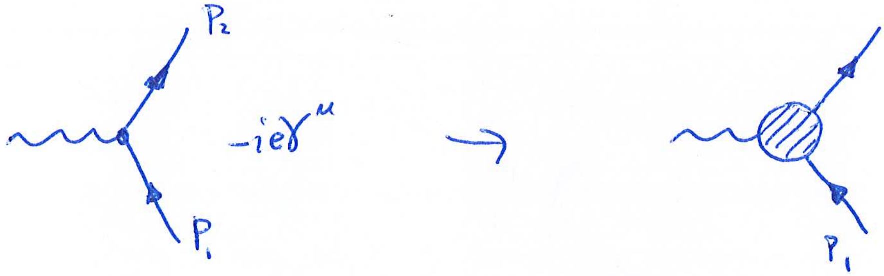
Electron vertex

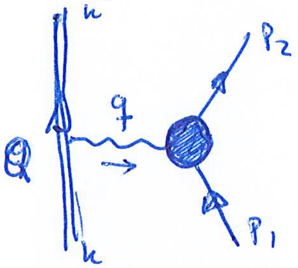


all orders.

$-ie\Gamma^\mu(p_1,p_2)$

$e\bar{\psi}\gamma^\mu\psi A_\mu$

e.g.



$q = p_2 - p_1$

$$iM_{fi} = (-ie)(-iQ)\left(\bar{u}^{s_2}_{p_2}\Gamma^\mu_{(p_2,p_1)}u^{s_1}_{p_1}\right)\left(\bar{u}^{s'}_{k'}\gamma^\nu u^{s}_{k}\right)\frac{(-i\eta^{\mu\nu})}{q^2}$$

$$\Gamma^\mu(p_1,p_2) = \alpha p_1^\mu + \beta p_2^\mu + \delta\gamma^\mu$$

γ^μ, p_1, p_2 $\quad \not{p}_1 u^{s_1}_{p_1} = m u^{s_1}_{p_1}$ $\quad \bar{u}^{s_2}_{p_2}\not{p}_2 = m\bar{u}^{s_2}_{p_2}$

$\Rightarrow \alpha, \beta, \delta$ ~~of~~ scalar functions of q^2.

$q_\mu\Gamma^\mu = 0$ longitudinal photon, gauge inv. $\quad A_\mu \to A_\mu + \partial_\mu\Lambda$

$A_\mu(q) + q_\mu\Lambda(q)$

$$q_\mu\Gamma^\mu = \alpha\,(p_2-p_1)p_1 + \beta\,(p_2-p_1)p_2 + \delta\,(\not{p}_2-\not{p}_1)$$

$$\bar{u}^{s_2}_{p_2}(\Gamma^\mu q_\mu)u^{s_1}_{p_1} = \left[\alpha\,(p_1p_2 - m^2) + \beta(m^2 - p_1p_2)\right]\bar{u}^{s_2}_{p_2}u^{s_1}_{p_1} = 0 \Rightarrow \boxed{\alpha = \beta}$$

$$\Gamma^\mu = \alpha(q^2)\,(p_1^\mu + p_2^\mu) + \delta(q^2)\,\gamma^\mu$$

$$-i\sigma^{\mu\nu} = \frac{1}{2}[\gamma^\mu, \gamma^\nu]$$

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\eta^{\mu\nu}$$

$$\bar{u}_{p_2}\gamma^\mu u_{p_1} = \bar{u}_{p_2}\left[\frac{p_1^\mu + p_2^\mu}{2m} + \frac{i\sigma^{\mu\nu}q_\nu}{2m}\right] u_{p_1}$$

$$-i\sigma^{\mu\nu}q_\nu = \frac{1}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)(p_2 - p_1)_\nu$$

$$= \frac{1}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)p_{2\nu} - \frac{1}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)p_{1\nu}$$

$$= \frac{1}{2}(-2\gamma^\nu\gamma^\mu + 2\eta^{\mu\nu})p_{2\nu} - \frac{1}{2}(2\gamma^\mu\gamma^\nu - 2\eta^{\mu\nu})p_{1\nu}$$

$$= -\not{p}_2\gamma^\mu + p_{2\cancel{\mu}}^{\mu} - \gamma^\mu\not{p}_1 + p_1^\mu$$

$$\bar{u}_{p_2}\gamma^\mu u_{p_1} = \frac{1}{2m}\bar{u}_{p_2}\left(\cancel{p_1^\mu} + \cancel{p_2^\mu} + \not{p}_2\gamma^\mu - \cancel{p_2^\mu} + \gamma^\mu\not{p}_1 - \cancel{p_1^\mu}\right)u_{p_1}$$

$$= \bar{u}_{p_2}\gamma^\mu u_{p_1} \quad ✓$$

$$\Gamma^\mu = F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2m}$$

form factors.

pert. theory $F_1 = 1 \quad F_2 = 0$

③

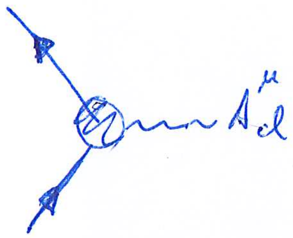
Form factors at $q^2=0$.

external field.

$$V = \int d^3x \; j_\mu A^\mu_{cl} = \int d^3x \; \bar\psi \gamma_\mu \psi A^\mu_{cl}.$$

or very large $\vec E, \vec B$ (many photons).

$A^\mu |\ldots\rangle \Rightarrow$ $a_k, a_k^\dagger$ replaced by classical amplitudes.



$$\mathcal{M}_{fi} = -ie \; \bar u^{s_2}_{p_2} \Gamma^\mu(p_2,p_1) u^{s_1}_{p_1} A_\mu^{cl}(q)$$

— · —

effective interaction ; low energy.

~~$V = \int d^3x$~~

$$q^2 \to 0 \qquad \Gamma^\mu \simeq F_1(0)\gamma^\mu + F_2(0)\frac{i\sigma^{\mu\nu}q_\nu}{2m}$$

$$e^{-ip_1x} e^{ip_2x} e^{-iqx}$$

$$\delta(-p_1+p_2-q)$$

~~V~~ $=$

$$\mathcal{M}_{fi} = -ieF_1(0)\; \bar u^{s_2}_{p_2}\gamma^\mu u^{s_1}_{p_1} A_\mu^{cl}$$
$$-ie F_2(0)\; \bar u^{s_2}_{p_2} \frac{i\sigma^{\mu\nu}q_\nu}{2m} u^{s_1}_{p_1} A_\mu^{cl}$$

effective vertex

$$V = e\int d^3x \; F_1(0)\, \bar\psi\gamma^\mu\psi A_\mu^{cl} + e\int d^3x\, F_2(0)\, \bar\psi \frac{\sigma^{\mu\nu}}{2m}\psi\, \partial_\nu A_\mu^{cl}$$

$eF_1(0)$ charge ; $F_1(0) = 1$

$$V = e\int d^3x\, F_2(0)\, \frac{1}{4m}\, \bar{\psi}\sigma^{\mu\nu}\psi\, F_{\mu\nu}$$

Take $\vec{B} = B_3\hat{z}$ $\qquad B_3 = \partial_x A_y - \partial_y A_x = F_{12} = -F_{21}$

$$V = \frac{eF_2(0)}{4m}\, \overset{{}^{12}{}_{12} + {}^{21}{}_{21}}{2} \int d^3x\, \langle\bar{\psi}\sigma^{12}\psi\rangle\, F_{12}$$

$$= \frac{eF_2(0)}{2m} \int d^3x\, \langle\bar{\psi}\sigma^{12}\psi\rangle\, B_3$$

$$\sigma^{12} = \frac{i}{2}[\gamma^1,\gamma^2] = \frac{i}{4}\left[\begin{pmatrix}0 & \sigma_1\\ -\sigma_1 & 0\end{pmatrix}\begin{pmatrix}0 & \sigma_2\\ -\sigma_2 & 0\end{pmatrix} - \begin{pmatrix}0 & \sigma_2\\ -\sigma_2 & 0\end{pmatrix}\begin{pmatrix}0 & \sigma_1\\ -\sigma_1 & 0\end{pmatrix}\right] = \frac{i}{4}\begin{pmatrix}-\sigma_1\sigma_2+\sigma_2\sigma_1 & 0\\ 0 & -\sigma_1\sigma_2+\sigma_2\sigma_1\end{pmatrix} = -\frac{ii}{4}2\begin{pmatrix}\sigma_3 & 0\\ 0 & \sigma_3\end{pmatrix}$$

$$= \cancel{1}\begin{pmatrix}\sigma_3 & 0\\ 0 & \sigma_3\end{pmatrix} \qquad\qquad u = \sqrt{m}\begin{pmatrix}\xi\\ \xi\end{pmatrix}$$

$$\int d^3x \langle 1_{p',s}|\bar{\psi}\sigma^{12}\psi|1_{p,s}\rangle = \cancel{(2\pi)}\int d^3x\, e^{i(p'x - px)}\, \bar{u}^s_{p'}\sigma^{12}u^s_p = (2\pi)^3\delta^{(3)}(\vec{p}\,' - p)\, \bar{u}^s_p\sigma^{12}u^s_p$$

$$= \langle 1_{p',s}|1_{p,s}\rangle \frac{\bar{u}^s_p\sigma^{12}u^s_p}{2E_p} \underset{\substack{p\\ \text{at rest}}}{=} \langle 1_{p',s}|1_{p,s}\rangle \frac{1}{\cancel{2m}}\cancel{2m}\,\cancel{2}\,\xi^{\dagger(s)}\frac{\sigma^3}{\cancel{2}}\xi^s$$

$$= \cancel{2}\vec{S}_3\, \langle 1_{p',s}|1_{p,s}\rangle$$

$$V = \frac{eF_2(0)}{2m}\cancel{2}S_3B_3 = \cancel{2}\frac{eF_2(0)}{m}(B_3S_3)$$

$$\boxed{\delta\mu \overset{?}{=} -\cancel{2}\frac{eF_2(0)}{m}}$$

$$\boxed{g = \cancel{\frac{eF_2(0)}{2m}}}$$

$$\mu = g\left(\frac{e}{2m}\right)S$$

$$g = 2 + 2F_2(0)$$

⑤

Also

$$\mathcal{L} = i\bar{\psi}\not{\partial}\psi - e\bar{\psi}\not{A}\psi - \frac{eF_2(0)}{4m}\bar{\psi}\sigma^{\mu\nu}\psi F_{\mu\nu} - m\bar{\psi}\psi$$

Dirac eqn.

$$i\not{\partial}\psi - e\not{A}\psi - \frac{eF_2(0)}{4m}\sigma^{\mu\nu}F_{\mu\nu}\psi - m\psi = 0$$

$$\sigma^{ij} = \frac{1}{2}\epsilon^{ijk}\begin{pmatrix}\sigma_k & 0\\ 0 & \sigma_k\end{pmatrix} \qquad \epsilon^{\overset{1}{i}\overset{2}{j}\overset{3}{k}}F_{ij} = 2B_k$$

$$\sigma^{ij}F_{ij} = 2\sigma_k B_k$$

at rest $\psi = \sqrt{m}\begin{pmatrix}\xi\\ \xi\end{pmatrix}$ $\quad \tilde{\psi} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}\xi\\ \xi\end{pmatrix} = \begin{pmatrix}*\\ 0\end{pmatrix};\ \tilde{\psi} = \begin{pmatrix}\overset{\text{large}}{\overset{\downarrow}{\varphi}}\\ \underset{\text{small}}{\chi}\end{pmatrix}$

$$\gamma^0 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} \qquad \gamma^j = \begin{pmatrix}0 & \sigma^j\\ -\sigma^j & 0\end{pmatrix} \leftarrow \text{same as before.}$$

$$i\begin{pmatrix}\dot{\varphi}\\ -\dot{\chi}\end{pmatrix} + i\begin{pmatrix}\sigma^j\partial_j\chi\\ -\sigma^j\partial_j\varphi\end{pmatrix} - e\begin{pmatrix}\sigma^j A_j\chi\\ -\sigma^j A_j\varphi\end{pmatrix} - \frac{eF_2(0)}{2m}\sigma_k B_k\begin{pmatrix}\varphi\\ \chi\end{pmatrix} -$$

$$-m\begin{pmatrix}\varphi\\ \chi\end{pmatrix} = 0$$

$$\varphi = e^{-imt}\Phi \qquad \chi = e^{-imt}X$$

$$i\dot{\Phi} = -i\sigma^j\partial_j X + e\sigma^j A_j X + \frac{eF_2(0)}{2m}\sigma_k B_k\Phi$$

$$i\dot{X} - 2mX = i\sigma^j\partial_j\Phi - e\sigma^j A_j\Phi + \frac{eF_2(0)}{2m}\sigma_k B_k X$$

$$X \simeq -\frac{1}{2m}\sigma^i(i\partial_j - eA_j)\Phi$$

$$\underset{\overset{\uparrow}{e^{iEt}}}{E}\Phi = -(-i\sigma^i\partial_j + e\sigma^i A_j)\frac{1}{2m}(i\sigma^l\partial_j - e\sigma^l A_l)\Phi +$$

$$+ \frac{eF_2(0)}{4m}\sigma_k B_k \Phi$$

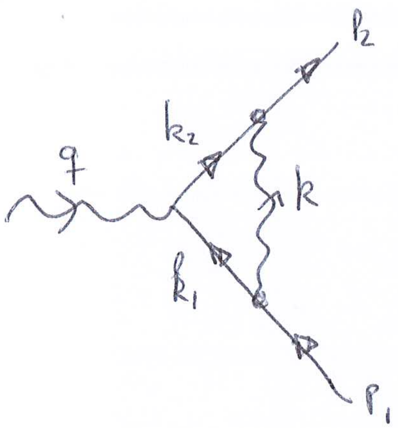
$$= \frac{1}{2m}(P_j + eA_j)(P_l + eA_l)\underbrace{\sigma^i\sigma^l}_{\delta^{il}+i\epsilon^{ilk}\sigma_k}\Phi + \frac{eF_2(0)}{4m}\sigma_k B_k\Phi$$

$$= \frac{(p+eA)^2}{2m} + \frac{ie}{2m}\underbrace{[P_j, A_l]}_{-i\partial_j A_l}\epsilon^{jlk}\sigma_k\Phi + \frac{eF_2(0)}{2m}(SB)\Phi$$

$$= \frac{1}{2m}(p+eA)^2 + \frac{e}{2m}B_k\sigma_k\Phi + \frac{eF_2(0)}{2m}(BS)\Phi$$

$$= \frac{1}{2m}(p+eA)^2 + \frac{e}{m}(BS)\Phi + \frac{eF_2(0)}{2m}(BS)\Phi$$

$$\boxed{\mu = -\frac{e}{m}\vec{S} - \frac{eF_2(0)}{2m}\vec{S}}$$



$k_1 = p_1 - k$

$k_2 = p_2 - k$

$$(-ie)^2 \; i^2 \int \frac{d^4k}{(2\pi)^4} \frac{\bar{u}^{s_2}_{p_2} \gamma^\alpha (\not{k}_2 + m)\gamma^\mu (\not{k}_1 + m)\gamma^\beta u^{s_1}_{p_1} (-i\eta_{\alpha\beta})}{(k_2^2 - m^2 + i\varepsilon)(k_1^2 - m^2 + i\varepsilon)(k^2 + i\varepsilon)}$$

$$\gamma^\alpha \gamma^\mu \gamma_\alpha = -\gamma^\alpha \gamma_\alpha \gamma^\mu + 2\delta^{\mu}_{\alpha}\gamma^\alpha = -4\gamma^\mu + 2\gamma^\mu = -2\gamma^\mu$$

$$\gamma^\alpha \gamma^\mu \gamma^\nu \gamma_\alpha = -\gamma^\alpha \gamma^\mu \gamma_\alpha \gamma^\nu + 2\gamma^\alpha \gamma^\mu \delta^\nu_\alpha$$

$$= 2\gamma^\mu \gamma^\nu + 2\gamma^\nu \gamma^\mu = 4\eta^{\mu\nu}$$

$$\gamma^\alpha \gamma^\mu \gamma^\nu \gamma^\rho \gamma_\alpha = -\gamma^\alpha \gamma^\mu \gamma^\nu \gamma_\alpha \gamma^\rho + 2\gamma^\alpha \gamma^\mu \gamma^\nu \delta^\rho_\alpha$$

$$= -4\eta^{\mu\nu}\gamma^\rho + 2\gamma^\rho \gamma^\mu \gamma^\nu = \cancel{-4\eta^{\mu\nu}\gamma^\rho} - 2\gamma^\rho \gamma^\nu \gamma^\mu + \cancel{4\gamma^\rho \eta^{\mu\nu}}$$

$$= -2\gamma^\rho \gamma^\nu \gamma^\mu$$

$$-ie^2 \int \frac{d^4k}{(2\pi)^4} \frac{\bar{u}^{s_2}_{p_2}\left(-2m^2\gamma^\mu + 4m(k_1^\mu + k_2^\mu) - 2\not{k}_1 \gamma^\mu \not{k}_2\right) u^{s_1}_{p_1}}{(k_2^2 - m^2 + i\varepsilon)(k_1^2 - m^2 + i\varepsilon)(k^2 + i\varepsilon)}$$

$$2ie^2\int\frac{d^4k}{(2\pi)^4}\,\frac{\bar{u}^{s_2}_{p_1}\left(m^2\gamma^\mu-2(k_1^\mu+k_2^\mu)m+\not{k}_1\gamma^\mu\not{k}_2\right)u^{s_1}_{p_1}}{\left((p_2-k)^2-m^2+i\varepsilon\right)\left((p_1-k)^2-m^2+i\varepsilon\right)\left(k^2+i\varepsilon\right)}$$

$$\int d\alpha\, d\beta\, d\gamma\;\frac{2\,\delta(\alpha+\beta+\gamma-1)}{\left(k^2\underset{(\alpha+\beta)}{-m^2}-2\alpha p_2k+\alpha p_2^2-2\beta p_1k+\beta p_1^2+i\varepsilon\right)^3}$$

$$2\int d\alpha\, d\beta\, d\gamma\;\frac{\delta(\alpha+\beta+\gamma-1)}{\left[\left(k-\alpha p_2-\beta p_1\right)^2+\alpha(1-\alpha)p_2^2+\beta(1-\beta)p_1^2-2\alpha\beta\, p_1\cdot p_2\overset{(\alpha+\beta)}{-m^2}+i\varepsilon\right)^3}$$

$$k\to k+\alpha p_2+\beta p_1$$

$$4ie^2\int\frac{d^4k}{(2\pi)^4}\int d\alpha\, d\beta\, d\gamma\;\frac{\delta(\alpha+\beta+\gamma-1)\;\bar{u}^{s_2}_{p_2}\left(m^2\gamma^\mu-2\left(p_1+p_2-2k-2\alpha p_2-2\beta p_1\right)^\mu m+(\not{p}_1-\not{k}-\alpha\not{p}_2-\beta\not{p}_1)\gamma^\mu(\not{p}_2-\not{k}-\alpha\not{p}_2-\beta\not{p}_1)\right)u^{s}_{p_1}}{\left[k^2+\alpha(1-\alpha)\underset{m^2}{p_2^2}+\beta(1-\beta)\underset{m^2}{p_1^2}-2\alpha\beta p_1p_2-(\alpha+\beta)m^2+i\varepsilon\right]^3}$$

$$(\alpha-\alpha^2+\beta-\beta^2\cancel{-\alpha}\cancel{-\beta})\,m^2\qquad(-\alpha^2-\beta^2)\,m^2$$

$$-2p_1p_2=(p_1+p_2)^2-p_1^2-p_2^2=q^2-2m^2$$

$$p_1p_2=-q^2/2+m^2$$

$$4ie^2\int\frac{d^4k}{(2\pi)^4}\int d\alpha\, d\beta\, d\gamma\;\frac{\delta(\alpha+\beta+\gamma-1)\;\bar{u}^{s_2}_{p_2}\left[m^2\gamma^\mu-2\left((1-2\alpha)p_2^\mu+(1-2\beta)p_1^\mu\cancel{-2k^\mu}\right)m+\left((1-\beta)\not{p}_1-\alpha\not{p}_2-\not{k}\right)\gamma^\mu\left((1-\alpha)\not{p}_2-\beta\not{p}_1-\not{k}\right)\right]u^{s_1}_{p_1}}{\left[k^2+(-\alpha^2-\beta^2-2\alpha\beta)m^2+\alpha\beta q^2+i\varepsilon\right]^3}$$

$$4ie^2\int\frac{d^4k}{(2\pi)^4}\int d\alpha\, d\beta\, d\gamma\;\delta(\alpha+\beta+\gamma-1)\,\frac{\bar{u}^{s_2}_{p_2}\Big[m^2\gamma^\mu-2(1-2\alpha)m(p_1^\mu+p_2^\mu)+\big((1-\beta)\not{p}_1-\alpha\not{p}_2\big)\gamma^\mu\big((1-\alpha)\not{p}_2-\beta\not{p}_1\big)+\not{k}\gamma^\mu\not{k}\Big]u^{s_1}_{p_1}}{\left[k^2-(\alpha+\beta)^2m^2+\alpha\beta q^2+i\varepsilon\right]^3}$$

$$\not{k}\gamma^\mu\not{k}=\underbrace{k_\alpha k_\beta}_{\int d^4k}\gamma^\alpha\gamma^\mu\gamma^\beta=\frac{1}{4}k^2\eta_{\alpha\beta}\gamma^\alpha\gamma^\mu\gamma^\beta=-\frac{1}{2}k^2\gamma^\mu$$

$$\bar{u}^{s_2}_{p_2}\Big[\big((1-\beta)\not{p}_1-\alpha m\big)\gamma^\mu\big((1-\alpha)\not{p}_2-\beta m\big)\Big]u^{s_1}_{p_1}$$

$$(1-\beta)(1-\alpha)\,\bar{u}^{s_2}_{p_2}\not{p}_1\gamma^\mu\not{p}_2u^{s_1}_{p_1}-\beta(1-\beta)m\,\bar{u}^{s_2}_{p_2}\not{p}_1\gamma^\mu u^{s_1}_{p_1}$$

$$-\alpha(1-\alpha)m\,\bar{u}^{s_2}_{p_2}\gamma^\mu\not{p}_2u^{s_1}_{p_1}+\alpha\beta m^2\,\bar{u}^{s_2}_{p_2}\gamma^\mu u^{s_1}_{p_1}$$

~~$-((1-\alpha)(1-\beta)\bar{u}^{s_2}_{p_2}\gamma^\mu\not{p}_1$~~

$$\not{p}_1\gamma^\mu\not{p}_2=-\gamma^\mu\not{p}_1\not{p}_2+2p_1^\mu\not{p}_2=\gamma^\mu\not{p}_2\underset{m}{\not{p}_1}-2(p_1p_2)\gamma^\mu+2p_1^\mu\underset{m}{\not{p}_2}$$

$$=-\not{p}_2\gamma^\mu m+2p_2^\mu m-2(p_1p_2)\gamma^\mu+2p_1^\mu m=-m^2\gamma^\mu+2m(p_1^\mu+p_2^\mu)-2(p_1p_2)\gamma^\mu$$

$$m\not{p}_1\gamma^\mu=-m^2\gamma^\mu+2mp_1^\mu$$

$$m\gamma^\mu\not{p}_2=-m^2\gamma^\mu+2mp_2^\mu$$

⑩

$$\cancel{-q^2/2} \qquad q^2-2m^2$$

$$\bar{u}^{s_2}_{p_2}\Big[(1-\alpha)(1-\beta)\big(\underbrace{-m^2\gamma^\mu} + 2m(p_1^\mu+p_2^\mu)\,\boxed{-2(p_1p_2)\gamma^\mu}\big) -$$

$$\underbrace{+\alpha(1-\alpha)m^2\gamma^\mu} - \alpha(1-\alpha)\,2mp_2^\mu \underbrace{+\beta(1-\beta)m^2\gamma^\mu} - \beta(1-\beta)\,2mp_1^\mu +$$

$$\underbrace{+\alpha\beta m^2\gamma^\mu}\Big]u^{s_1}_{p_1}$$

$$-1+\alpha+\beta\cancel{-\alpha\beta}+\alpha-\alpha^2+\beta-\beta^2\cancel{+\alpha\beta}$$

$$m^2\bar{u}^{s_2}_{p_2}\gamma^\mu u^{s_1}_{p_1}\Big(-(1-\alpha)(1-\beta)+\alpha(1-\alpha)+\beta(1-\beta)+\alpha\beta\Big)$$

$$-2(p_1p_2)\,\bar{u}^{s_2}_{p_2}\gamma^\mu u^{s_1}_{p_1}\,(1-\alpha)(1-\beta)$$

$$+2m\,\bar{u}^{s_2}_{p_2}(p_1^\mu+p_2^\mu)\Big((1-\alpha)(1-\beta)-\alpha(1-\alpha)\Big)$$

$$1-\alpha-\beta+\alpha\beta-\alpha+\alpha^2$$

$$4ie^2\int\frac{d^4k}{(2\pi)^4}\int\frac{d\alpha\,d\beta\,d\gamma\,\delta(\alpha+\beta+\gamma-1)\times}{[k^2-(\alpha+\beta)^2m^2+\alpha\beta q^2+i\varepsilon]^3}\times$$

$$-3(1-\alpha)(1-\beta)$$

$$\times\Big\{\bar{u}^{s_2}_{p_2}\gamma^\mu u^{s_1}_{p_1}\Big[+\frac{q^2}{\cancel{2}}(1-\alpha)(1-\beta)+m^2\big(1+\alpha(1-\alpha)+\beta(1-\beta)+\alpha\beta\big)\Big]$$

$$+2m\,\bar{u}^{s_2}_{p_2}(p_1^\mu+p_2^\mu)u^{s_1}_{p_1}\Big(-(1-2\alpha)+(1-\alpha)(1-\beta)-\alpha(1-\alpha)\Big)$$

$$-\frac{1}{2}k^2\,\bar{u}^{s_2}_{p_2}\gamma^\mu u^{s_1}_{p_1}\Big\}$$

$$1-3(1-\alpha)(1-\beta)+\alpha(1-\alpha)+\beta(1-\beta)+\alpha\beta$$

$$\boxed{1-3}\,\boxed{+3\alpha}\,\boxed{+3\beta}-3\alpha\beta+\boxed{\alpha}-\alpha^2\boxed{+\beta}-\beta^2+\alpha\beta$$

$$-2+4\alpha+4\beta-2\alpha\beta-\alpha^2-\beta^2$$

$$-2+4(\alpha+\beta)-(\alpha+\beta)^2=-2+4(1-\gamma)-(1-\gamma)^2$$

$$=\boxed{-2}\,\boxed{+4}-4\gamma\boxed{-1}+2\gamma-\gamma^2=1-2\gamma-\gamma^2$$

$$\cancel{-1}\,\cancel{+2\alpha}\,\cancel{+1}\,\cancel{-\alpha}-\beta+\alpha\beta\,\cancel{-\alpha}+\alpha^2$$

$$-\beta+\alpha\beta+\alpha^2\underset{\substack{\uparrow\\ \text{symmetrize}}}{\longrightarrow}\frac{1}{2}\alpha^2+\frac{1}{2}\beta^2+\alpha\beta-\frac{1}{2}\beta-\frac{1}{2}\alpha=\frac{1}{2}(\alpha+\beta)^2-\frac{1}{2}(\alpha+\beta)$$

$$=\frac{1}{2}(1-\gamma)^2-\frac{1}{2}(1-\gamma)=\cancel{\frac{1}{2}}-\gamma+\frac{1}{2}\gamma^2\cancel{-\frac{1}{2}}+\frac{1}{2}\gamma$$

$$=-\frac{1}{2}\gamma+\frac{1}{2}\gamma^2=-\frac{1}{2}\gamma(1-\gamma)$$

$$4ie^2\int\frac{d^4k}{(2\pi)^4}\int d\alpha\, d\beta\, d\gamma\,\frac{\delta(\alpha+\beta+\gamma-1)}{[k^2-(1-\gamma)^2m^2+\alpha\beta q^2+i\varepsilon]^3}\times$$

$$\times\Big\{\bar{u}^{s_2}_{p_2}\gamma^\mu u^{s_1}_{p_1}\left[q^2(1-\alpha)(1-\beta)+m^2(1-2\gamma-\gamma^2)\right]$$

$$-m\,\boxed{\bar{u}^{s_2}_{p_2}(p_1^\mu+p_2^\mu)u^{s_1}_{p_1}}\,\gamma(1-\gamma)-\frac{1}{2}k^2\,\bar{u}^{s_2}_{p_2}\gamma^\mu u^{s_1}_{p_1}\Big\}$$

$$\hookrightarrow=2m\,\bar{u}_{p_2}\gamma^\mu u_{p_1}-i\bar{u}_{p_2}\sigma^{\mu\nu}q_\nu u_{p_1}$$

$(-ie)$ ✓

$$4ie^2\int\frac{d^4k}{(2\pi)^4}\int d\alpha d\beta d\gamma \frac{\delta(\alpha+\beta+\gamma-1)}{[k^2-(1-\gamma)^2m^2+\alpha\beta q^2+i\varepsilon]^3}$$

$$\times\Big\{\bar{u}^{s_2}_{p_2}\gamma^\mu u^{s_1}_{p_1}\Big(-\frac{1}{2}k^2+q^2(1-\alpha)(1-\beta)+m^2(\underbrace{1-2\gamma-\gamma^2-2\gamma+2\gamma^2}_{1-4\gamma+\gamma^2})\Big)$$

$$+im\gamma(1-\gamma)\ \bar{u}^{s_2}_{p_2}\sigma^{\mu\nu}q_\nu u^{s_1}_{p_1}\Big\}$$

$$F_1(q^2)=4ie^2\int\frac{d^4k}{(2\pi)^4}\int d\alpha d\beta d\gamma\frac{\delta(\alpha+\beta+\gamma-1)}{[k^2-(1-\gamma)^2m^2+\alpha\beta q^2+i\varepsilon]^3}\times$$

$$\times\Big(-\frac{1}{2}k^2+q^2(1-\alpha)(1-\beta)+m^2(1-4\gamma+\gamma^2)\Big)$$

$$F_2(q^2)=4ie^2\int\frac{d^4k}{(2\pi)^4}\int d\alpha d\beta d\gamma\frac{\delta(\alpha+\beta+\gamma-1)\ 2m^2\gamma(1-\gamma)}{[k^2-(1-\gamma)^2m^2+\alpha\beta q^2+i\varepsilon]^3}$$

$$\Delta=-\alpha\beta q^2+(1-\gamma)^2m^2$$

$$=4ie^2\int d\alpha d\beta d\gamma\ \delta(\alpha+\beta+\gamma-1)\ 2m^2\gamma(1-\gamma)\frac{(-i)\ \Gamma(3-2)}{(4\pi)^2\ \Gamma(3)}\frac{1}{\Delta}$$

$$=\frac{4e^2\,2m^2}{(4\pi)^2\,2}\int d\alpha d\beta d\gamma\frac{\gamma(1-\gamma)\delta(\alpha+\beta+\gamma-1)}{((1-\gamma)^2m^2-\alpha\beta q^2)}$$

$$=\frac{e^2m^2}{4\pi^2}\int_0^1 d\gamma\int_0^{1-\gamma}d\alpha\int_0^{1-\gamma-\alpha}d\beta\frac{\gamma(1-\gamma)\ \delta(\alpha+\beta+\gamma-1)}{((1-\gamma)^2m^2-\alpha\beta q^2)}$$

$$F_2(q^2) = \frac{e^2 m^2}{4\pi^2} \int_0^1 d\gamma \int_0^{1-\gamma} d\alpha \; \frac{\gamma(1-\gamma)}{\left[(1-\gamma)^2 m^2 - \alpha(1-\alpha-\gamma)q^2\right]}$$

$$F_2(q^2=0) = \frac{e^2 \cancel{m^2}}{4\pi^2} \int_0^1 d\gamma \int_0^{1-\gamma} d\alpha \; \frac{\gamma\cancel{(1-\gamma)}}{(1-\gamma)^2 \cancel{m^2}}$$

$$= \frac{e^2}{4\pi^2} \int_0^1 d\gamma\, \gamma = \frac{e^2}{8\pi^2}$$

$\alpha = e^2/4\pi = \frac{1}{137}$

$$F_2(0) = \frac{\alpha}{2\pi}$$

$g = 2 + \alpha/\pi$

$g_{exp} = 2.002319\,3043617(15)$

$\delta g = 0.00232343$

0.0023228 (exp.)