Problem 1

Consider the Hamiltonian:

\[ H = \frac{p^2}{2m} + \lambda x^4 \]  

(0.1)

Show that, by an appropriate rescaling of the \( x \) coordinate, \( x = a \xi \), the corresponding Schrödinger equation can be put in the form

\[ -\xi^2 \frac{d^2 \psi}{d \xi^2} + \xi^4 \psi = \epsilon \psi \]  

(0.2)

a) Obtain the constant \( a \) and the rescaled energy \( \epsilon \) in terms of \( \hbar, m, \lambda \) and the energy \( E \).

b) Use the WKB method to obtain the approximate eigenvalues \( \epsilon_n \) and evaluate numerically the result for \( n = 0, 2, 4, 40 \) where \( n = 0 \) is the ground state. Compare with the values obtained from a numerical solution of the equation:

\[ E_0 = 1.060361945, \ E_2 = 7.4557 \ E_4 = 16.2618267 \ E_{40} = 303.912074247522]; \]  

(0.3)

For which states is the approximation better and why?

c) Challenge: Can you evaluate numerically the energy \( E_{20} \) and compare it with the WKB approximation?

Problem 2

Estimate the mean life of the nuclei \( ^{238}_{92}\text{U} \) and \( ^{212}_{84}\text{Po} \). See problem 8.4 in Griffiths.