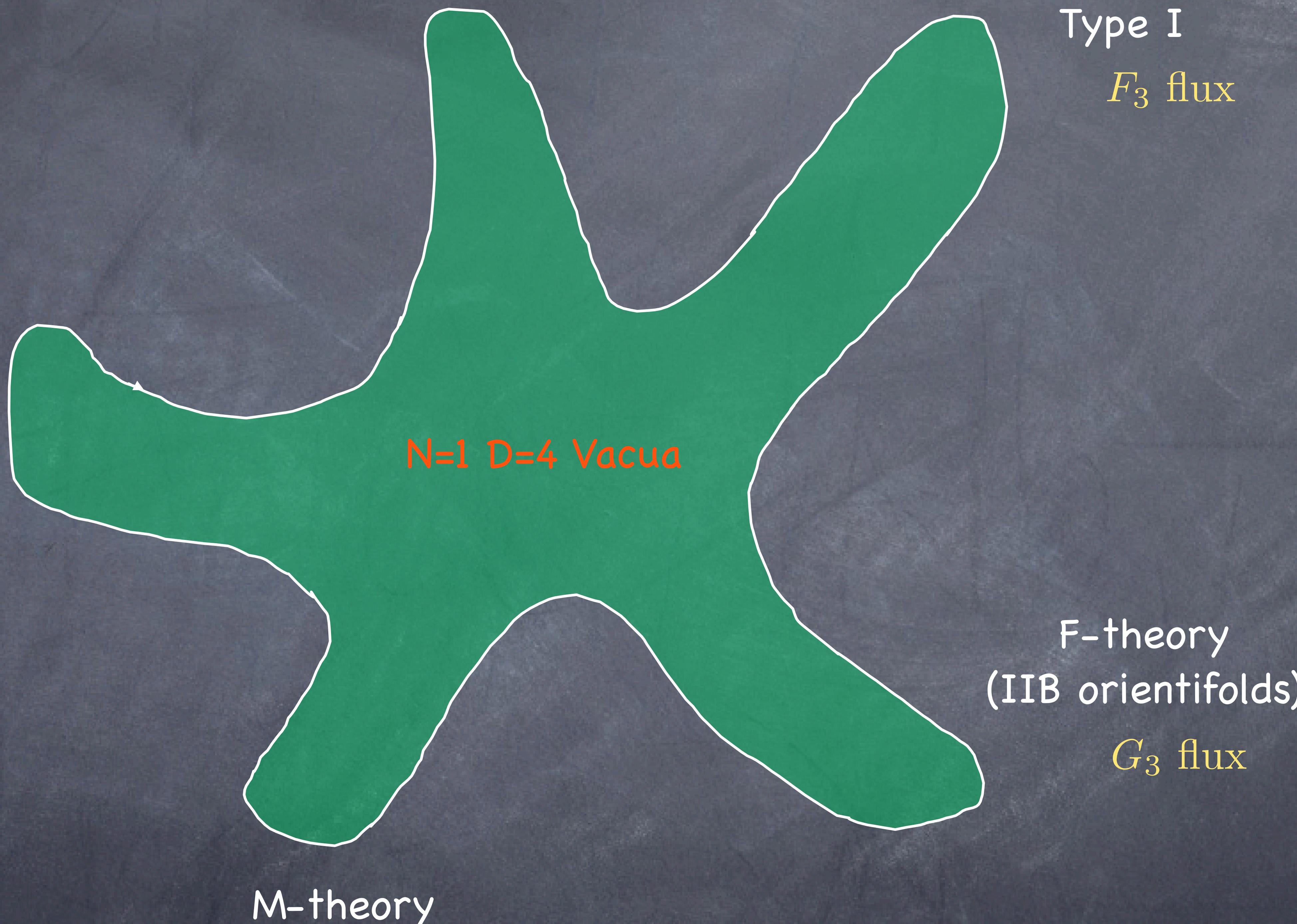
G_3 flux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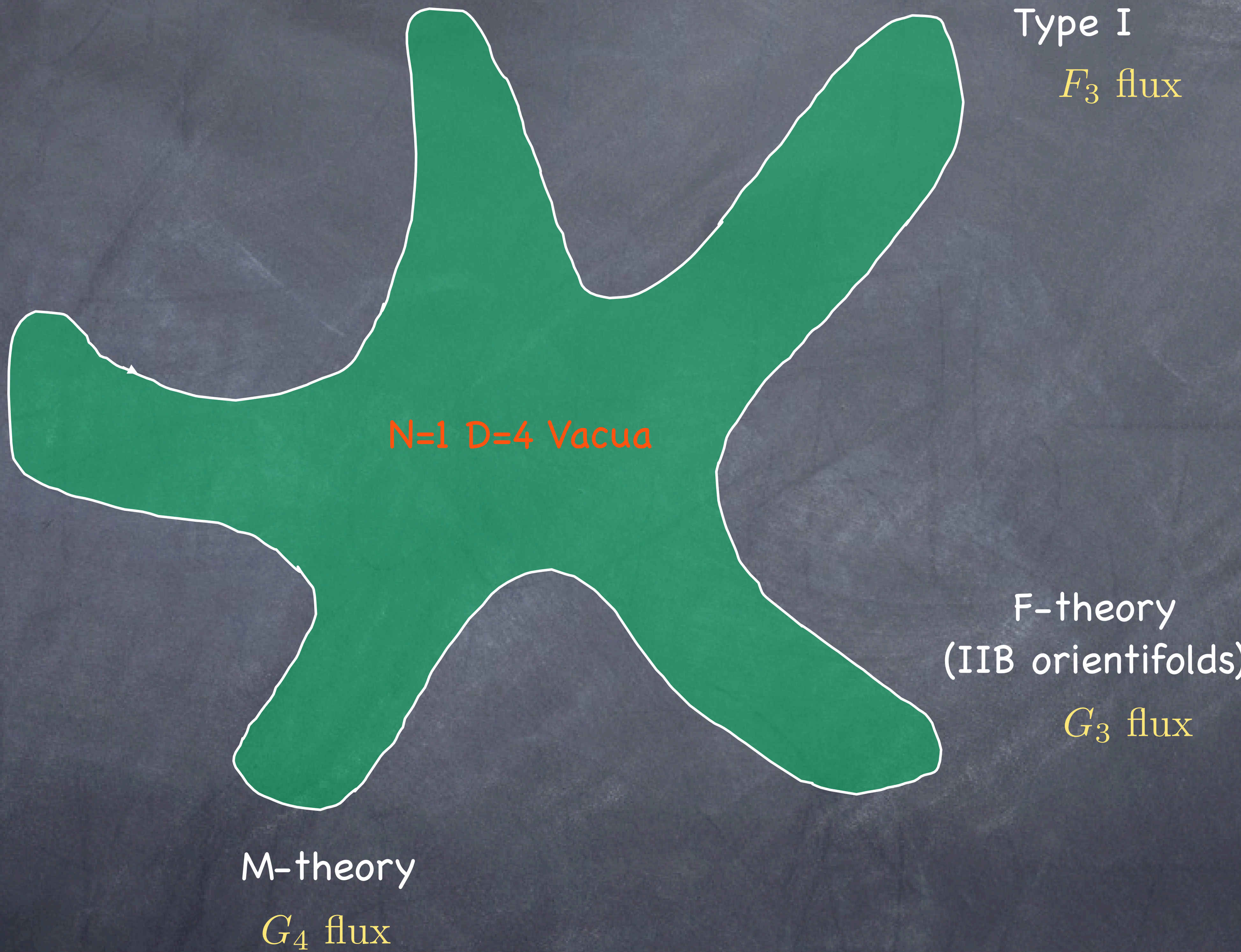
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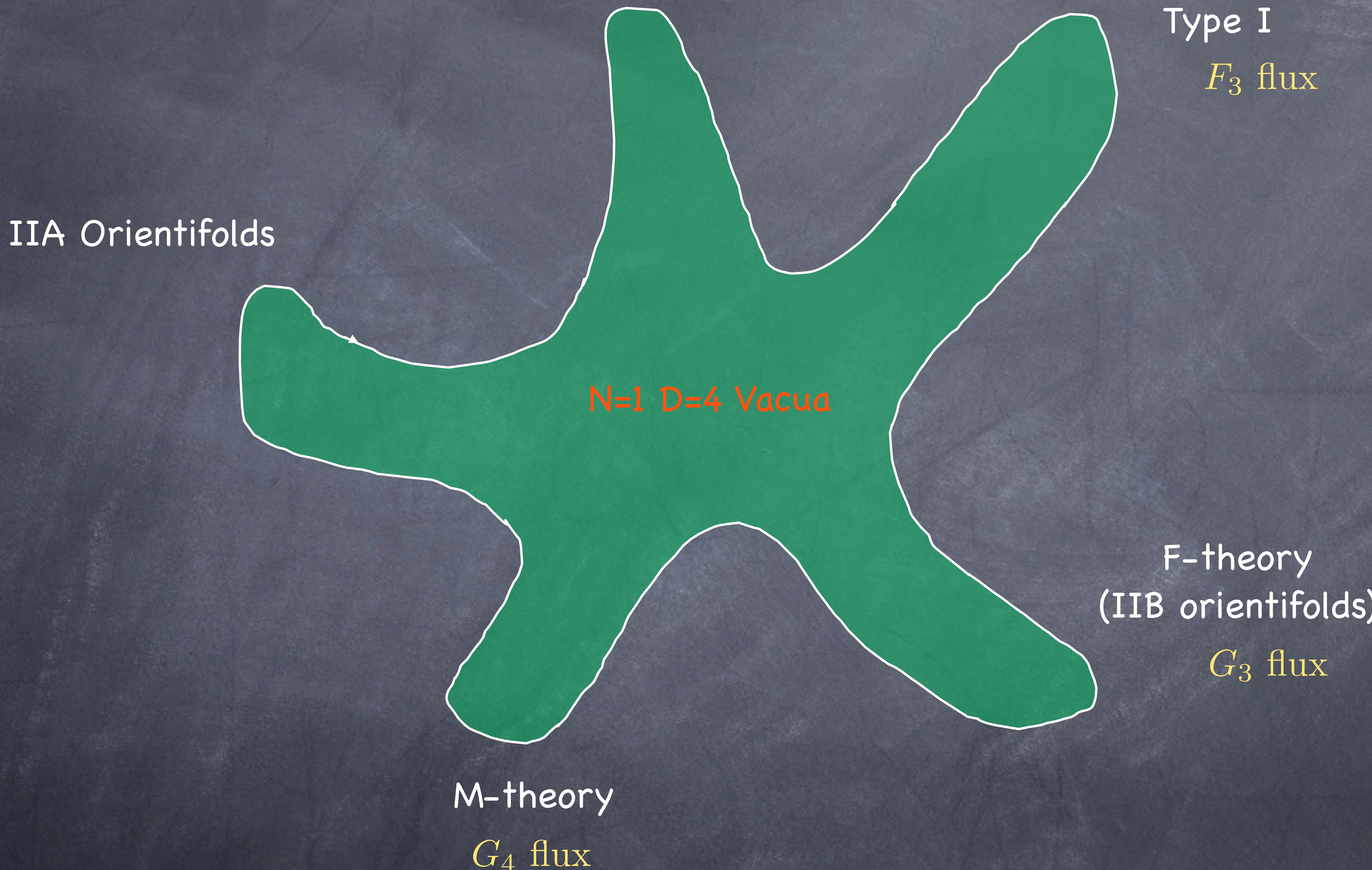
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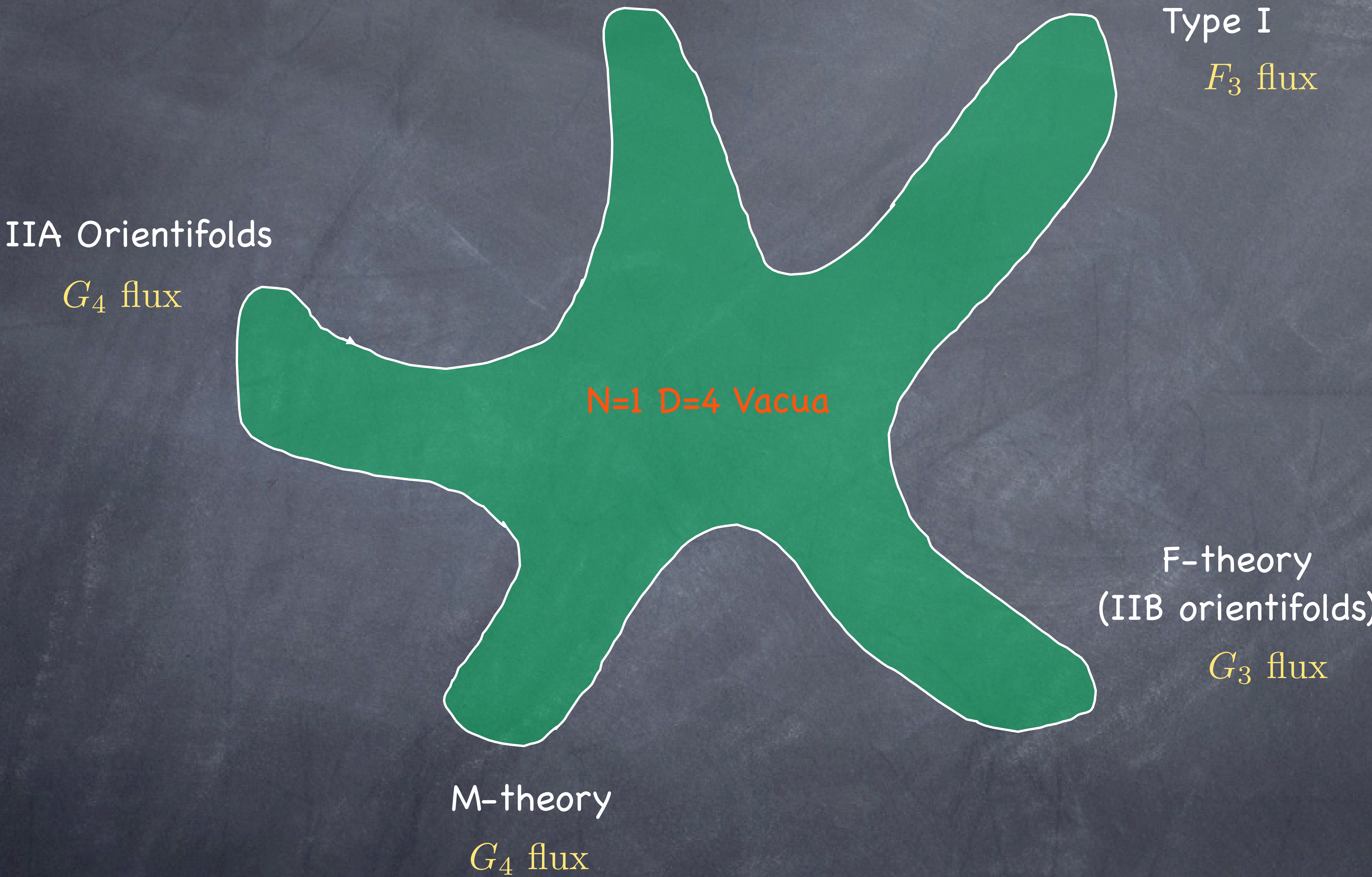
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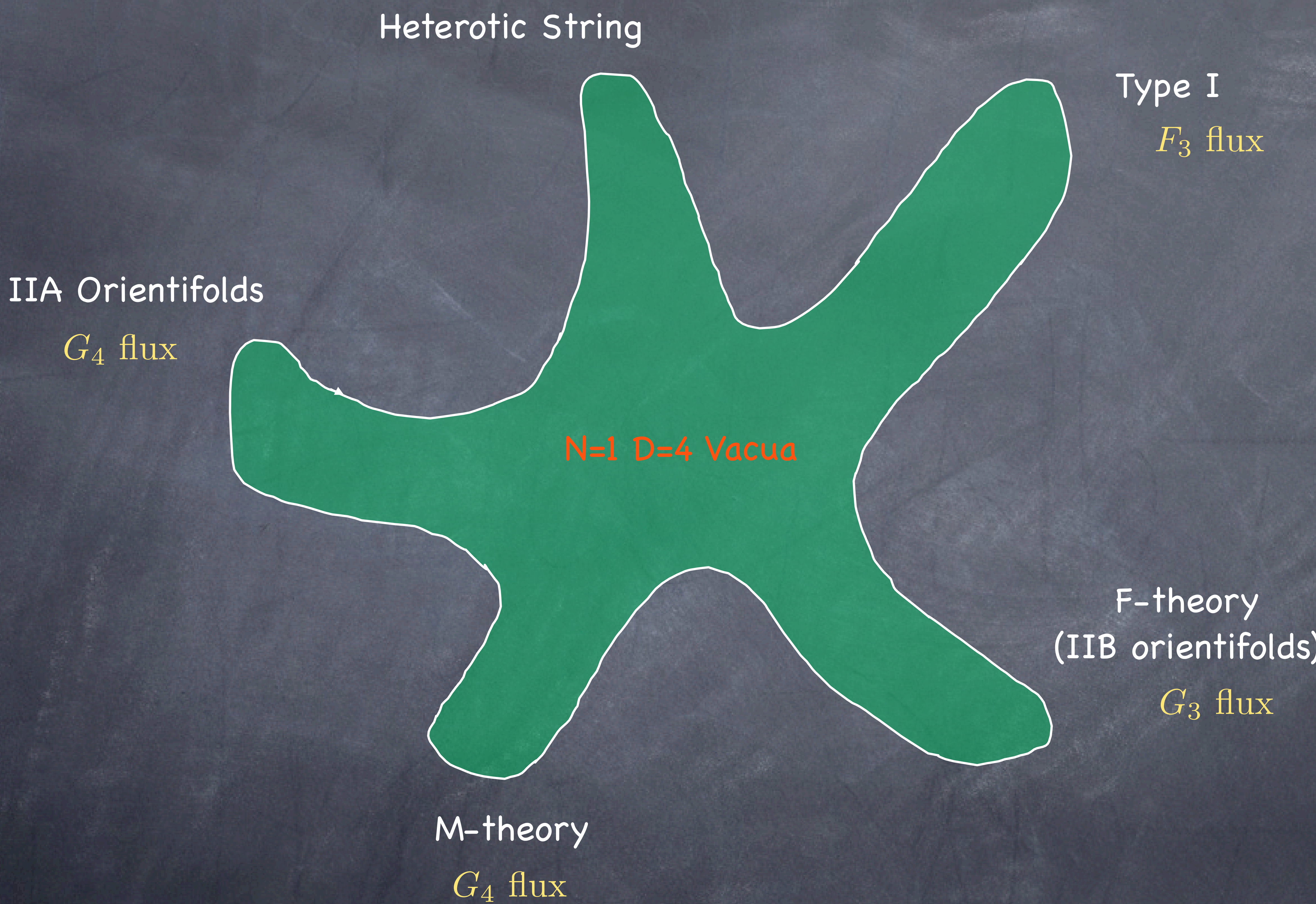
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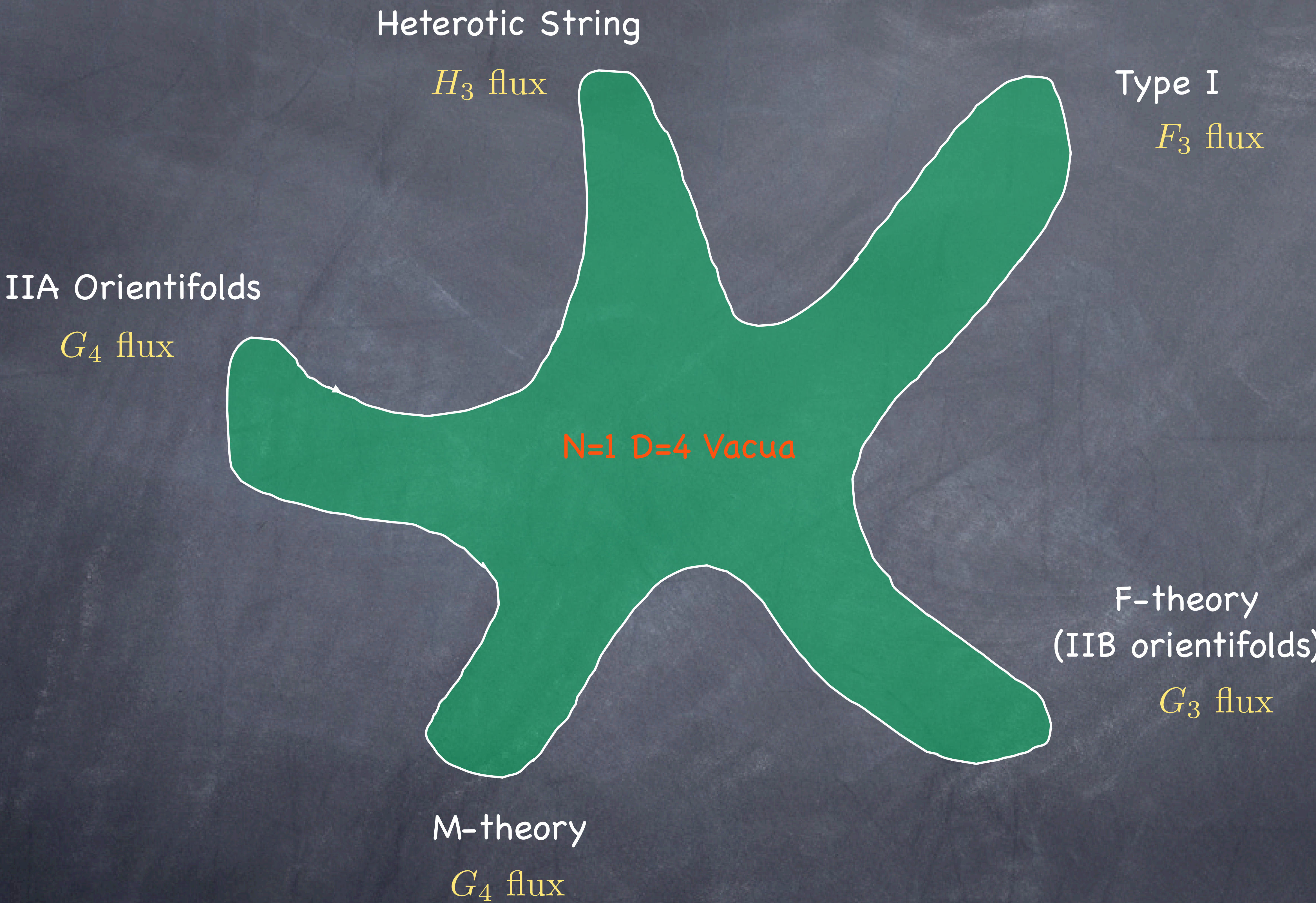
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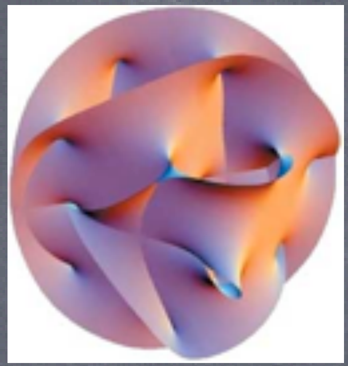

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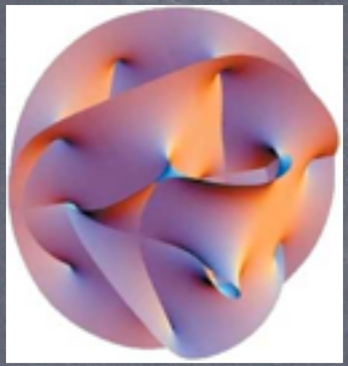

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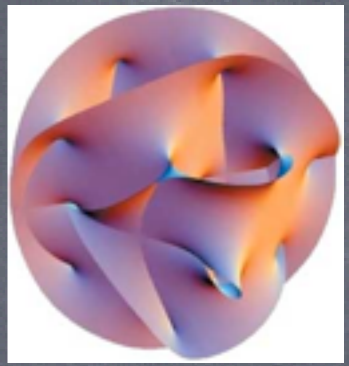

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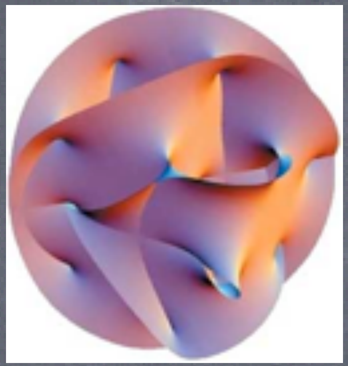

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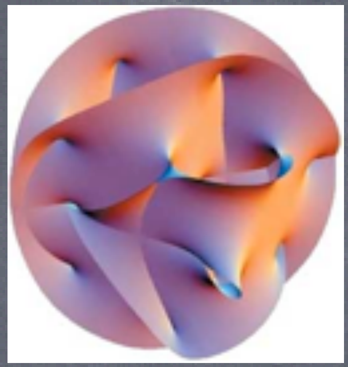

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- Only in the heterotic string is the required data purely NS with no RR fields.

- For models with RR fields, not much is known beyond the SUGRA approximation.

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The primary constraint is the Bianchi identity which has a gravitational correction:

$$dH = \frac{\alpha'}{4} \{ \mathrm{tr}(R \wedge R)(\omega_+) - \mathrm{tr}(F \wedge F) \}$$

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They are likely to be a very special subset of generic heterotic compactifications which will typically have torsion:

$$R_{\mu\nu} \sim H_{\mu\rho\lambda} H_{\nu}^{\rho\lambda} + \ldots$$

Generic compactifications should have few if any moduli other than the string dilaton.

What we want: a linear framework analogous to the linear sigma model (Witten) that allows us to build analogues of the quintic Calabi-Yau.

$$\sum_i z_i^5 = 0 \subset \mathbb{P}^4$$

We will need to discover new geometries since very few examples of torsional spaces are known.

Non-Compact Models

Basics: we will restrict to (0,2) theories built from chiral superfields

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Let's recall that the simplest (2,2) non-linear sigma models are defined by a choice of Kahler potential:

$$\mathcal{L} = \int d^4\theta \, K(\Phi, \bar{\Phi}) \qquad g_{i\bar{\jmath}} = \partial_i \partial_{\bar{\jmath}} K$$

For a (0,2) theory, the analogous data is a collection of one-forms:

$$\begin{aligned}\mathcal{L} &\sim \int d^2\theta \left(K_i(\Phi,\bar{\Phi})\partial_-\Phi^i + c.c.\right) \\ &\sim -g_{i\bar{\jmath}}\,\partial_\alpha\phi^i\partial^\alpha\phi^{\bar{\jmath}} + b_{i\bar{\jmath}}\,\epsilon^{\alpha\beta}\partial_\alpha\phi^i\partial_\beta\phi^{\bar{\jmath}} + \ldots.\end{aligned}$$

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Kahler => $K_i = \partial_i K$

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$$\mathcal{L}_{FI} \sim \frac{t}{4} \int d\theta^{+} \Upsilon + c.c. \sim -rD + \frac{\theta}{2\pi} F_{01}.$$

$e \to \infty$

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The moduli space is a toric variety:

$$D^{-1}(0)/U(1)$$

realized as a symplectic quotient of $\mathbb{C}^d$ for d charged fields by U(1) with moment map D.

In the IR limit, we can solve for the gauge field:

$$A_\mu = \frac{i}{2} \frac{\sum q_i \left(\bar{\phi}^i \partial_\mu \phi^i - \phi^i \partial_\mu \bar{\phi}^i \right)}{\sum q_i^2 |\phi^i|^2}.$$

This gives the space-time B-field:

$$B = \frac{\theta}{2\pi} dA = \epsilon^{\mu\nu} B_{i\bar{j}} \partial_\mu \phi^i \partial_\nu \phi^{\bar{j}}$$

If we can make θ effectively vary, we can generate a non-zero H=dB.

Modify the FI term which is a superpotential coupling:

$$\mathcal{L}_{FI} \sim \frac{t}{4} \int d\theta^+ f(\Phi) \Upsilon + c.c.$$

This has the following effect:

$$\frac{\theta}{2\pi} \to \frac{\theta}{2\pi} + \mathrm{Im}(f(\Phi))$$

$$V(\phi) \to \frac{e^2}{2} \left(\sum q_i |\phi^i|^2 + \mathrm{Re}(f) - r \right)^2$$

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Example: Conifold with Torsion

A single U(1) gauge group with charged matter:

$$\phi^i (i = 1, 2) \quad q_i = +1, \qquad \phi^m (m = 1, 2) \quad q_a = -1$$

$$|\phi^i|^2 - |\phi^m|^2 = r$$

Take a quadratic $f \sim f_{im} \phi^i \phi^m$. Higher powers are possible but the dilaton appears to blow up.

$$\phi_i = \bar{\phi}^i, \qquad \phi_{\bar{\imath}} = \phi^i, \qquad \tilde{\phi}_i = f_{im} \phi^m, \qquad \tilde{\phi}_{\bar{\imath}} = \bar{f}_{im} \bar{\phi}^m.$$

This leads to a B-field and metric which depend on a tunable deformation:

$$
\begin{aligned}
G_{i\bar{\jmath}} &= \delta_{i\bar{\jmath}} - \frac{\phi_i \phi_{\bar{\jmath}} - \widetilde{\phi}_i \widetilde{\phi}_{\bar{\jmath}}}{\sum |\phi|^2}, \\
B_{i\bar{\jmath}} &= -\frac{\phi_i \widetilde{\phi}_{\bar{\jmath}} - \phi_{\bar{\jmath}} \widetilde{\phi}_i}{\sum |\phi|^2}, \ldots
\end{aligned}
$$

This is a beautiful collection of non-compact torsional spaces.

Compact Models

The previous approach never involves quantized fluxes. Yet we expect flux quantization to play a central role:

$$\frac{1}{2\pi\alpha'}\int H \in 2\pi\mathbb{Z}$$

How do we build compact models?

Let's draw an analogy with N=1 D=4 gauge theory:

$$\int d^2x d\theta^+ \, \Upsilon \quad \Leftrightarrow \quad \int d^4x \, d^2\theta \, W^\alpha W_\alpha$$

$$\mathrm{Im} \int d^4x \, d^2\theta \, (\tau W^\alpha W_\alpha) \quad \rightarrow \quad \frac{1}{4g^2} F^2 + \frac{\theta}{32\pi^2} F \wedge F$$

$$\tau = \frac{8\pi}{g^2} + i\theta$$

Renormalization is tightly controlled by holomorphy,

$$\Lambda^b \rightarrow e^{2\pi i} \Lambda^b, \qquad \tau \sim \tau + 1$$

$$\tau(\mu) = \frac{b}{2\pi i} \log(\Lambda/\mu) + f(\Lambda^b, \phi)$$

We will allow log interactions for Υ in the fundamental theory.

Note that no scale is need to define the log in two dimensions.

$$\mathcal{L}_{FI} = \frac{i}{8\pi} \int d\theta^+ \, N_i^a \log\left(\Phi^i\right) \Upsilon^a + c.c.$$

Integers — Different gauge factors

We could also add additional single valued functions but let's focus on the log which has all the novelty.

This model is not classically gauge-invariant! Under a $U(1)^b$ gauge transformation:

$$\Phi^i \to e^{iQ_i^b \Lambda^b} \Phi^i$$

$$\delta\mathcal{L}_{FI} = -\left(\frac{N_i^a Q_i^b}{8\pi} \int d\theta^+ \, \Lambda^b \Upsilon^a + c.c. \right).$$

The antisymmetric part of this anomaly (in a,b) can be canceled by the classical coupling

$$\mathcal{L}_2 = \frac{1}{4\pi} \int d^2\theta^+ \, T^{ab} A^a V_-^b$$

where T^{ab} is antisymmetric. This coupling shifts by

$$\delta\mathcal{L}_2 = \left(-\frac{1}{8\pi} T^{ab} \int d\theta^+ \, \Lambda^a \Upsilon^b + c.c. \right).$$

On the other hand, the gauge theory is generally anomalous with a symmetric one-loop anomaly:

$$\mathcal{A}^{ab} = \sum_i Q_i^a Q_i^b - \sum_\alpha Q_\alpha^a Q_\beta^b$$

Right-movers (curvature) Left-movers (NS5-branes & bundle)

$$\delta \mathcal{L} = \left(\frac{\mathcal{A}^{ab}}{8\pi} \int d\theta^+ \, \Lambda^a \Upsilon^b + c.c. \right).$$

Choosing

$$T^{ab} + Q_i^{[a} N_i^{b]} = 0, \qquad \sum_i Q_i^{(a} N_i^{b)} - \mathcal{A}^{ab} = 0.$$

gives a quantum gauge invariant theory. These are intrinsically quantum models.

We can now add superpotentials to carve out surfaces in these generalizations of toric varietes. Introduce a left-moving charged fermionic superfield:

$$\bar{\mathfrak{D}}_{+}\Gamma = \sqrt{2}E(\Phi).$$

The superpotential couplings

$$\mathcal{L}_J = -\frac{1}{\sqrt{2}}\int d\theta^{+}\,\Gamma\cdot J(\Phi) + c.c.$$

give a bosonic potential

$$V = |E|^2 + |J|^2.$$

For a suitable choice of fields and charges, these give conformal models generalizing Calabi-Yau spaces.

Classical and Quantum Geometries

Let's get a feel for the structures that arise, starting with the classical geometries that generalize projective space.

Let's start with the case of one field (NS5-brane-like):

$$Q|\phi|^2 - N \log|\phi| = r.$$

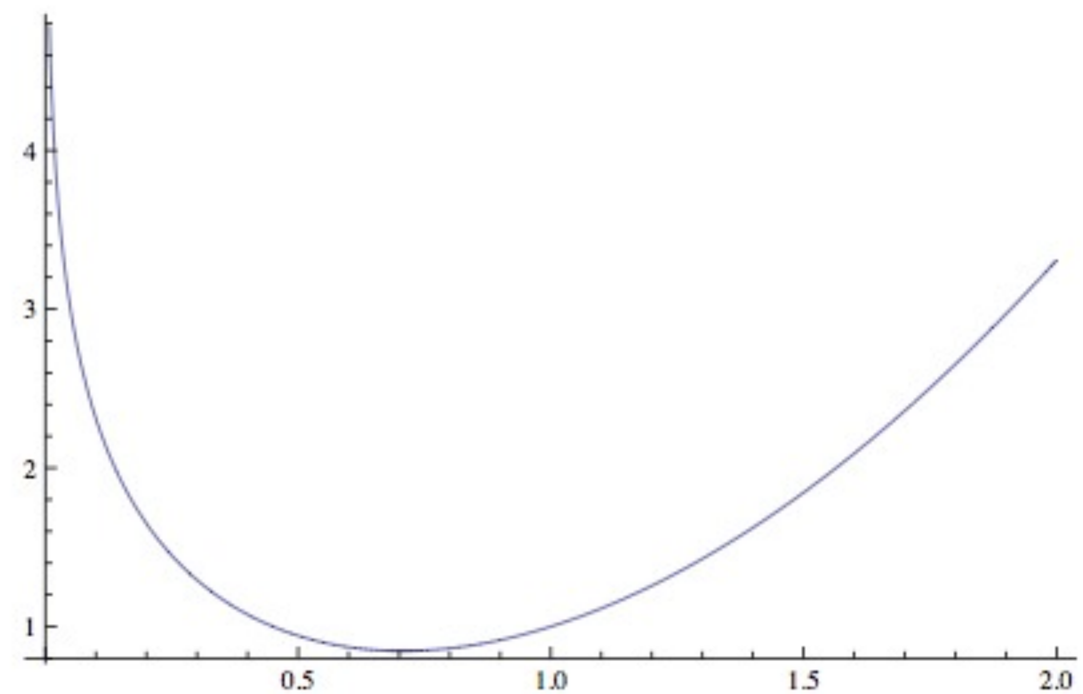

Figure 1: *A plot of* $|\phi|^2 - \log|\phi|$ *against* $|\phi|$.

There is a minimum at $|\phi|^2 = \frac{N}{2Q}$ which defines an r_{min}.

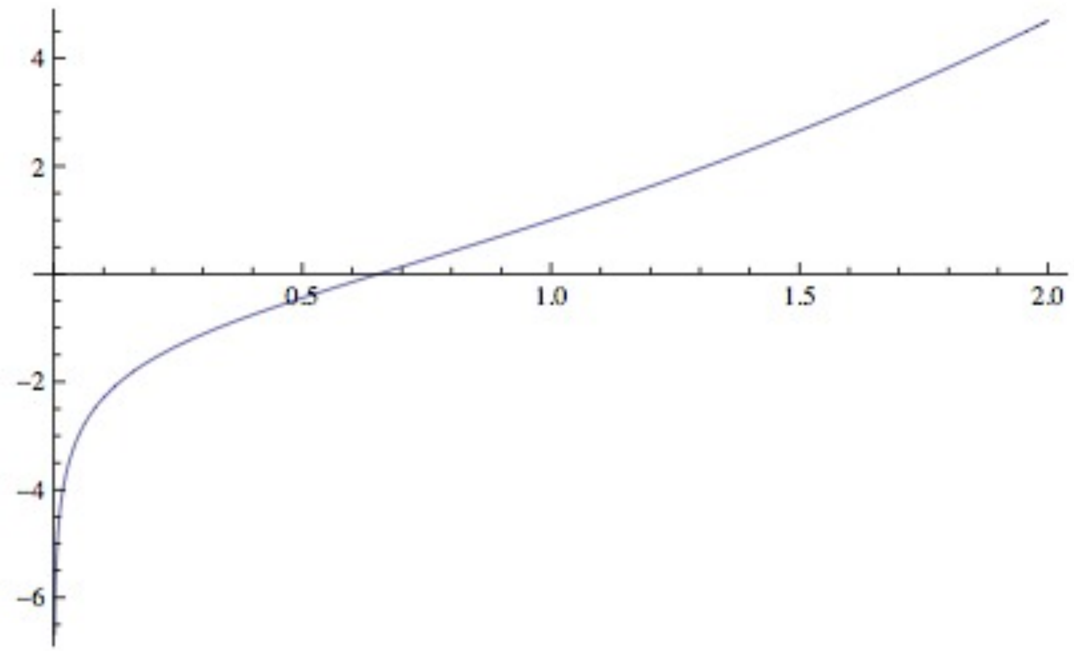

Figure 2: *A plot of* $|\phi|^2 + \log|\phi|$ *against* $|\phi|$.

For the other sign (anti-brane sign), there are solutions for all r.

Note that the log field cannot vanish!

Moving to the case of two fields:

No log interactions: $|\phi^0|^2 + |\phi^1|^2 = r$

Let's define the skeleton for this space to be the contour in the $(|\phi^0|, |\phi^1|)$ plane solving this equation.

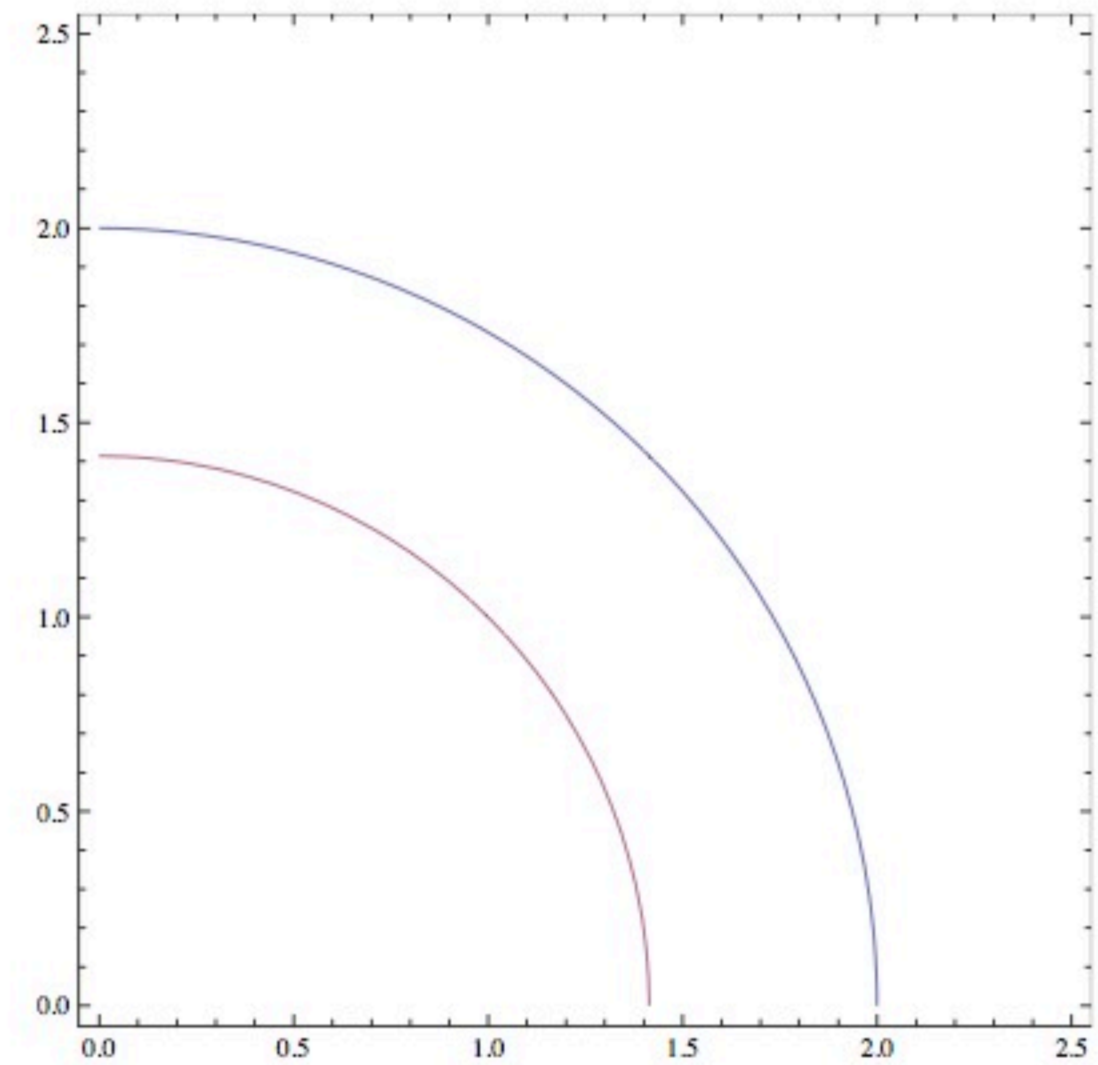

Figure 3: *A contour plot of* $|\phi^1|$ *versus* $|\phi^0|$ *for* $r = 2$ *and* $r = 4$.

The skeleton for the case giving S^2.

One log interaction: $|\phi^0|^2 + |\phi^1|^2 - N_0 \log |\phi^0| = r.$

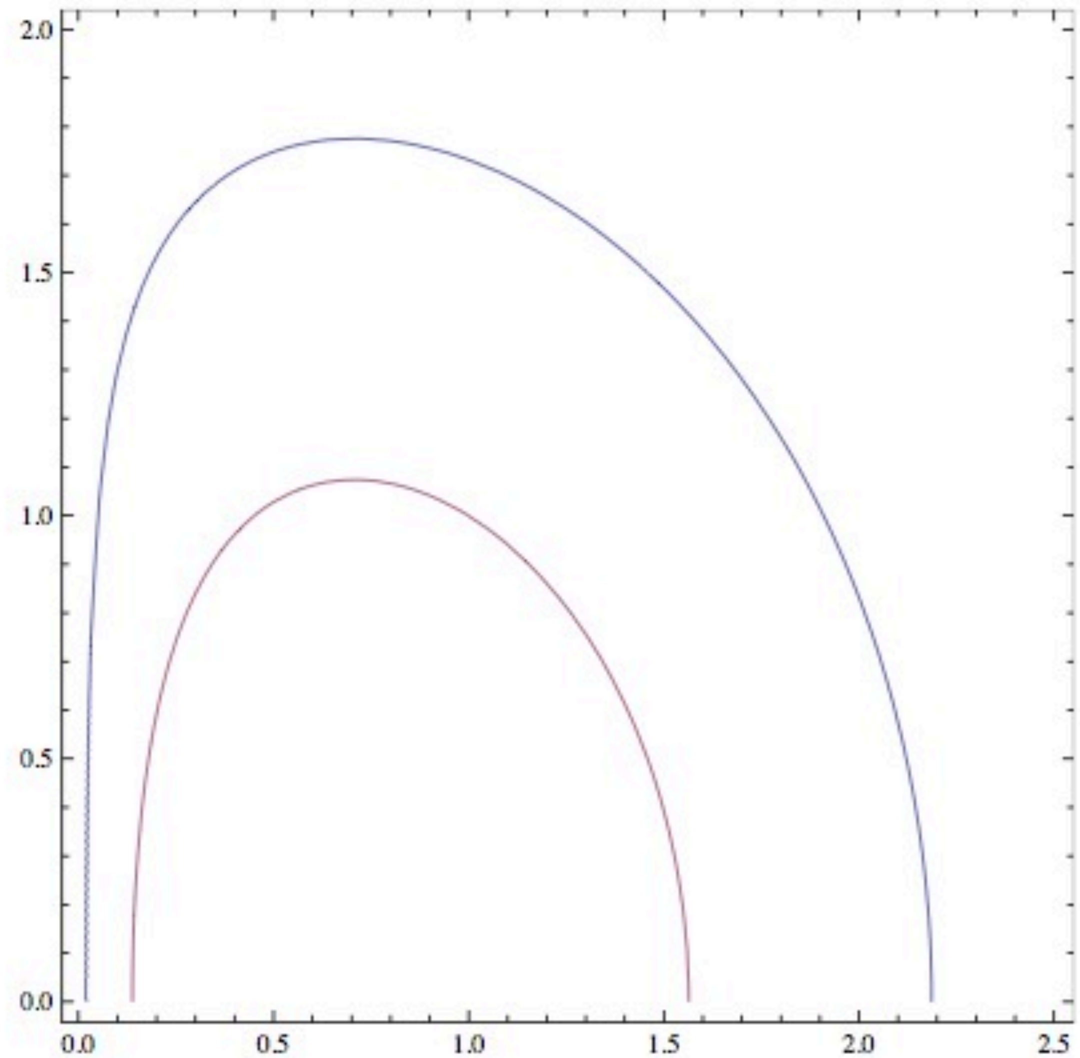

Figure 4: *A contour plot of* $|\phi^1|$ *versus* $|\phi^0|$ *for* $r = 2$ *and* $r = 4$*, with a single log interaction.*

Two log interactions:

$$|\phi^0|^2 + |\phi^1|^2 - N_0 \log|\phi^0| - N_1 \log|\phi^1| = r,$$

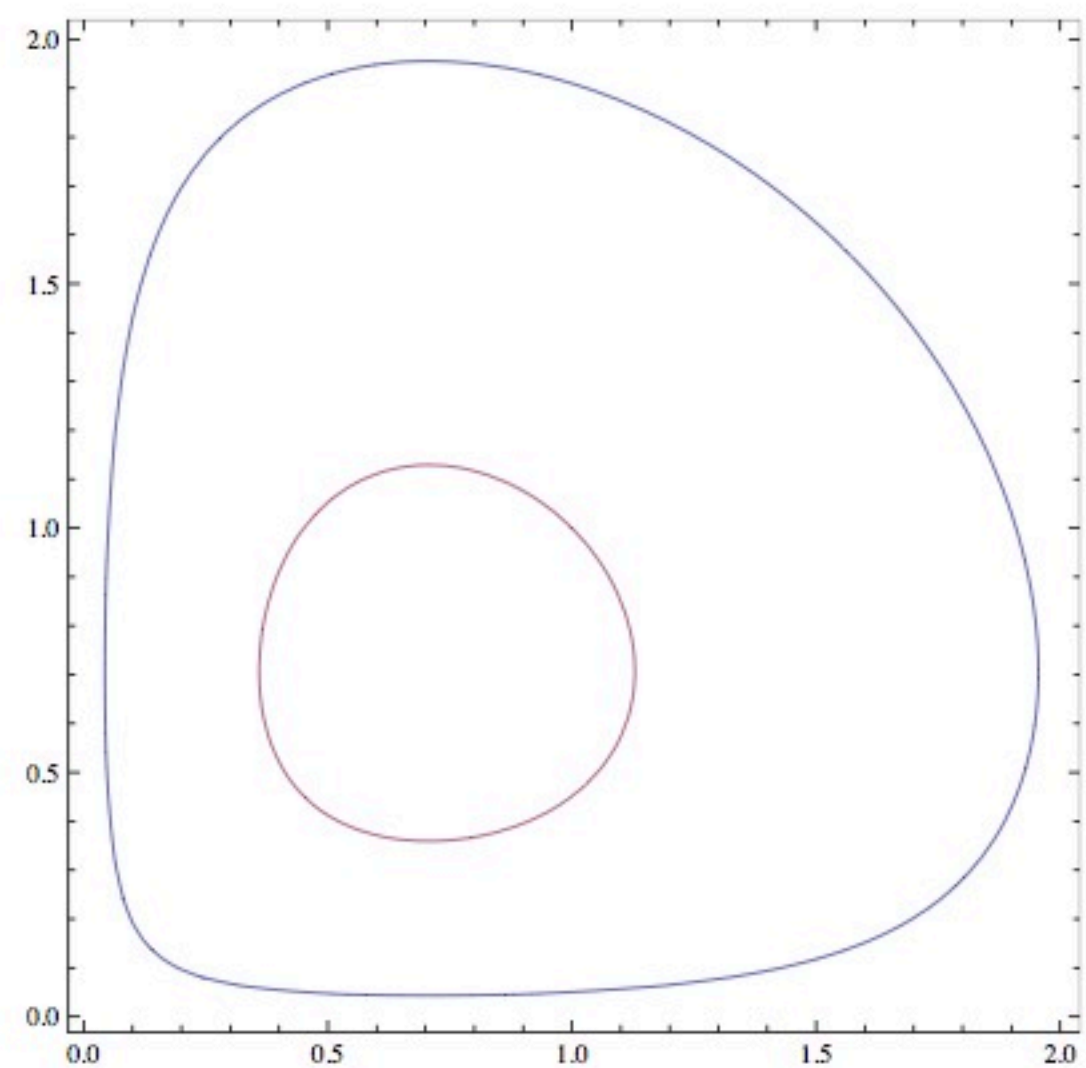

Figure 5: *A contour plot of* $|\phi^1|$ *versus* $|\phi^0|$ *for* $r = 2$ *and* $r = 4$*, with* $N_0 = N_1 = 1$.

For the case with many fields:

$$\sum_{i=0}^{d-1} |\phi^i|^2 - \sum_i N_i \log |\phi^i| = r$$

The moduli spaces take the form for 0 to d-1 log interactions,

$$\mathbb{P}^{d-1},\ S^{2d-2},\ S^{2d-3} \times S^1,\ S^{2d-4} \times (S^1)^2, \ldots,\ S^{d-1} \times (S^1)^{d-1}.$$

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