Momentum-carrying waves on D1-D5 microstate geometries

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Based on 1112.6413 and 1202.6421

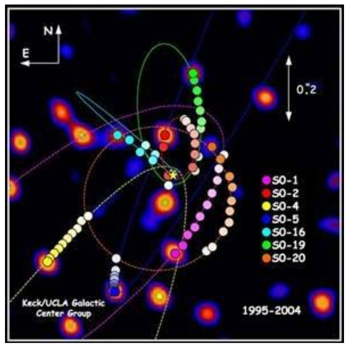
with Samir Mathur

Outline

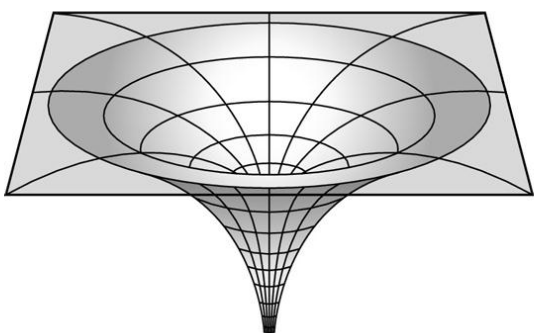
1. Black holes in string theory
2. D1-D5 system
3. Momentum-carrying perturbations
   on D1-D5 geometries

What is a Black Hole?

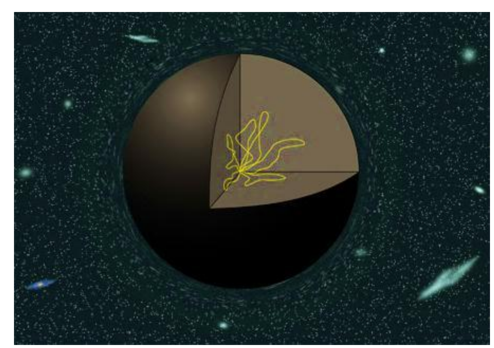
Physical: dark, heavy, compact bound state of matter



Classical: geometry with horizon



Quantum: bound state in quantum gravity theory

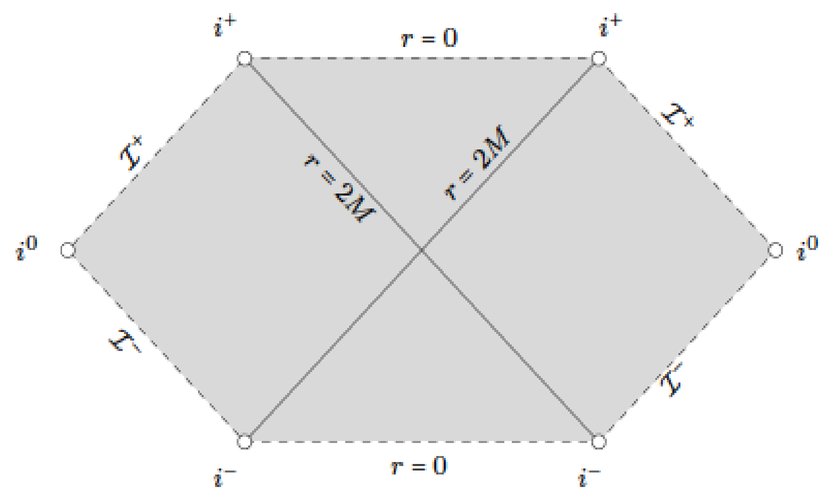


Why Black Holes?

- Strong evidence for astrophysical Black holes
- Seek unified theory of Nature $\rightarrow$ quantum gravity (QG)
- String theory: leading candidate
- Major tests of any QG theory:

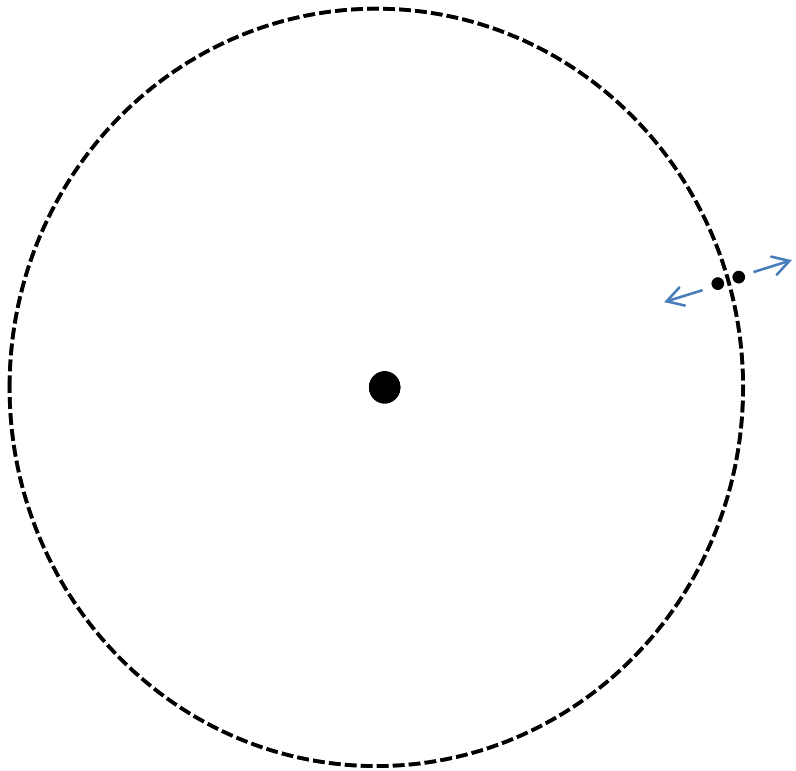
  i) UV finiteness

  ii) Information paradox



The Information Paradox

BH Horizon:
normal lab physics
(small curvature)



Hawking radiation:
pair creation

$\rightarrow$ entangled pair

- Endpoint of process: violation of unitarity or exotic remnants. Hawking '75

The Information Paradox

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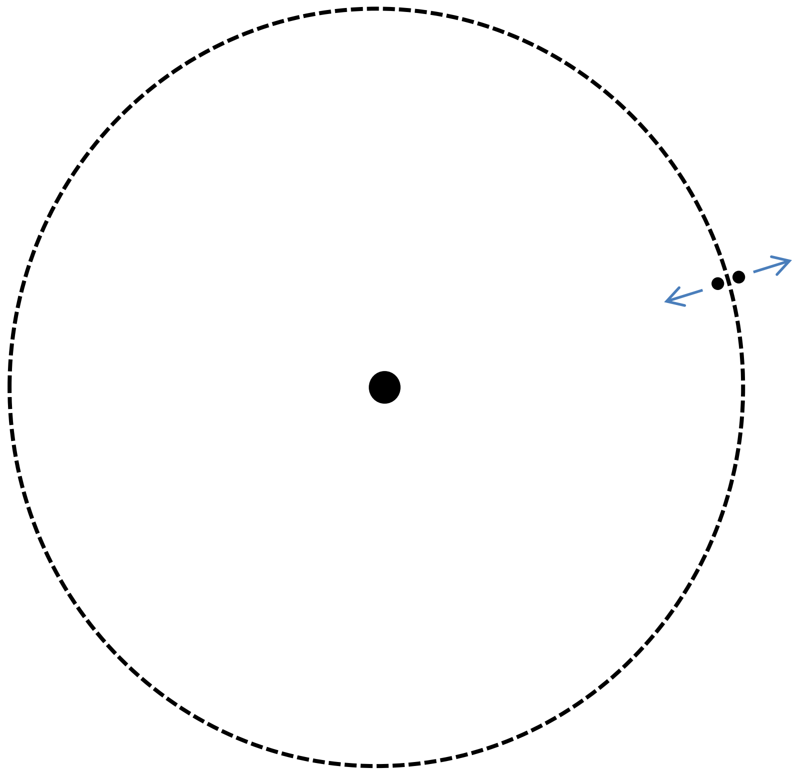

Hawking radiation:
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- Endpoint of process: violation of unitarity or exotic remnants. Hawking '75
- Conclusions robust against arbitrary small corrections Mathur '09

Black hole hair

- Bekenstein-Hawking entropy $S \;\; \rightarrow \;\; e^S$ microstates
- Can physics of individual microstates modify Hawking calculation?
- Many searches for Black hole 'hair': deformations at the horizon.
- In classical gravity, many 'no-hair' theorems resulted

Israel '67, Carter '71, Price '72, Robinson '75,...

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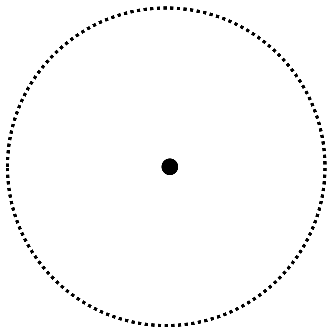
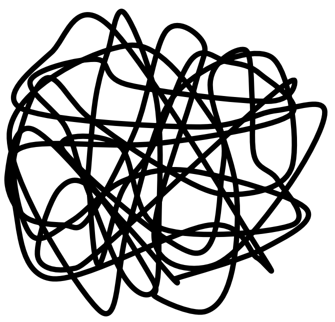
Israel '67, Carter '71, Price '72, Robinson '75,…

In String theory, we do find hair. Suggests that

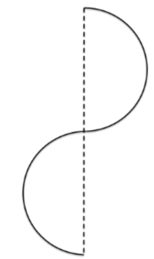
- Quantum effects important at would-be-horizon (fuzz)
- Bound states have non-trivial size (ball)





"Fuzzball"

Two-charge Black hole



- Simplest example: multiwound fundamental string + momentum
- Entropy reproduced by microscopic string states

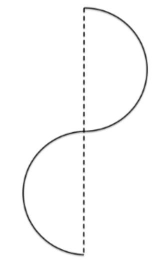
Sen '94

- For classical profiles, string sources good supergravity background

Dabholkar, Gauntlett, Harvey, Waldram '95, Callan, Maldacena, Peet '95
Lunin, Mathur '01

- Transverse vibrations only $\rightarrow$ non-trivial size
- Classical profiles $\leftrightarrow$ coherent states in usual way
- No horizons; source at location of string

Two-charge Black hole



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- NS1-P is U-dual to D1-D5 bound state

Lunin, Mathur '01

- Configurations are everywhere smooth in D1-D5 frame

Lunin, Maldacena, Maoz '02

- Caveat: two-charge Black hole is string-scale sized.

D1-D5-P: three charges

- Add momentum to D1-D5 $\rightarrow$ macroscopic black hole
- Entropy reproduced from microscopic degrees of freedom

Strominger, Vafa '96

- Many three-charge states constructed

Giusto, Mathur, Saxena, Srivastava,…
Bena, Bobev, de Boer, Shigemori, Wang, Warner,…

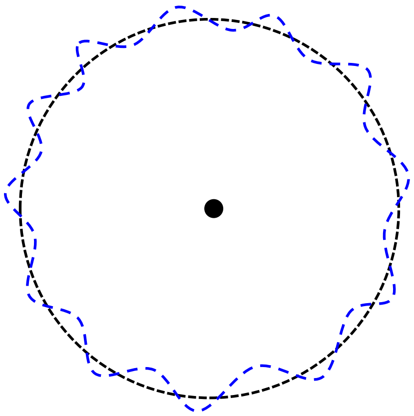
- Non-extremal states: unitary Hawking radiation

Das, Mathur '96
Jejjala, Madden, Ross, Titchener '05, Cardoso, Dias, Hovdebo, Myers '05
Chowdhury, Mathur '07, '08

- Open question: how large a subset of the BH degrees of freedom are well-described by smooth horizonless supergravity solutions?

Black string hair: early attempts

- Black string (with horizon) + travelling waves – heterotic / D1-D5-P



Larsen, Wilczek '95

Cvetic & Tseytlin '95, Tseytlin '96

Horowitz & Marolf '96

- Turns out that these solutions generically have curvature singularities at horizon

Kaloper & Myers '96, Horowitz & Yang '97

- An instance of the 'no hair' theorem

This talk

- Construct D1-D5-P states as perturbations of D1-D5 backgrounds
- Perturbations carry momentum in compact directions
- Analogue of early attempts at D1-D5-P hair,

  $\rightarrow$ this time no horizon so no singularities

- Construction motivated by old work on AdS_3 and asymptotic symmetries

  Brown, Henneaux '84

- Connect to ideas of states localized at boundary of AdS_3

  $\rightarrow$ $U(1)$ in $\mathrm{AdS}_5/\mathrm{CFT}_4$ & Singletons

  Witten '98

D1-D5 system: setup

We work in type IIB string theory on $\mathbb{R}^{1,4} \times S^1 \times T^4$

$$\begin{array}{ccccc} \mathbb{R}^{1,4} & \times & S^1 & \times & T^4 \\ t, x^\mu & & y & & z^i \end{array}$$

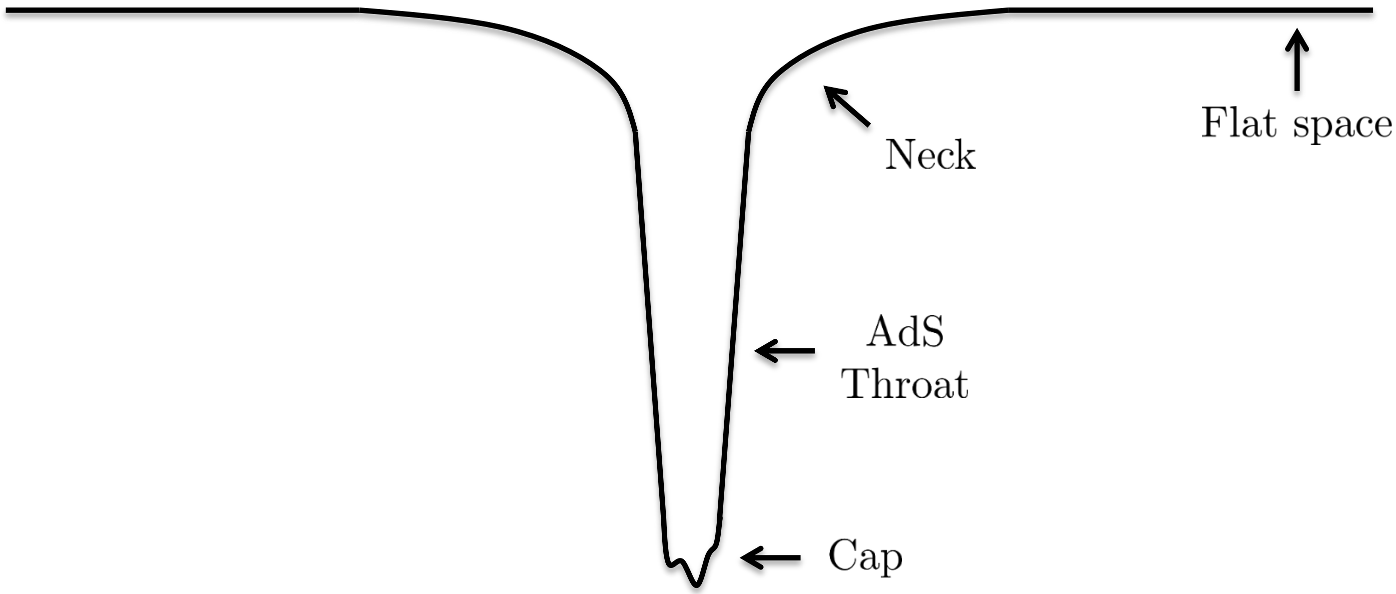
- Radius of S^1 : R_y
- Wrap n_1 D1 branes on S^1
- Wrap n_5 D5 branes on $S^1 \times T^4$

The bound state creates a geometry with D1 and D5 charges

$$Q_1 = \frac{1}{V}(2\pi)^4 g\alpha'^3\, n_1 \,, \qquad\qquad Q_5 = g\alpha'\, n_5$$

For simplicity, set $Q_1 = Q_5 = Q$.

To get an AdS throat, take $\sqrt{Q} \ll R_y$. Structure of geometry is then:



The throat is locally $AdS_3 \times S^3 \times T^4$.

D1-D5 CFT

- Worldvolume gauge theory on D1-D5 bound state flows in IR to a (4,4) SCFT.

- Orbifold point in moduli space: Free SCFT on $(T^4)^N/S_N$, $N = n_1 n_5$.

Symmetry generators:	(L)		
• $\text{Virasoro}_\text{L} \times \text{Virasoro}_\text{R}$	L_{-n}		
• R symmetries $SU(2)_\text{L} \times SU(2)_\text{R}$	J^a_{-n}	$a = 1,2,3$	
• U(1) currents of T^4 translations	J^i_{-n}	$i = 5,\ldots,8$	←

U(1) currents J^i

- Orbifold CFT on $(T^4)^N/S_N$: N copies of T^4 sigma model, fields

$$X^i\,, \qquad \psi^i\,, \qquad \bar{\psi}^i\,.$$

- U(1) currents of T^4 translations in copy r :

$$J^{(r),i} \;=\; \partial X^{(r),i} \qquad \Rightarrow \qquad J^{(r),i}_{-n} \;=\; \alpha^{(r),i}_{-n}\,.$$

→ Total U(1) currents:

$$J^i_{-n} \;=\; \sum_{r=1}^{N} \alpha^{(r),i}_{-n}\,.$$

- Modes of U(1) currents are simplest excitations of the CFT: a bosonic oscillator, symmetrized over the N copies.

Asymptotic Symmetries

- Asymptotic symmetry group (ASG): symmetries preserving asymptotics of the space
- ASG of AdS_3 : $\mathrm{Virasoro}_{\mathrm{L}} \times \mathrm{Virasoro}_{\mathrm{R}}$.

Brown, Henneaux '84

- D1-D5 AdS/CFT version:

$$
\begin{array}{ccccc}
AdS_3 & \times & S^3 & \times & T^4 \\
\updownarrow & & \updownarrow & & \updownarrow \\
L_{-n} & & J^a_{-n} & & J^i_{-n} \\
 & & & & \uparrow
\end{array}
$$

Asymptotic Symmetries

- Virasoro generator L_{-n} : diffeomorphism generated by $(v = t - y)$

$$\xi^L_{-n} \;=\; e^{-inv}\partial_v \qquad + \qquad \text{(corrections)}$$

$$\left[\xi^L_m, \xi^L_n\right] \;=\; i(m-n)\xi^L_{m+n}\,.$$

Brown, Henneaux '84

- T^4 translations – choose one torus direction z :

$$\xi^z_{-n} \;=\; e^{-inv}\partial_z \qquad\qquad \left[\xi^L_m, \xi^z_n\right] \;=\; i(m-n)\xi^z_{m+n}\,.$$

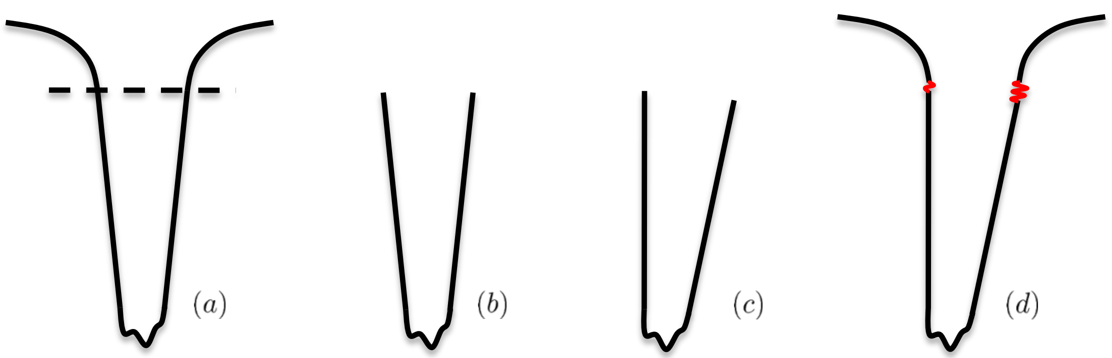
$\rightarrow$ ξ^z_{-n} generates metric component h_{vz} .

Form of the perturbation

- Take a D1-D5 ground state $|0\rangle_R$ and seek dual of $J^z_{-n}|0\rangle_R$
- In the $AdS_3 \times S^3 \times T^4$ throat+cap, J^z_{-n} is diffeomorphism along T^4
- However $J^z_{-n}|0\rangle_R$ has higher energy than $|0\rangle_R$

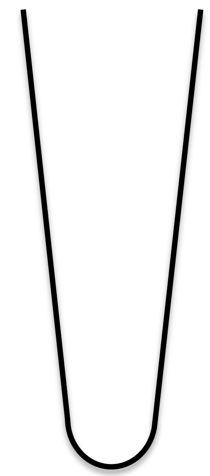
  $\rightarrow$ perturbation can't be a diffeomorphism everywhere

- Perturbation is non-trivial in the neck region between throat and flat asymptotics.



The background geometry

The background geometry we use is the simplest one:

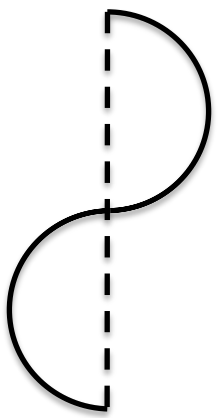


- Cap geometry is global AdS
- Corresponds to particular Ramond ground state obtained from spectral flow of NS vacuum

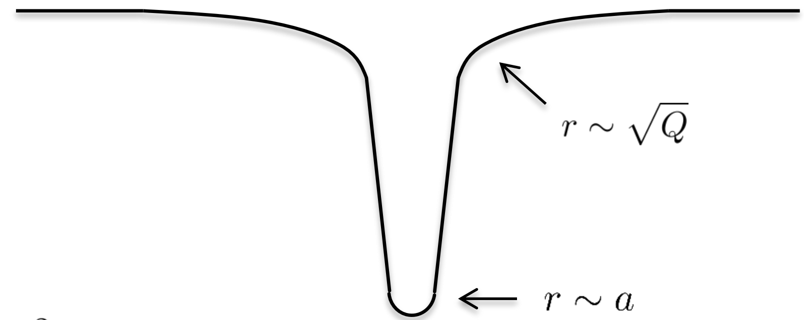
Maldacena, Maoz '00,

Balasubramanian, de Boer, Keski-Vakkuri, Ross '00

This background is U-dual to an NS1-P profile which is a single helix.



Lunin, Mathur '01



Full background fields:

$$
\begin{aligned}
ds^2 &= -\frac{1}{h}\left(dt^2 - dy^2\right) + hf\left(d\theta^2 + \frac{dr^2}{r^2+a^2}\right) \\
&\quad + h\left[\left(r^2 + \frac{a^2Q^2\cos^2\theta}{h^2f^2}\right)\cos^2\theta d\psi^2 + \left(r^2 + a^2 - \frac{a^2Q^2\sin^2\theta}{h^2f^2}\right)\sin^2\theta d\phi^2\right] \\
&\quad \frac{2Q}{Q+f}\left[\left(-a\cos^2\theta\, d\psi\right)dy + \left(-a\sin^2\theta\, d\phi\right)dt\right] + dz^i dz^i\,,
\end{aligned}
$$

$$
C^{(2)}_{ty} = -\frac{Q}{Q+f}\,, \qquad C^{(2)}_{\phi\psi} = Q\cos^2\theta + \frac{Qa^2\sin^2\theta\cos^2\theta}{Q+f}
$$

$$
C^{(2)}_{t\psi} = \frac{Q}{Q+f}\left(-a\cos^2\theta\right) \qquad C^{(2)}_{y\phi} = \frac{Q}{Q+f}\left(-a\sin^2\theta\right)
$$

where

$$
a = \frac{Q}{R_y}, \qquad f = r^2 + a^2\cos^2\theta, \qquad h = 1 + \frac{Q}{f}\,.
$$

Field equations

- Perturbation:

$$g = \bar{g} + \hat{\epsilon}\, h\,, \qquad C^{(2)} = \bar{C}^{(2)} + \hat{\epsilon}\, C\,, \qquad F^{(3)} = dC^{(2)}$$

- Equations of motion:

$$\begin{aligned} R_{AB} &= \frac{1}{4} F^{(3)}_{ACD} F_B^{(3)CD}\,, \\ F^{(3)}_{MNP}{}^{;P} &= 0 \end{aligned}$$

- Ansatz: choose one torus direction z and switch on $h_{Az}\,,\, C_{Az}\,.$

- Field equations separate into one eqn for

  $h_{Az} + C_{Az}$ and one for $h_{Az} - C_{Az}\,.$

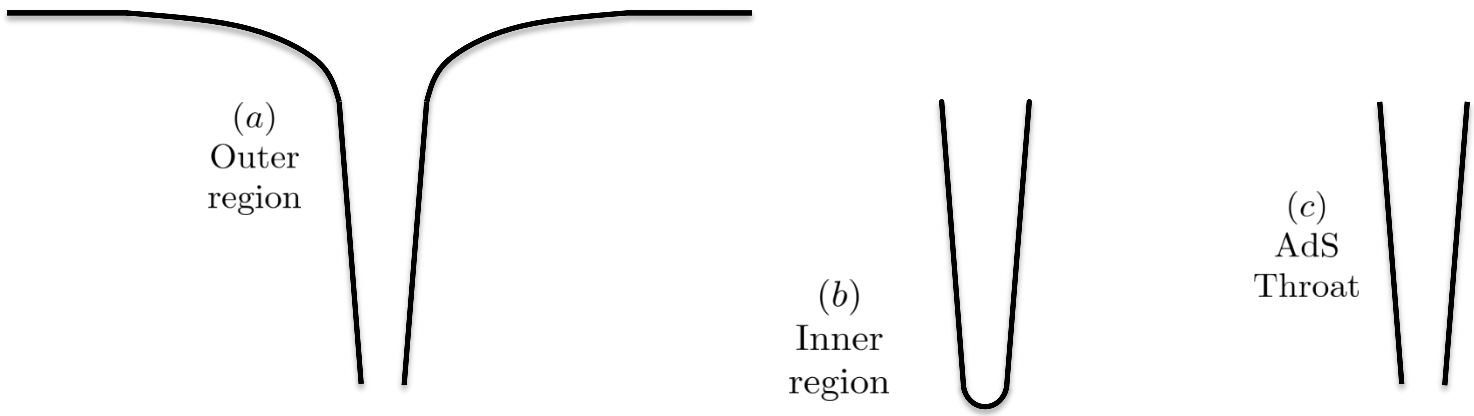
- Desired solution has $h_{Az} + C_{Az} = 0\,.$

Approximation procedure

Method for constructing the perturbation: Mathur, Saxena, Srivastava '03

- Solve equations of motion in 'outer' and 'inner' regions separately
- Match the solutions in the 'throat'

Mathur, Turton 1112.6413



- Extend to closed-form perturbation on full background

Mathur, Turton 1202.6421

J^z perturbation

- The perturbation is given by:

$$
\begin{aligned}
h_{vz} &= e^{-in\frac{v}{R_y}} \left(\frac{r^2}{r^2+a^2}\right)^{\frac{n}{2}} \frac{Q}{Q+f}\,, \\
h_{rz} &= e^{-in\frac{v}{R_y}} \left(\frac{r^2}{r^2+a^2}\right)^{\frac{n}{2}} \frac{i\,a\,Q}{r(r^2+a^2)}\,, \\
h_{\psi z} &= e^{-in\frac{v}{R_y}} \left(\frac{r^2}{r^2+a^2}\right)^{\frac{n}{2}} \frac{Q}{Q+f}\left(-a\cos^2\theta\right), \\
h_{\phi z} &= e^{-in\frac{v}{R_y}} \left(\frac{r^2}{r^2+a^2}\right)^{\frac{n}{2}} \frac{Q}{Q+f}\left(-a\sin^2\theta\right)
\end{aligned}
$$

and

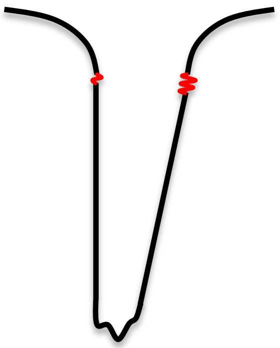
$$C_{Az} = -h_{Az}\,,$$

where

$$f = r^2 + a^2\cos^2\theta\,.$$

Properties of the perturbation

- Smooth at $r = 0$ and normalizable as $r \to$ infinity
- To leading order, perturbation reduces in the throat+cap to diffeomorphism generated by ξ
& gauge trans of $C^{(2)}$ generated by Λ :



$$\begin{aligned} \xi_z &= i\frac{R_y}{n} e^{-in\frac{v}{R_y}} \left(\frac{r^2}{r^2 + a^2} \right)^{\frac{n}{2}} , \\ \Lambda_z &= -\xi_z \, . \end{aligned}$$

- At top of throat, ξ is just a translation in z
as a function of v :

$$\xi_z \sim e^{-in\frac{v}{R_y}} \, .$$

Properties of the perturbation

- Energy = momentum = $\dfrac{n}{R_y}$ above ground state

- Upon quantization, only $n > 0$ will be physical excitations

- Generalizes to a class of other background geometries labelled by parameter

$$k = 1, 2, \ldots, N \,, \qquad N = n_1 n_5 \,.$$

- Can also examine in 'black ring' regime of parameters $\sqrt{Q} \gg R_y$ relevant to D1-D5-P black ring

Elvang, Emparan, Mateos, Reall '04

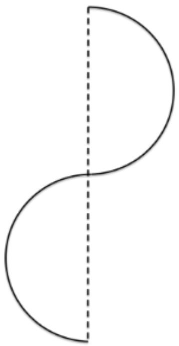
$\rightarrow$ find perturbation spreads out to scale of ring

Quadratic order (in progress)

- At quadratic order, expect to see momentum charge along $v \rightarrow h_{vv}$
- How will this be determined?
- Example: fundamental string with momentum

  $\rightarrow$ at location of source, no momentum along string

  Dabholkar, Gauntlett, Harvey, Waldram '95

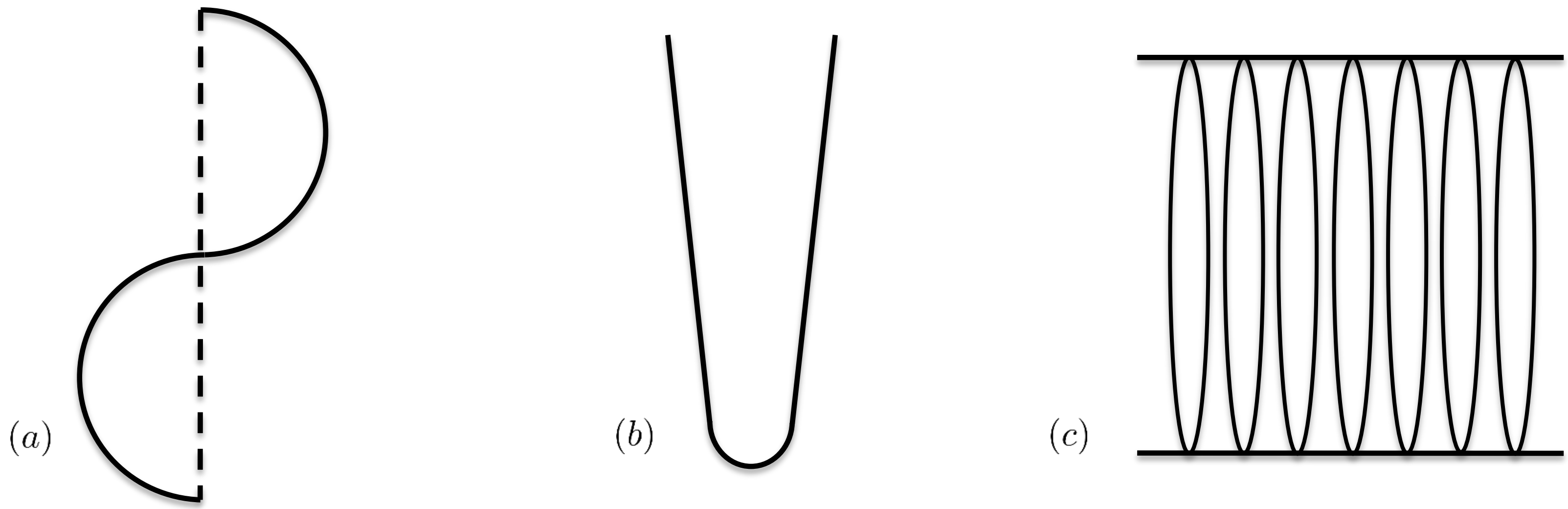


- In our case, D1-D5 geometries have smooth caps, no sources

  $\rightarrow$ smoothness of perturbation may fix momentum charge.

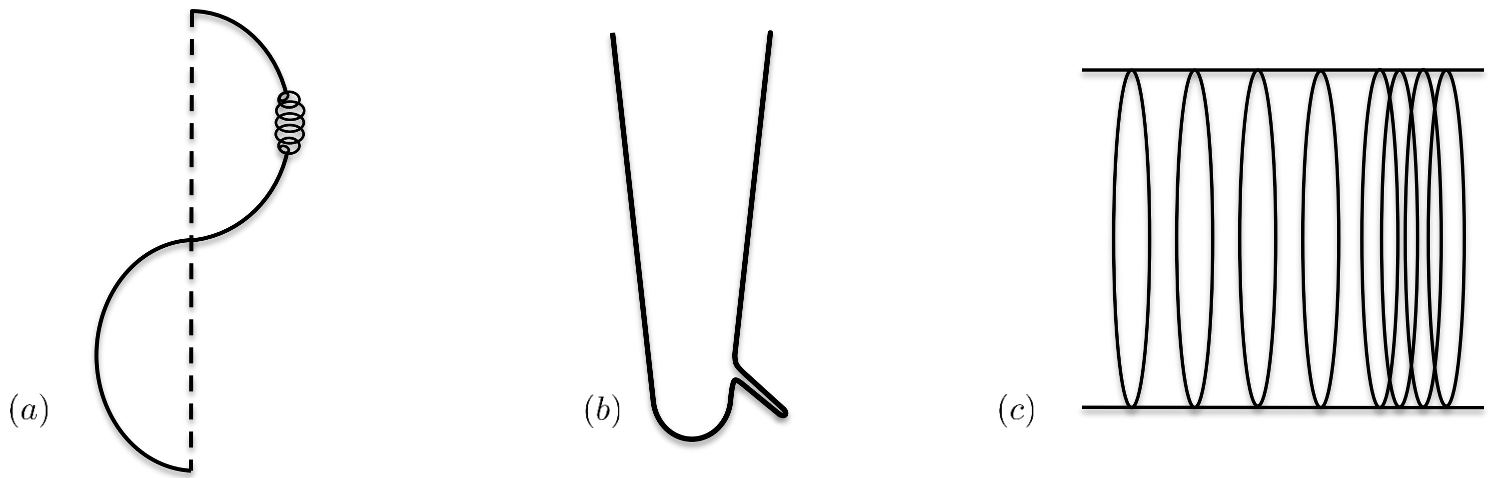
Comments

- Background geometry used in this talk:



- Saw that global J^z_{-n} perturbation gave perturbation at neck
- Suggests a way to understand the states which live at the cap (majority of states)

- Background with small sub-throat:

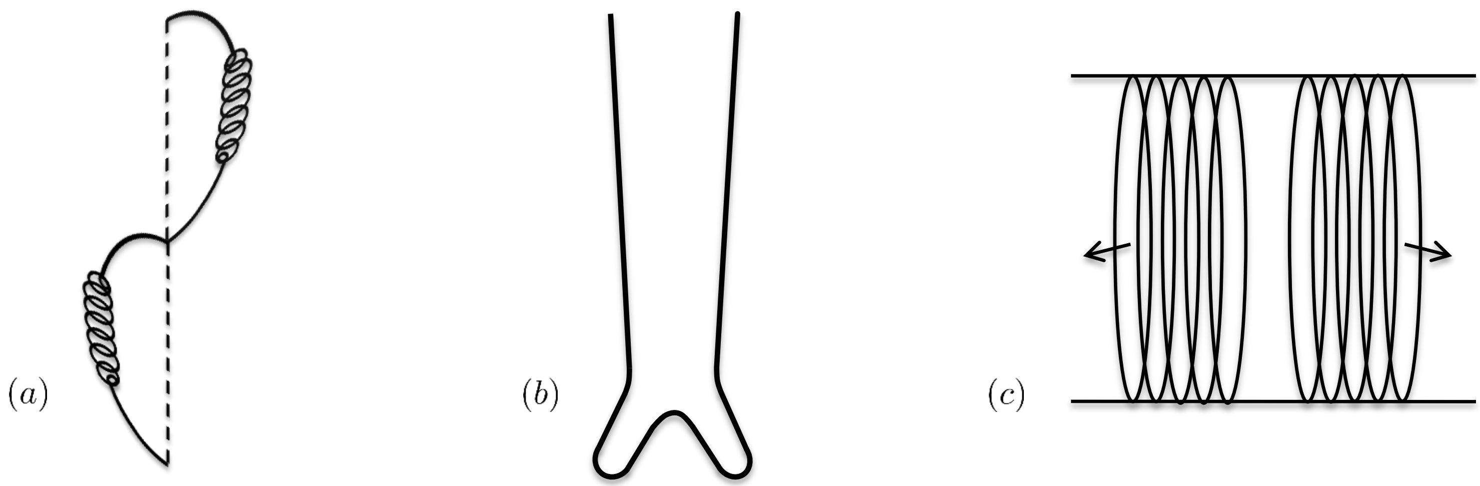


- Neck of sub-throat $\sim$ flat space (on scale of sub-throat)

- J^z_{-n} perturbation of the sub-throat should be the state

$$|0_1\rangle_R \otimes |0_2\rangle_R \ldots |0_n\rangle_R \otimes \sum_{k=1}^{K} J^{z\ (k)}_{-n} \left(\prod_{k=1}^{K} |0^s_k\rangle_R \right)$$

- Background with two large sub-throats:



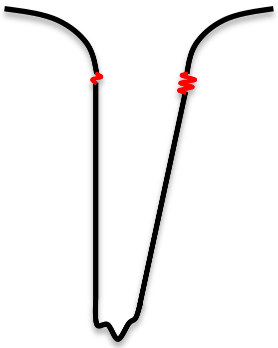
- Antisymmetric combination of J^z_{-n} perturbations of sub-throats:

$$\left(\sum_{k=1}^{\frac{n_1 n_5}{2}} J^{z\,(k)}_{-n} - \sum_{k=\frac{n_1 n_5}{2}+1}^{n_1 n_5} J^{z\,(k)}_{-n} \right) \left(\prod_{k=1}^{\frac{n_1 n_5}{2}} |0_k^+\rangle_R \otimes \prod_{k=\frac{n_1 n_5}{2}+1}^{n_1 n_5} |0_k^-\rangle_R \right)$$

Conclusions

- D1-D5 microstate geometries support 'hair' where a horizon didn't
- Constructed perturbations which carry momentum in compact directions
- Suggests a picture of more general three-charge states

Future:

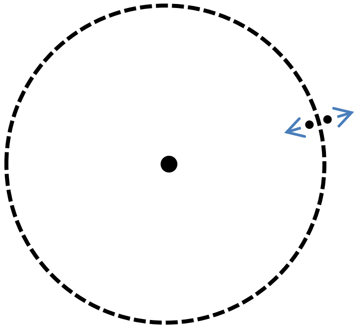


- Construct perturbation to quadratic order, fully non-linear order?
- Make perturbation on more general backgrounds $\rightarrow$ non-extremal?

Bonus slides

The Information Paradox

- Each Hawking pair increases entanglement entropy by $\ln 2$



When does process terminate? If no new physics enters problem until BH becomes Planck-sized,

1. BH evaporates completely $\Rightarrow$ ~~unitarity~~
2. Planck-sized remnant $\Rightarrow$ exotic objects
   (arbitrarily high entanglement with surroundings)

Allowing arbitrary small corrections to this process, conclusions robust

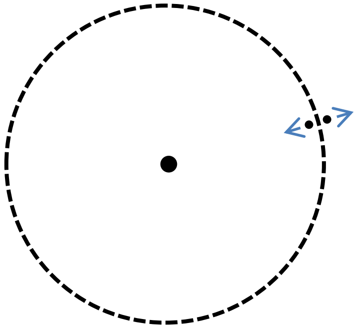
Mathur '09

If a traditional horizon forms and persists, the theory either violates unitarity or has exotic remnants.

The Information Paradox

More precise version:

- Allow corrections to Hawking pair creation, controlled by parameter ϵ



- Then each Hawking pair increases entanglement entropy by $(\ln 2 - 2\epsilon)$.

- If normal physics at horizon, $\epsilon \ll 1$, and entanglement entropy always increases $\Rightarrow$ ~~unitarity~~ / remnants

Proof uses "strong subadditivity" theorem of quantum information theory

Mathur '09

Nonextremal: Hawking radiation

- Emission rate from D1-D5 system matches Hawking radiation rate

  Das, Mathur '96

- Class of non-extremal microstate geometries known

  Jejjala, Madden, Ross, Titchener (JMaRT) '05

- Ergoregion emission – classical instability

  Cardoso, Dias, Hovdebo, Myers '05

- Matches (Hawking) emission rate from these states

  Chowdhury, Mathur '07, '08

- In fuzzball scenario, Hawking radiation is ordinary quantum emission

$\Rightarrow$ unitary

Black ring limit