Euclidean Wilson Loops in AdS/CFT

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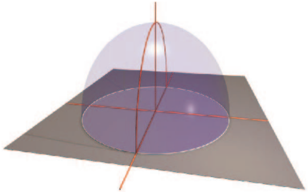
Based on arXiv:1104.3567v2
(w/ R. Ishizeki and M. Kruczenski)
+ New stuff
(w/ M. Kruczenski)

Great Lakes Strings Conference 2012

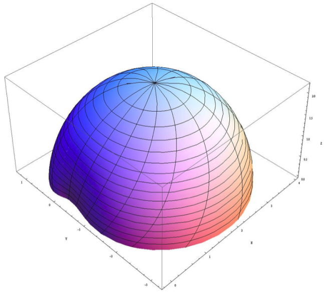
March 3, 2012

- Euclidean Wilson loops with constant scalar $\rightarrow$ Minimal area surfaces in Euclidean AdS_3 [Maldacena,Rey,Yee]



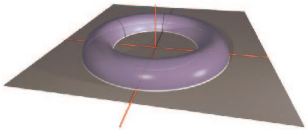
Circular

Berenstein Corrado Fishler
Maldacena Gross Ooguri,
Erickson Semenoff Zarembo
Drukker Gross, Pestun



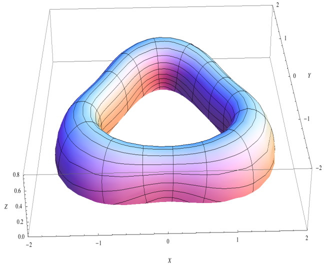
This talk

Multiple Curves



Concentric circles

Drukker Fiol



This talk

Minimal surfaces in Euclidean AdS_3

- $EAdS_3$ space may be described by embedding coordinates X_μ, with $\mu = 0, 1, 2, 3$. $X_0^2 - X_1^2 - X_2^2 - X_3^2 = 1$
- Sigma model: $S = \frac{1}{2}\int(\partial X_\mu \bar{\partial} X^\mu - \Lambda(X_\mu X^\mu - 1))d\sigma d\tau$
- Equation of motion: $\partial\bar{\partial}X_\mu = \Lambda\, X_\mu, \;\; \Lambda = -\partial X_\mu \bar{\partial} X^\mu$.
- Virasoro Constraint: $\partial X_\mu \partial X^\mu \;=\; 0 \;=\; \bar{\partial} X_\mu \bar{\partial} X^\mu$.
- Pohlmeyer reduction/Symmetry and choice of gauge: $\partial\bar{\partial}\alpha \;=\; e^{2\alpha} \;+\; f\,\bar{f}\,e^{-2\alpha}$
- $dw = \sqrt{f(z)}dz$ then $\alpha \rightarrow \alpha + \frac{1}{4}ln(f\bar{f})$, $\partial\bar{\partial}\alpha \;=\; 2cosh(2\alpha)$

Solution to Cosh-gordon equation

Solution: $e^{2\alpha} = \frac{\theta^2(\zeta)}{\hat{\theta}^2(\zeta)}$, $\zeta = 2\omega(p_1)\bar{z} + 2\omega(p_3)z$

Take $\tau \in \mathbb{C}^g$, $Im\,\tau > 0$, $\theta(\zeta) = \sum_{n\in\mathbb{Z}^g} e^{i\pi[n^t\tau n + 2n\zeta]}$

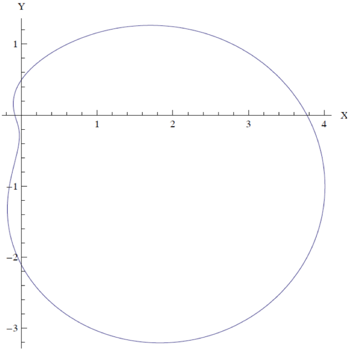
In this talk $g = 3$

[Babich Bobenko, Baker]

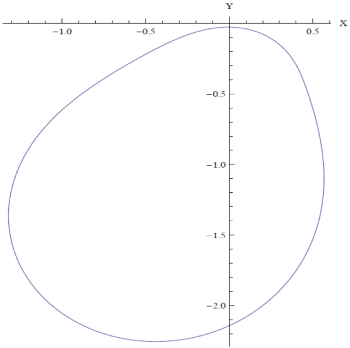
Poincaré coordinates: $X + i\,Y = \frac{X_1 + iX_2}{X_0 - X_3}$, $\quad Z = \frac{1}{X_0 - X_3}$

$Z \propto \theta(\zeta)\,\hat{\theta}(\zeta) \quad Z = 0 \implies \theta(\zeta) = 0$ or $\hat{\theta}(\zeta) = 0$

Wilson loops:



$\lambda = i$

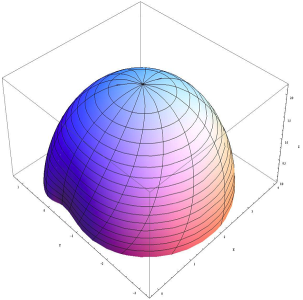


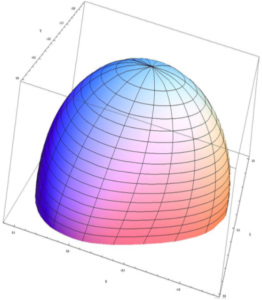
$\lambda = -\frac{1+i}{\sqrt{2}}$

Surface dual to Wilson loop

Renormalized area: $\boxed{A = \frac{L}{\epsilon} + A_f}$

$$A_f = 4D_{p_1p_3}\log\theta(0)\oint(\sigma d\tau - \tau d\sigma) + \frac{1}{2}\oint\frac{\nabla^2\hat{\theta}}{|\nabla\theta|}\,dl$$





$\lambda = i$ $\qquad\qquad$ $\lambda = -\frac{1+i}{\sqrt{2}}$

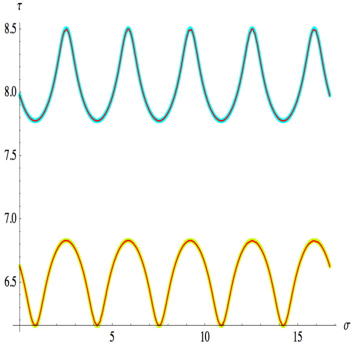
$A_f = -6.598$ for both.

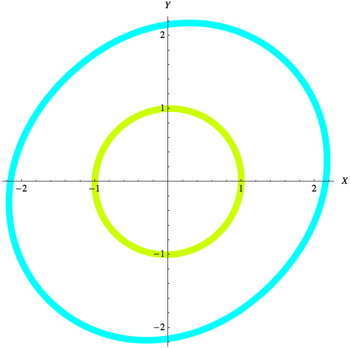
Theta function and the complex torus

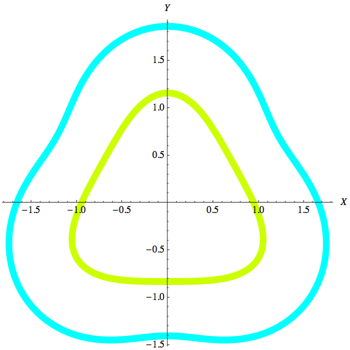
- Consider a g dim complex vector space V and $\Lambda \subset V$.
- $X = V/\Lambda$ is called a complex torus. $\Lambda = ker\,\pi$, $\pi : V \to X$
- $h = f \circ \pi$ is the pullback of a meromorphic function f on X.
- How to construct meromorphic functions on X?
- Try ratios of Λ-periodic holomorphic functions; $g(z+\lambda) = g(z),\ \lambda \in \Lambda = \mathbb{Z} + \mathbb{Z}\tau$. You will get a constant. Oops!!
- Instead, take ratios $R(z) = \frac{\prod_{i=1}^{m} \theta^{(x_i)}(z)}{\prod_{j=1}^{n} \theta^{(y_j)}(z)}$, $\theta^{(x)}(z) = \theta(z - \epsilon'/2 - \tau\epsilon/2 - x)$
- When $m = n$ and $\sum x_i - \sum y_j \in \mathbb{Z}$, this works! $R(z+\tau) = R(z)$.

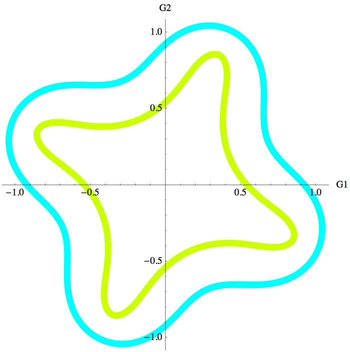
Periodic solutions

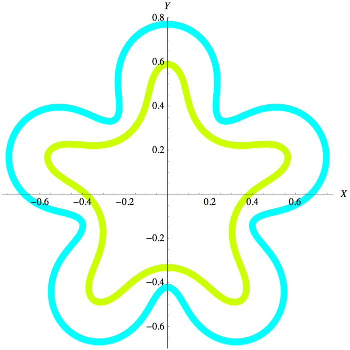
- Along the Wilson loop; $X - i\,Y = -e^{2(\mu z+\nu\bar{z})}\frac{\hat{\theta}(\zeta+\int_1^4)}{\hat{\theta}(\zeta-\int_1^4)}$

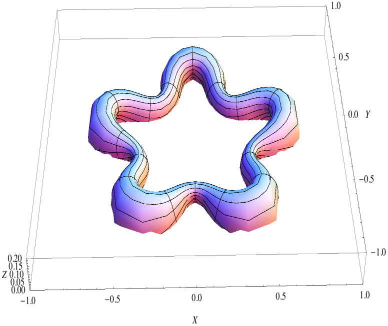





Y
X
1.5
1.0
0.5
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.5
1.0
1.5


G2
G1
1.0
0.5
−0.5
−1.0
−1.0
−0.5
0.5
1.0





$$A_{finite} = -2\Im\left\{ D_{13}\log\theta(0)\oint z d\bar{z} + \int_{top-bottom} D_1 \log\theta(\zeta) d\bar{z} \right\}$$

Conclusion

- We have found a new infinite class of Euclidean Wilson loops for which the minimal surfaces of their gravity dual can be obtained analytically.
- We also showed that Euclidean Wilson loops may be deformed via theta function periodicity.
- Integrability?
- Mathematics?