

662, Homework III, (2 problems)

Problem 1

Consider a real scalar field with a ϕ^4 interaction, namely

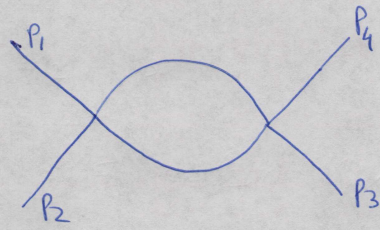
$$S = \int d^d x \left[\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{4!} \phi^4 \right] \quad (0.1)$$

- a) Compute the bubble diagram of the figure for $d = 4 - \epsilon$ dimensions. Obtain the $1/\epsilon$ divergent term and the ϵ independent term. Define $p = p_1 + p_2$ and consider the cases $p^2 > 4m^2$ and $p^2 < 4m^2$. The result should be in terms of elementary functions (ln, arcsin, square root).
- b) Determine for what values of p^2 the diagram has an imaginary part and compute the imaginary part explicitly from your previous result.

Problem 2

For a scalar field theory determine the superficial degree of divergence for each of the diagrams in the figure.

P1



P2

