On $\mathcal{N} = 2$ Truncations of IIB on $T^{1,1}$

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based on: 1111.6567 [Liu, PS, Halmagyi]

Outline

Consistent Truncations of Supergravity Theories

$\mathcal{N} = 2$ Truncations on $T^{1,1}$

Important features from $\mathcal{N} = 2$ matter coupled gauged supergravity

Recap

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Today I'll focus on another motivation...

- Constructing and understanding string solutions

Conifold Solutions

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What can we learn by studying these reductions within actual supergravity truncations?

- Many times one directly reduces theory to one dimension (the "cone" coordinate over the compact manifold) and analyzes equations there. Embedding these into 5d supergravities gives another tool for analysis.
- Supergravity techniques allow for systematic construction of a scalar "superpotential."
- Perhaps knowledge of (5d) supergravity scalar coset will give insight into dualities/solution generating techniques.

Philosophy of consistent truncations

Dimensional Reduction

- Would like to dimensionally reduce IIB (or M-theory or whatever) on a compact manifold to an effective lower dimensional theory – in present case a five-dimensional supergravity.
- Usual procedure – KK reduction, gives infinite tower of states.
- Truncating the KK reduction to a subset of fields in such a way that the higher dimensional equations are satisfied is termed a "consistent truncation."

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Truncation procedure

- A convenient way to do this is to reduce on a set of forms defined on internal manifold which close under exterior differentiation and wedge products.
- Recently, this has been applied to many reductions, (nearly Kahler manifolds, cosets, SE_5 in M-theory and IIB, and various flux compactifications)

$SU(2) \times SU(2)$ Singlet Reduction on $T^{1,1}$

Structure of $T^{1,1}$ allows for many deformations

[Cassani,Faedo;Bena,Giecold,Graña,Halmagyi,Orsi]

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- Metric is $U(1)$ fiber over Kahler base – $\mathbb{CP}^1 \times \mathbb{CP}^1$ allows for "twisting" and "squashing", also there are two individual Kahler two-forms (one for each $\mathbb{CP}^1$.), and a holomorphic $(2,0)$ form.

$$ds_{10}^2 = e^{2u_3-2u_1} ds_5^2 + e^{2u_1+2u_2} E_1 \bar{E}_1 + e^{2u_1-2u_2} E_2' \bar{E}_2' + e^{-6u_3-2u_1} E_5 E_5$$

$$E_2' = E_2 + v \bar{E}_1$$

$$J_1 = \tfrac{i}{2} E_1 \wedge \bar{E}_1, \qquad J_2 = \tfrac{i}{2} E_2 \wedge \bar{E}_2, \qquad \Omega = E_1 \wedge E_2, \qquad E_5 = g_5 + A_1$$

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- Expanding forms yields eight vectors and eleven scalars and allows for three form flux:

$$\begin{aligned} \widetilde{F}_5 \;\; = \;\; & (1+*)[e^Z J_1 \wedge J_2 \wedge E_5 + K_1 \wedge J_1 \wedge J_2 + K_{21} \wedge J_1 \wedge E_5 \\ & + K_{22} \wedge J_2 \wedge E_5 + 2\Re(L_2 \wedge \Omega \wedge E_5)] \end{aligned}$$

$$B_2^i = b_2^i + b_1^i \wedge E_5 + c_0^i J_+ + e_0^i J_- + 2\Re(b_0^i \Omega),$$

$$F_3^i = dB_2^i + j_0^i J_- \wedge E_5.$$

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The three $\mathcal{N} = 2$ Truncations

Full reduction – $\mathcal{N} = 4$ coupled to three vector multiplets

[Cassani,Faedo;Bena,Giecold,Graña,Halmagyi,Orsi]

$\underline{\mathcal{N} = 4 \text{ theory}}$

gravity: metric $+ \, 6 \times$ 1-forms $+ \, 1$ scalar (u_3),

3 vectors ($\mathcal{N} = 4$): $3 \times$ 1-forms $+ \, (u_1, u_2, k, c_0^i, e_0^i, b_0^i, \bar{b}_0^i, \tau, \bar{\tau}, v, \bar{v})$.

$$\text{scalar-coset:} \frac{SO(5,3)}{SO(5) \times SO(3)} \times SO(1,1)$$

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$$\text{scalar-coset:} \frac{SO(5,3)}{SO(5) \times SO(3)} \times SO(1,1)$$

- This contains the Papadopoulos-Tseytlin ansatz as a subtruncation. Which has been used to discuss structure of many solutions on the conifold.

The three $\mathcal{N} = 2$ Truncations

1. **Betti-vector sector - 2 hyper-multiplets & 2 vector multiplets**

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	Betti-vector truncation
gravity + 2 vectors:	$(g_{\mu\nu}; A_1, k_{11}, k_{12}; u_2, u_3)\,,$
2 hypers:	$(u_1, k, \tau, \bar{\tau}, b_0^i, \bar{b}_0^i)\,.$

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- This can be further truncated to the universal SE_5 sector. Which in turn allows a truncation to pure $\mathcal{N} = 2$ supergravity.

The three $\mathcal{N} = 2$ Truncations

2. Betti-hyper sector - 3 hyper-multiplets & 1 vector multiplets

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	Betti-hyper truncation
gravity + vector:	$(g_{\mu\nu}; A_1, k_{11} + k_{12}; u_3)\,,$
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- This also can be further truncated to the universal SE_5 sector and to pure $\mathcal{N} = 2$ supergravity.
- Contains the Klebanov-Strassler solution.

The three $\mathcal{N} = 2$ Truncations

3. NS sector - 2 hyper-multiplets & 2 vector multiplets

$$\text{scalar-coset:} \frac{SO(4,2)}{SO(4) \times SO(2)} \times SO(1,1) \times SO(1,1)$$

	NS truncation
gravity + 2 vectors:	$(g_{\mu\nu}; A_1, b_1^2, b_2^2; \phi + 4u_1, u_3)$,
2 hypers:	$(\phi - 4u_1, u_2, c_0^2, e_0^2, b_0^2, \bar{b}_0^2, v, \bar{v})$.

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- Same scalar coset as Betti-vector, but with different gauging.
- Does not allow a truncation to minimal $\mathcal{N} = 2$ supergravity.
- Contains the Maldacena-Nunez solution.
- More generally, there is an interpolating solution which demonstrates a geometric transition and can be related to the baryonic branch of the Klebanov-Strassler solution through a TST transformation [Maldacena,Martelli].

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- Idea: use techniques of 5d gauged supergravity to understand various features.
- In particular, we wish to understand the existence of scalar superpotentials and the constraints imposed by supersymmetry.

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But first, I should explain the relevant details of 5d gauged supergravity...

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Review of $5d$ $\mathcal{N} = 2$ matter coupled gauged supergravity

Supersymmetry Variations [Ceresole,Dall'agata]

Gravity multiplet coupled to vector and hyper matter.

$$\begin{aligned}
\delta\psi_{\mu\, i} &= [D_\mu + \frac{i}{24}X^I(\gamma_\mu{}^{\nu\rho} - 4\delta^\nu_\mu\gamma^\rho)F_{I\,\nu\rho}]\epsilon_i + \frac{i}{6}X^I(P_I)_i{}^j\epsilon_j \\
\delta\lambda^x_i &= (-\frac{i}{2}\gamma\cdot D\phi^x - \frac{1}{4}g^{xy}\partial_y X^I\gamma^{\mu\nu}F_{I\,\mu\nu})\epsilon_i - g^{xy}\partial_y X^I(P_I)_i{}^j\epsilon_j \\
\delta\zeta^A &= f^{i\,A}_X(-\frac{i}{2}\gamma\cdot Dq^X + \frac{1}{2}X^I K^X_I)\epsilon_i
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Important Features

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Important Features

- Theory "specified" by scalar manifold and killing vectors K^X_I or prepotentials $(P_I)_i{}^j \equiv P^r_I(\sigma^r)^i{}_j$.
- Can define a "superpotential" : $W = \sqrt{P^r P^r}$ $(P^r \equiv X^I P^r_I)$ – which under certain conditions satisfies:

$$V = 2g^{\Lambda\Sigma}\partial_\Lambda W\partial_\Sigma W - \frac{4}{3}W^2$$

BPS Domain Wall Equations

Domain Wall Ansatz & BPS Equations

Break 5d space-time metric into a warped product of radial coordinate and Minkowski space,

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$$\partial_x Q^r = 0\,, \text{where } Q^r \equiv P^r / \sqrt{P^r P^r}$$

which can be shown to be equivalent to all prepotentials being “parallel”

$$\vec{P}_I \times \vec{P}_J = 0\,, \qquad I, J = 1, .., n_V + 1.$$

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True superpotential

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- In which case, solutions of BPS conditions are necessarily SUSY

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⋆ In addition W_{fake} is often constructed by considering the action of a reduced one-dimensional system in r but relation to P_3 not completely understood

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Fake superpotential

- $W_{fake} \equiv P^3$ (because it works)
- In general, W_{fake} satisfies superpotential/potential relation and yields BPS-like system of first order equations
- Solutions of "BPS" flow equations not necessarily SUSY
- However, when $\vec{P}_I \times \vec{P}_J = 0$ is enforced, solutions satisfy BPS conditions

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Lets look at BPS conditions for domain wall solutions in the Betti-hyper and NS truncations...

Betti-hyper sector

Prepotentials:

$$
\begin{aligned}
P_0 &= -i[(\tfrac{3}{2\rho_2}(1+|\rho|^2) - \tfrac{1}{2}e^{-4u_1}e^{Z})\sigma_3 \\
&\quad + \tfrac{1}{2\rho}e^{-2u_1}v_i(3(\bar\rho - i)^2\bar b_0^i + 3(\bar\rho + i)^2 b_0^i - (1 - i\bar\rho)(1 + i\bar\rho)j_0^i)\sigma_+ \\
&\quad - \tfrac{1}{2\rho}e^{-2u_1}\bar v_i(3(\rho + i)^2 b_0^i + 3(\rho - i)^2\bar b_0^i - (1 + i\rho)(1 - i\rho)j_0^i)\sigma_-]\,, \\
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P_1 &= -2ie^{-4u_1}\sigma_3\,.
\end{aligned}$$

A fake superpotential emerges from $P^3 (= X^I P_I^3)$ (reproduces potential but is known to have non-SUSY solutions!):

$$W_{fake} \equiv P^3 = -\frac{1}{2}e^{-4u_1+4u_3}e^{Z} + 2e^{-4u_1-2u_3} + \frac{3}{2\tau_2}(1+|\tau|^2)e^{4u_3}\,.$$

Betti-hyper sector

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A fake superpotential emerges from $P^3(= X^I P_I^3)$ (reproduces potential but is known to have non-SUSY solutions!):

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- This slightly generalizes superpotential constructed in literature for the KS ansatz.

Betti-hyper sector

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- Constraint $\vec{P}_I \times \vec{P}_J = 0$ in general not satisfied for solutions to the W_{fake} system.
- W_{fake} also exists in Betti-vector and generic SE reductions.

A non-supersymmetric solution

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f_{KS}(t) &= \frac{(-t\coth t + 1)}{2\sinh t}(-1 + \cosh t) \\
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\end{aligned}$$

- $SL(2,R)$ rotated version of a solution of Kuperstein and Sonnenschein
- $\vec{P}_0 \times \vec{P}_1 \neq 0$ unless $C_1 = \tilde{C}_1 = 0$
- Also, unless $C_i = \tilde{C}_i = 0$ the solution is singular

NS-sector

Prepotentials:

$$\begin{aligned} P_0 &= i\Big[\Big(3-\tfrac{1}{2}e^{\phi/2-2u_1}(e^{-2u_2}((1+|v|^2)P-6(vb_0+\bar{v}\bar{b}_0))-e^{2u_2}P)\Big)\sigma_3 \\ &\qquad -\Big(3\bar{v}+2ie^{\phi/2-2u_1}(3ib_0-\tfrac{i}{2}\bar{v}P)\Big)\sigma_+-(c.c)\sigma_-\Big]\,, \\ P_1 &= i\Big[e^{\phi/2-2u_1}(e^{-2u_2}(1-|v|^2)+e^{2u_2})\sigma_3-2\bar{v}e^{\phi/2-2u_1}\sigma_+-2ve^{\phi/2-2u_1}\sigma_-\Big] \\ P_2 &= 0\,. \end{aligned}$$

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- However, can construct $W_{true} = \sqrt{P^r P^r}$ while imposing paraller constraint.
- W_{true} system then reproduces an ansatz of Maldacena and Martelli which maps to the baryonic branch of the KS gauge theory via TST transformation.

Outline

Consistent Truncations of Supergravity Theories

$\mathcal{N} = 2$ Truncations on $T^{1,1}$

Important features from $\mathcal{N} = 2$ matter coupled gauged supergravity

Recap

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In short, I think consistent truncations of maximal supergravity theories provide a useful tool in the study of string/M-theory solutions.

Thank you!