Towards a Gravity Dual of Charmonium in the Strongly Coupled Plasma

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Outline

- Introduction/Motivation
- A New Model at T=0
- Finite Temperature
- Results
- Conclusion

How can one know that they have a QGP?

- One way is through J/ψ suppression. Masui and Satz, 1986
- Charmonium states in the presence of a deconfined state can dissociated via color Debye screening.
- The observation of J/ψ suppression could be a tell-tale experimental signal for the presence of a QGP.

- However, suppression is a complicated process, thus we will focus only on the dissociation of charmonium.

Field Theory

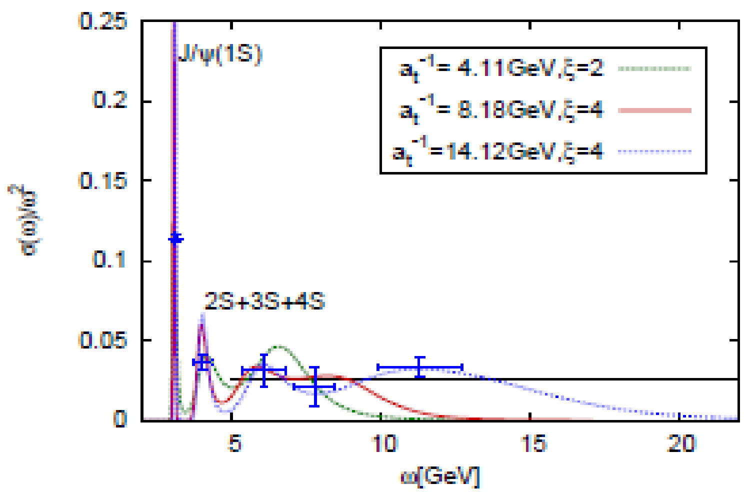
- Consider the charm current operator. $J^\mu = \bar{c}\gamma^\mu c$
  - The 1^{--} state of this operator is J/ψ.
- Construct the current-current correlator and its spectral function.
- The correlator and the spectral function are well defined in field theory and can be calculated.

- At T=0, the spectral function (in the large Nc limit) consists of delta functions at the location of the mass states.

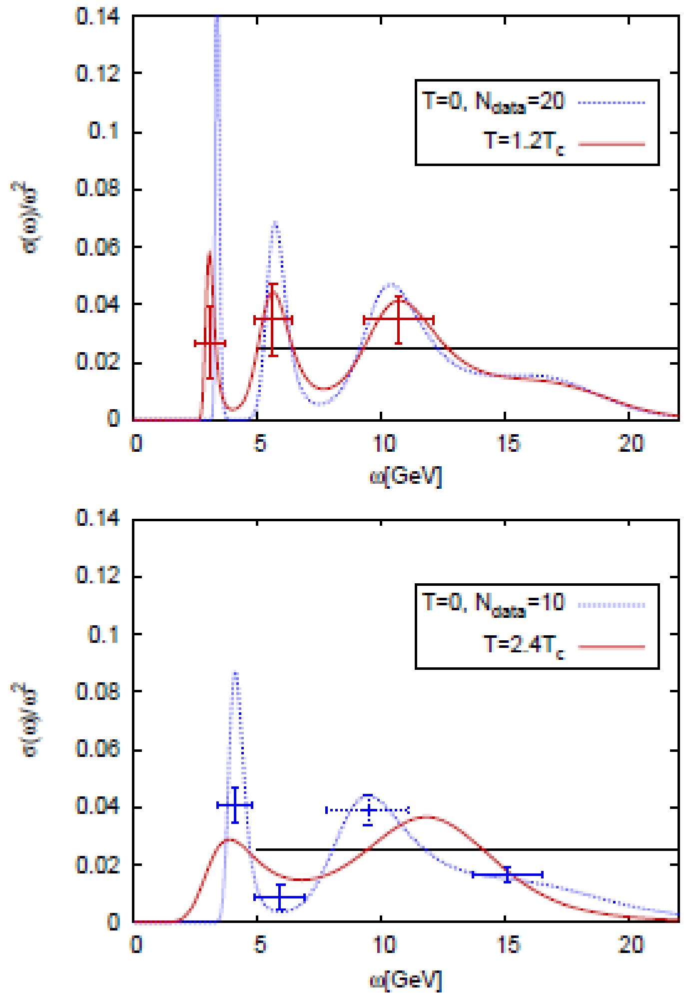
- At finite T, these spikes broaden with the strength of the peaks reducing.
- At some point, this broadening process makes the peak unrecognizable.
- Attribute the lack of a spectral peak to the state dissociating.

Lattice Results

T=0



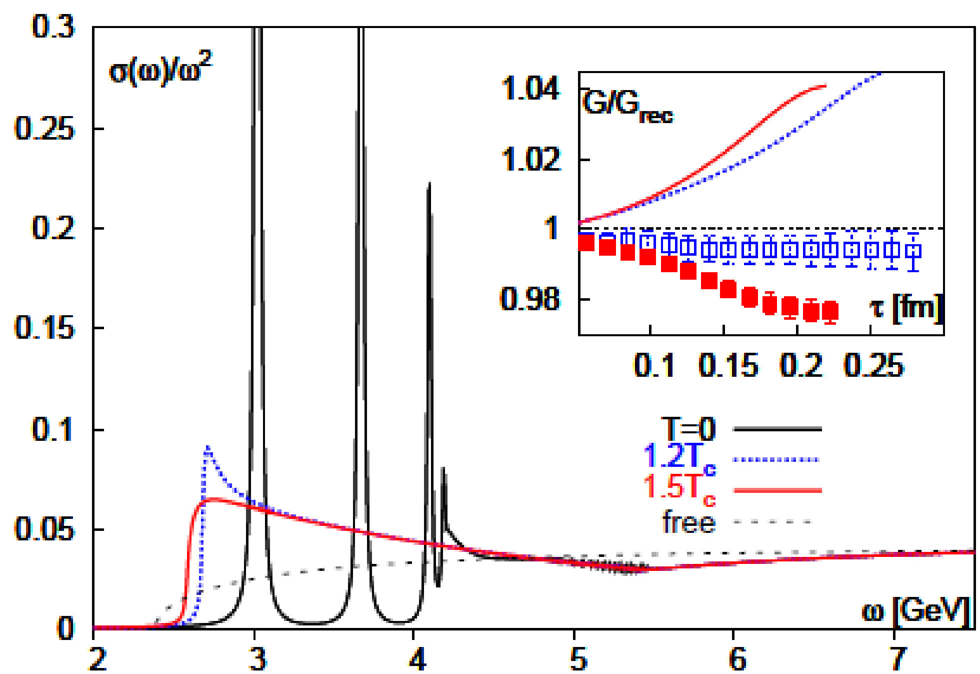
Finite T



Spectral peak corresponding to J/ψ appears to be present up to 2.4 T_c.

A. Jakovac, P. Petreczky, K. Petrov and A. Velytsky, Phys. Rev. D 75, 014506 (2007)

Potential Models



A. Mocsy and P. Petreczky, Phys. Rev. Lett. 99, 211602 (2007)

J/ψ spectral peak disappears at a much lower temperature, ~1.2-1.5 T_c.

Holography and AdS/CFT Correspondence

Maldecena; Gubser, Klebanov, and Polyakov; Witten (1998)

- Relationship between a strongly coupled field theory and a weakly couple theory of gravity.
  - N=4 SYM and 5D Anti-deSitter gravity.
- Operators of the field theory correspond to 5D fields in the bulk space.

$$J^\mu \rightarrow V^\mu$$

- Expectation values of the 4D operators can be found by calculating derivatives of the 5D action wrt the boundary value of a bulk field.

$$G_R(q) = \left. \frac{\partial^2 S}{\partial V_0(q) \partial V_0(-q)} \right|_{z \rightarrow 0}$$

“Top-Down” perspective

- A string theory provides a background geometry upon which fluctuations are investigated.
- Flavor can be introduced to the gravity theory by introducing probe D7 branes. Karch and Katz, 2002
- Heavy meson systems have been investigated in the context of a D3/D7 construction where the branes are separated from one another by some gap, $L \sim m_q$.
- Only one scale in the problem! Kruczenski et al. 2003
  - All states in mass spectrum set by this scale.
    - Charmonium spectrum is set but 2 scales, m_q and Λ_{QCD}
  - Dissociation temperature also set by same scale.

“Bottom-up” approach

- Construct a gravity theory with desirable properties.
- Such as the soft wall model of light vector mesons.
- Rescaled ρ

Fujita et al, 2009

  - The standard soft wall model for light vector mesons has one parameter which is fit to the ρ mass.
  - Instead the parameter is fixed to the J/ψ mass
  - Reasonable procedure since both light and heavy vector mesons exhibit a Regge behavior in spectrum.
- T_D is smaller, 1.2 Tc.
- Again there is only one scale in the problem.
  - Both the mass of J/ψ and Regge behavior is fixed from the same parameter
  - However, separation energy between excited states is parametrically Λ_{QCD} and not m_q

- To understand the dissociation of charmonium states, we will need to understand the finite T spectral functions (correlators).
- But the finite T spectral function is related to the T=0 spectral function (correlator).
  - So we should get this correct as well.

- At T=0, the correlator can be written as a series of poles located at the mass states, with residues equal to the decay const.

$$\Pi_V(-q^2) = \sum_n \frac{f_n^2}{q^2 - m_n^2 + i\epsilon}$$

- Therefore to reproduce the correlator (spectral function), both the masses and the decay consts of each state should be reproduced.

- Goal 1: Construct a new “bottom-up” model at T=0 for a U(1) field whose correlator approximates that of charmonium.

- Goal 2: Turn on temperature, and watch J/ψ disocciate.
  - Corollary: We will apply some new techniques in order to analyze the resulting spectral functions.

The model

Action

$$S = -\frac{1}{4g_5^2}\int d^5x\sqrt{g}e^{-\Phi}V_{MN}V^{MN}$$

$$V_{MN} = \partial_M V_N - \partial_N V_M$$

Metric

$$ds^2 = e^{2A(z)}\left[\eta_{\mu\nu}dx^\mu dx^\nu - dz^2\right]$$

A(z) is a warping factor

Φ(z) is a “dilaton” field

Both fields are initially unspecified and will be determined from charmonium considerations.

$$B(z) = A(z) - \Phi(z)$$

$$\partial_z \left[e^{B(z)} \partial_z V \right] + q^2 e^{B(z)} V = 0$$

Normalizable modes $\quad V = v_n \qquad V|_{z=0} = 0$

Discrete spectrum $\quad q^2 = m_n^2$

Decay constants

$$\langle 0 | J_\mu(0) | n \rangle = f_n m_n \epsilon_n \qquad f_n = \frac{1}{g_5 m_n} \, v_n' e^{B(z)} \Big|_{z \to 0}$$

Schrödinger Equation $\quad -d^2\Psi/dz^2 + U(z)\Psi = q^2\Psi$

Schrödinger Potential $\quad U(z) = \frac{B''(z)}{2} + \left(\frac{B'}{2}\right)^2$

- The properties of charmonium which we are interested in calculating, namely the masses and the decay constants depend on the Schrödinger potential.

- Therefore we will chose U(z) such that the resulting masses and decay constants for J/ψ and ψ' are consistent with experiment.

- This is considered the inverse-scattering problem.

- The potential is uniquely determined from the spectrum and the derivative of the eigenfunctions (decay const.) for the Dirichlet problem.

Poeschel and Trubowitz, 1987

Determining the Schrödinger Potential

- Prerequisites
  - Want the potential to approach AdS potential near UV boundary.

$$U \sim \frac{3}{4z^2} \qquad B(z) \sim -\log z$$

  - For Regge like behavior, want soft-wall near the IR.

$$U \sim z^2$$

- Spectrum
  - Need a mass shift in the ground state which is independent of the excited state spacings.

$$U(z) = \underbrace{\frac{3}{4z^2} + a^4 z^2}_{\text{Standard soft wall}} + c^2$$

Shift

Decay constants

Observable	Experiment (MeV)	$U_{(a)}$ (MeV)	$U_{(a,c)}$ (MeV)
$m_{J/\psi}$	3096	3096*	3096*
$m_{\psi'}$	3685	4378	3685*
$f_{J/\psi}$	416	348	145
$f_{\psi'}$	296	348	173

Decay constants are too small, we need to add a new feature to the potential to increase them.

$$f_n = \frac{1}{g_5 m_n} v_n''(0) = \frac{1}{g_5 m_n} (\sqrt{z}\psi_n)''\Big|_{z\to 0}$$

We need to include something localized near the UV boundary which makes the wavefunction steeper.

We will introduce a "Dip!"

A Dip is a region of finite width of stronger attraction.

We will consider the simplest Dip possible, namely a delta function.

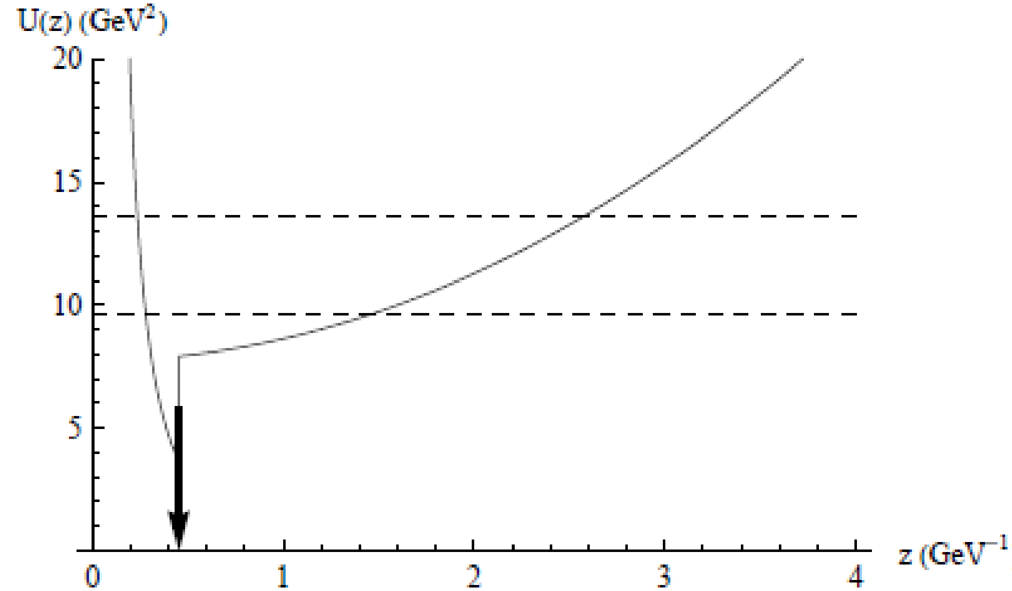
$$U(z) = \frac{3}{4z^2}\,\theta(z_d - z) + \left((a^2 z)^2 + c^2\right)\,\theta(z - z_d) - \alpha\delta(z - z_d)$$

AdS Soft wall "Shift" "Dip"

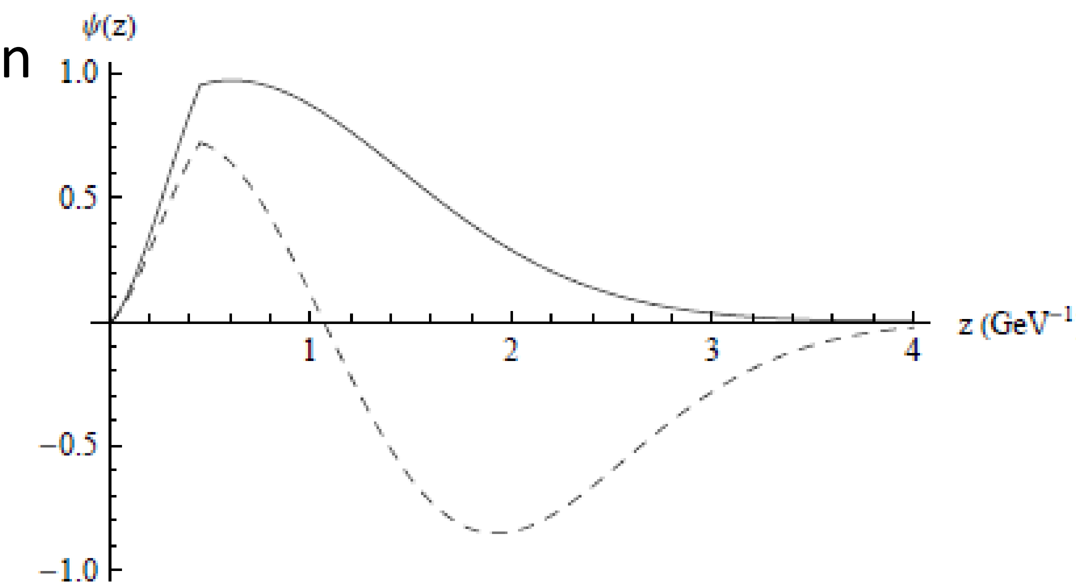
4 parameters which are fitted to the masses and decay constants of J/ψ and ψ'.

$$a = 0.970\ \mathrm{GeV},\ \ c = 2.781\ \mathrm{GeV}, \alpha = 1.876\ \mathrm{GeV},\ \ z_d^{-1} = 2.211\ \mathrm{GeV}$$

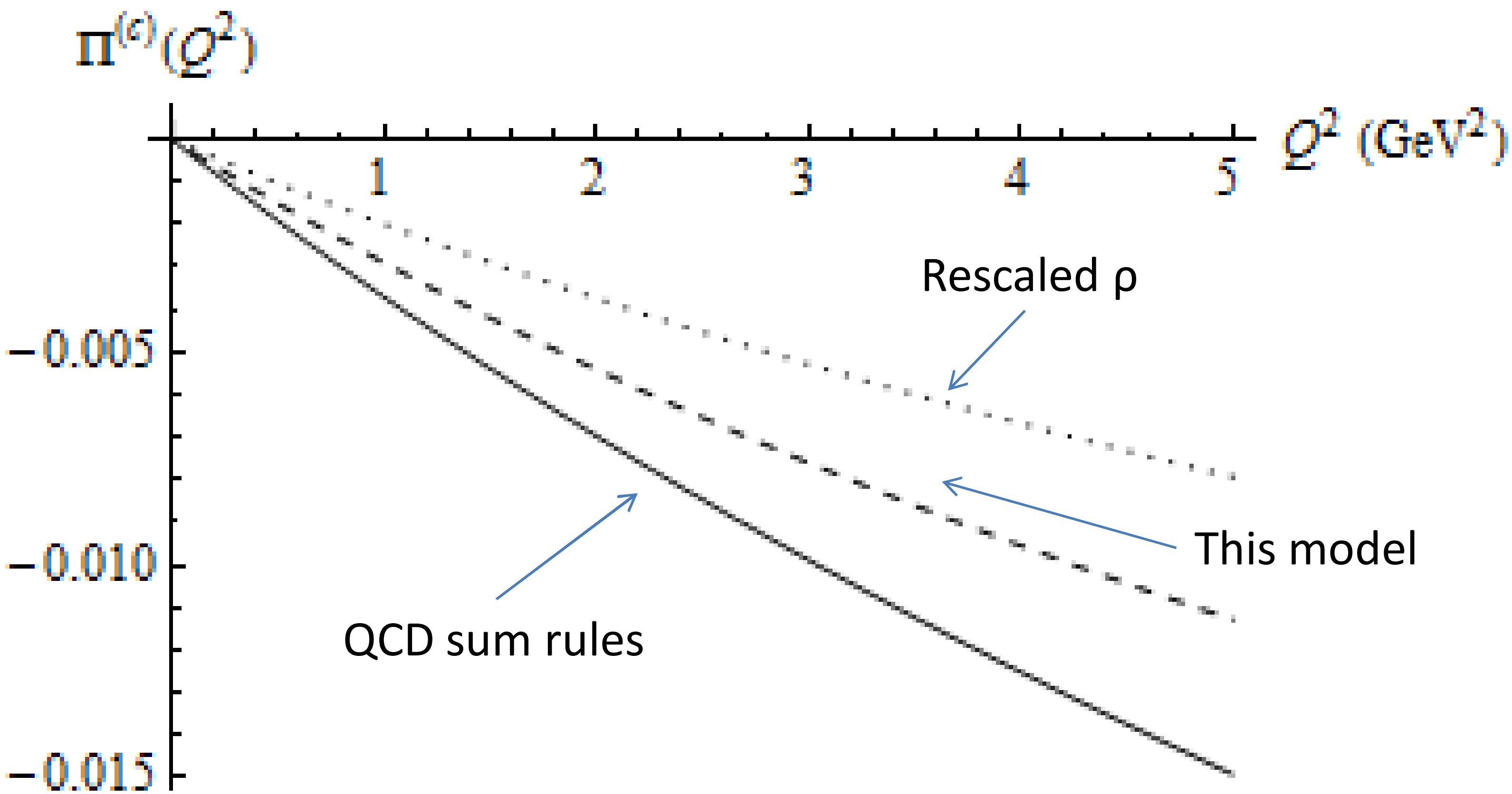
Schrödinger Potential



Normalized Wavefunction



Does this model respect QCD sum rules?



Finite Temperature

Metric with Black hole ansatz

$$ds^2 = e^{2A(z)}\left[hdt^2 - dx^2 - h^{-1}dz^2\right]$$

$$h(z_h) = 0 \qquad\qquad T = \frac{1}{4\pi}|h'(z_h)|$$

We will consider the simplest form for h(z).

$$h(z) = 1 - (z/z_h)^4$$

Ideally, h(z) should be determined dynamically.
In turn, B(z) should have some temperature dependence.
But we haven’t modeled any backgrounds dynamically.
And this allows us to focus on the effects of the new potential.

$$\partial_z \left(h e^{B(z)} \partial_z V \right) + \omega^2 h^{-1} e^{B(z)} V = 0$$

Look for solutions with the b.c.

$$V(\omega, \epsilon) = 1$$
$$V(\omega, z) \xrightarrow{z \to z_h} C(\omega)(1 - z/z_h)^{-i\omega/(4\pi T)}$$

Green's function

$$G_R(\omega) = -\frac{1}{g_5^2} h e^B V'(z,\omega) \Big|_{z=\epsilon} = -\frac{1}{g_5^2} \frac{V'(\epsilon,\omega)}{\epsilon}$$

Schrödinger Equation

$$-d^2\Psi/d\zeta^2 + U_T(\zeta)\Psi = \omega^2 \Psi$$

Schrödinger Potential

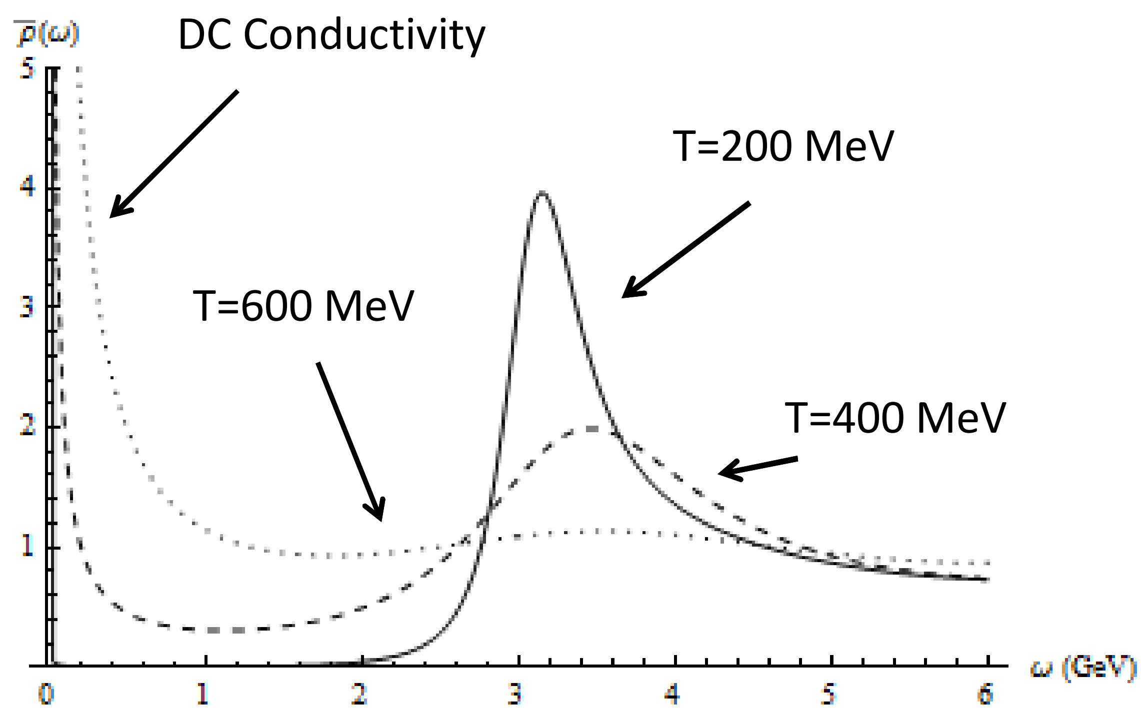
$$U_T(\zeta) = \left[\frac{B''(z)}{2} + \left(\frac{B'}{2} \right)^2 + \frac{B'(z)h'(z)}{2h(z)} \right] h(z)^2$$

$$U_T(\zeta) = \frac{d^2B/d\zeta^2}{2} + \left(\frac{dB/d\zeta}{2} \right)^2$$

Results

Spectral function

$$\bar{\rho}(\omega) \equiv \frac{\rho/\omega^2}{(\rho/\omega^2)|_{\omega\to\infty}} = \frac{2g_5^2}{\pi}\frac{\rho}{\omega^2} \qquad \bar{\rho}(\infty) = 1$$



Quasinormal modes

States corresponding to the poles of the finite T Green's function.

Satisfy the b.c.

$$v_n(\epsilon) = 0 \qquad v_n(z) \xrightarrow{z \longrightarrow z_h} c_n(1 - z/z_h)^{-i\omega/(4\pi T)}$$

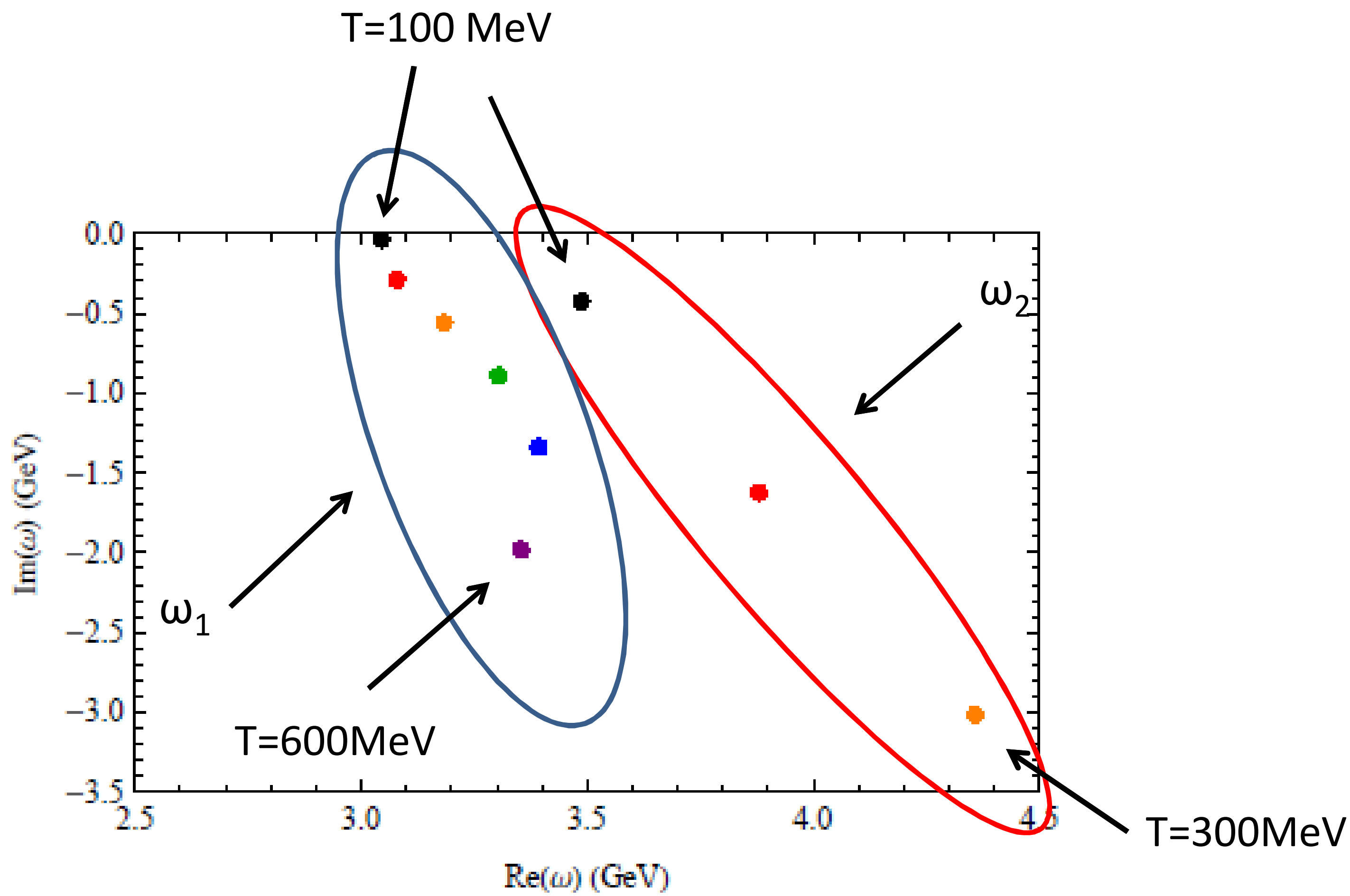
Look like the normal modes at T=0.

Form a discrete spectrum. ω_n

Analytic continuation of the mass at finite T.

For the spectral function,
Re ω_n corresponds to the peak location,
Im ω_n corresponds to the peak width.

Quasinormal modes



Residues

The residues of the poles can also be calculated.

$$r_n \equiv \lim_{\omega \to \omega_n} (\omega - \omega_n) G_R(\omega)$$

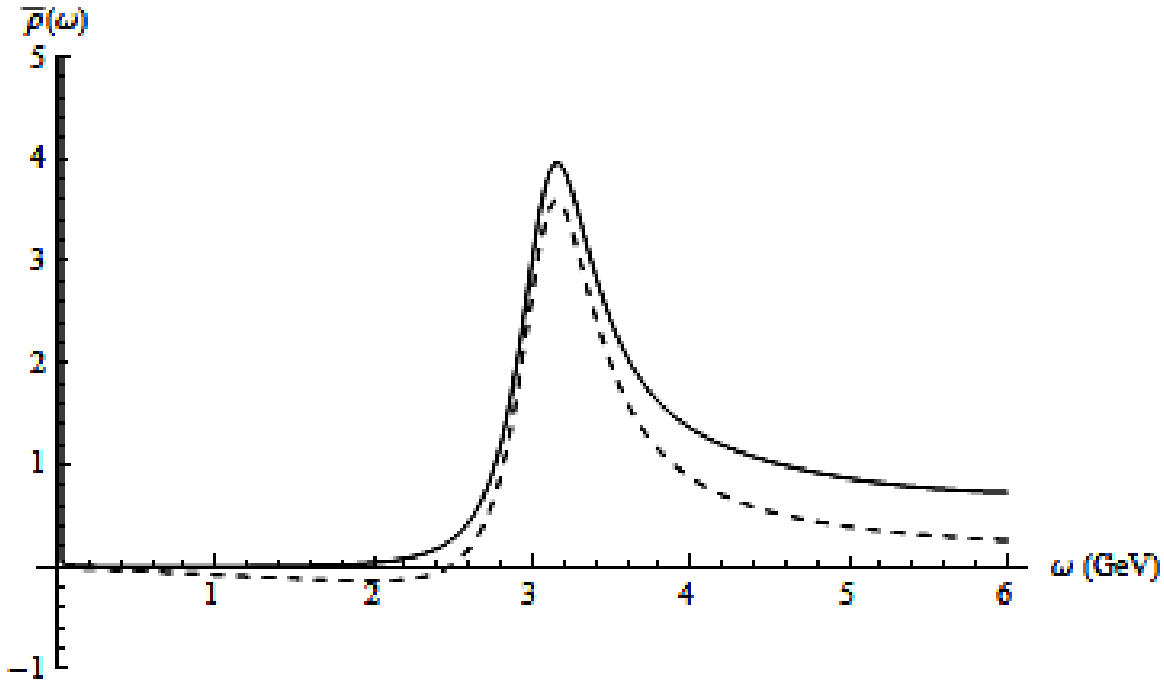
Can be expressed in terms of the quasinormal wavefunction.

$$r_n = \frac{1}{g_5^2} \frac{(v_n'(\epsilon)/\epsilon)^2}{2\omega_n}$$

T=0 $\quad r_n = \frac{f_n^2 m_n^2}{2\omega_n}$

$$\lim_{\delta \to 0} \left[\int_\epsilon^{(1-\delta) z_h} \frac{dz}{h} e^B v_n^2 + \frac{i}{2\omega_n} e^B v_n^2 \Big|_{z=(1-\delta) z_h} \right] = 1$$

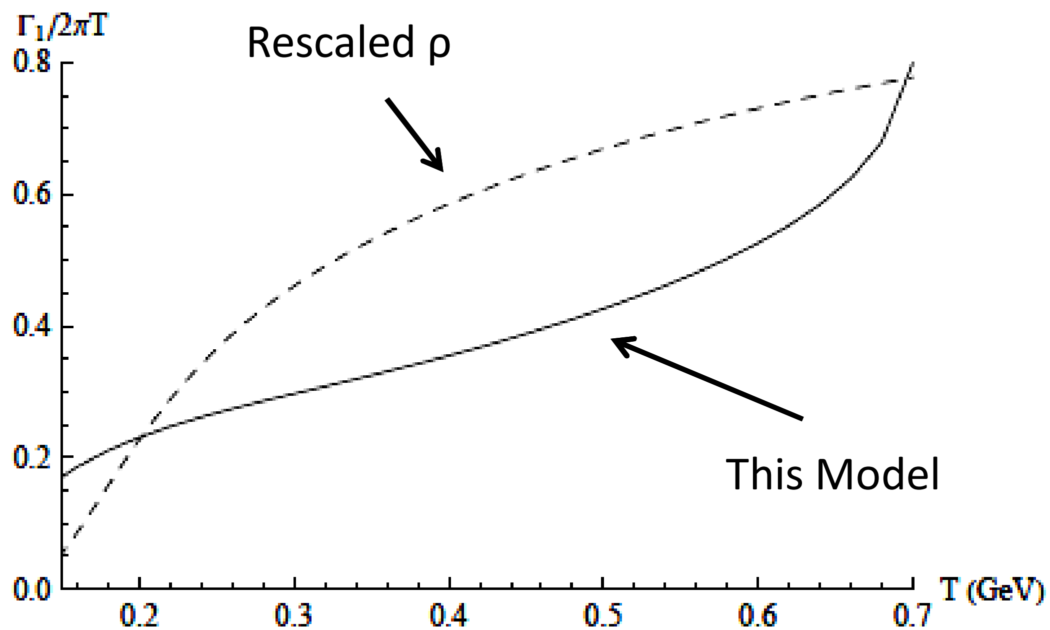
Spectral peak deconstruction



$$\bar{\rho}_n(\omega) \equiv \mathrm{Im}\left(\frac{\bar{r_n}}{\omega - \omega_n}\right) \qquad \bar{r}_n = \frac{2g_5^2}{\pi}\,\frac{r_n}{\omega_n^2}$$

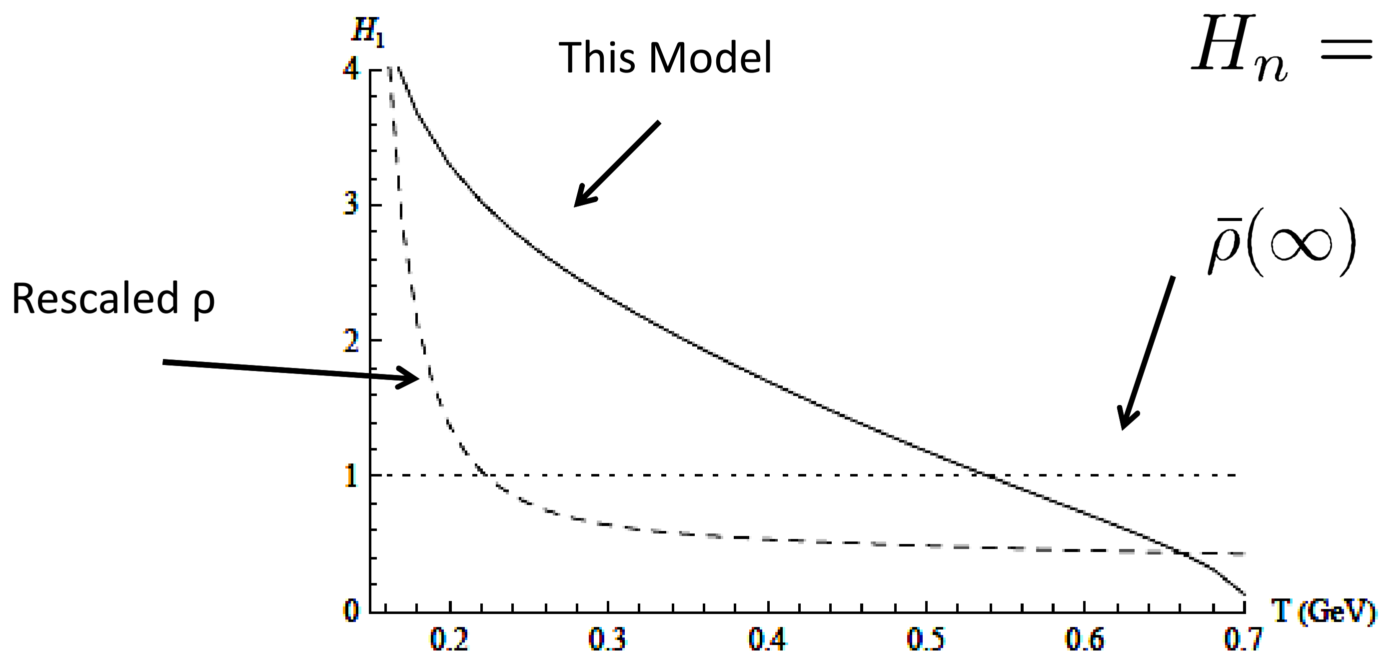
Comparison of the spectral peak widths

$$\Gamma_n = -\mathrm{Im}\,\omega_n$$



Comparison of the spectral peak heights

$$H_n = \frac{\mathrm{Re}\bar{r}_n}{\mathrm{Im}\omega_n}$$



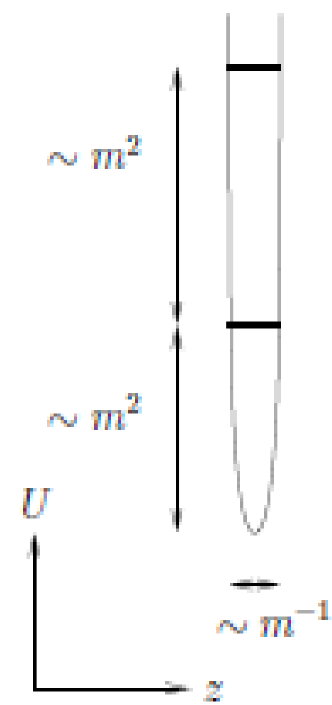
Criteria for melting: A state dissociated when its spectral peak becomes smaller than the spectral functions limiting value.

Tc=190 MeV

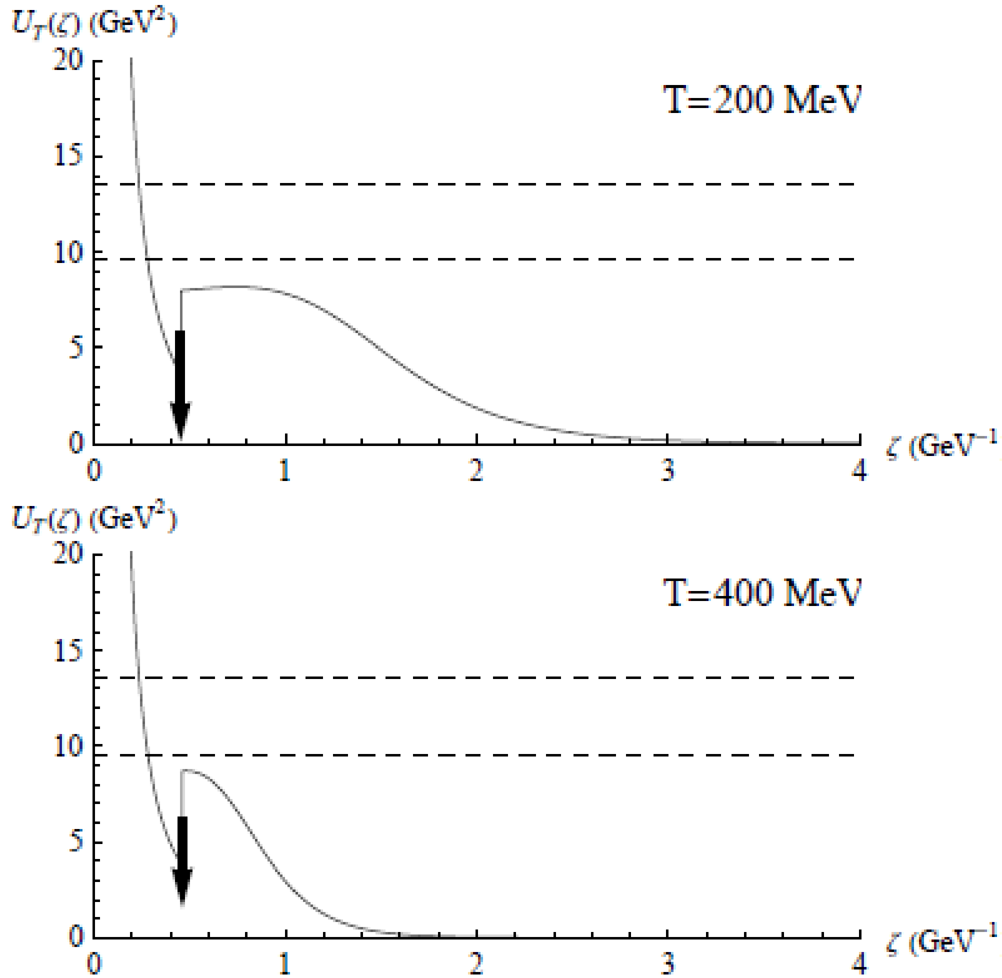
	Rescaled ρ	This Model
Dissociation Temp:	230 MeV = 1.2 Tc	540 MeV = 2.8 Tc

How can one explain the melting temperatures?

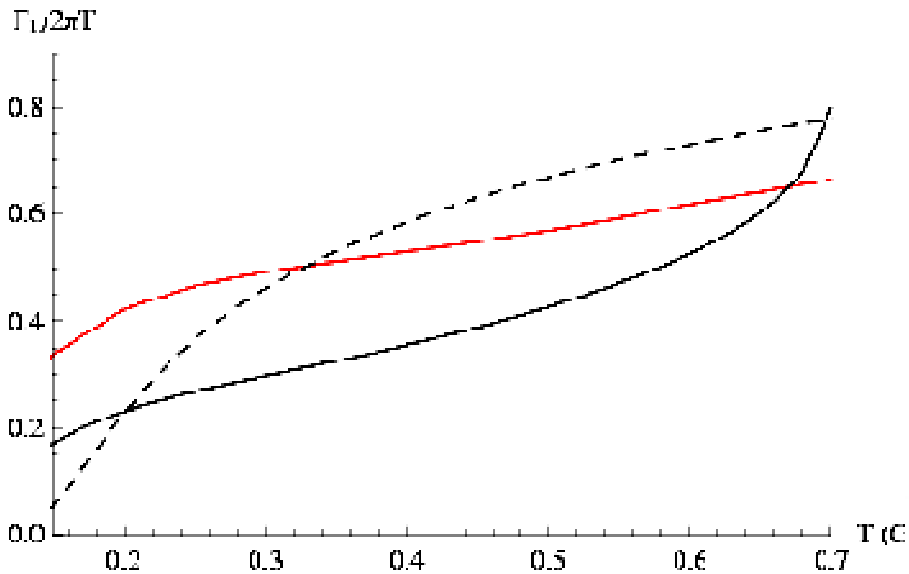
- In D3/D7 constructions, melting temperatures is fixed by heavy quark mass.
- In the bottom-up models, as the temperature increases the soft wall in the potential “melts.”
- The model with “dip” has weaker soft wall, melt widths grow initially faster than rescaled ρ model.
- However, for higher temperatures, the “dip” stabilizes the state, which prevents dissociation.

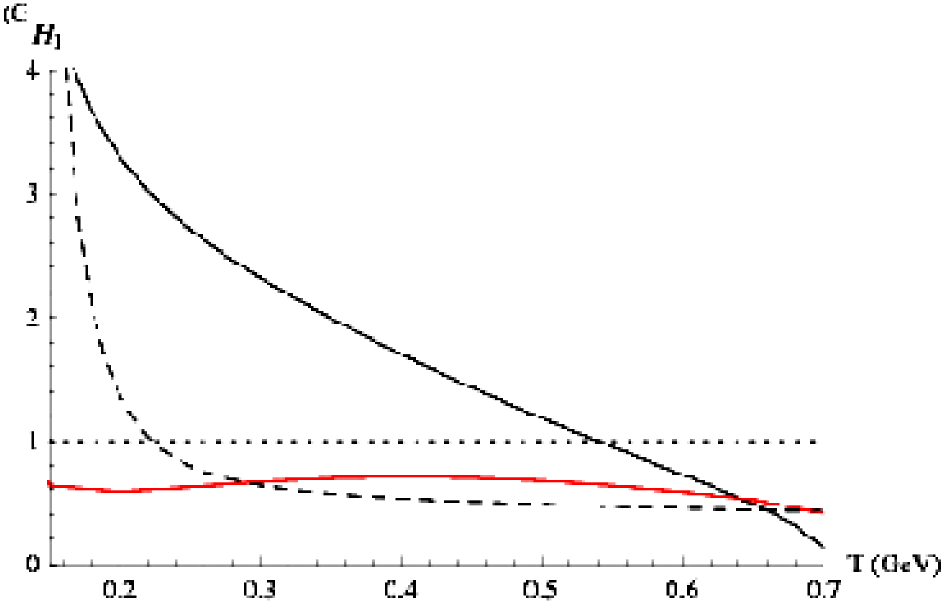


Potentials at finite temperature



Effects of delta function “dip” on width and height of spectral peaks.





Summary

- We've constructed a new gravity dual theory to describe J/ψ.
  - In the new model, charmonium dissociates ~540 MeV.
- We've applied new techniques to analysis the spectral function.
  - Analyzed spectral function by calculating quasinormal modes and residues of poles.
- Fertile area for future exploration
  - Fleshing out how such a system could arise dynamically. (Including considering if top-down constructions can be used to realize the features described here.)