662, Homework IV, (1 problem)

Problem 1

Consider a real scalar field with a ϕ^4 interaction in terms of the bare parameters and the renormalized ones, namely

$$\begin{aligned} S &= \int d^d x \left[\frac{1}{2}\partial_\mu \phi_0 \partial^\mu \phi_0 - \frac{1}{2} m_0^2\, \phi_0^2 - \frac{\lambda_0}{4!}\phi_0^4 \right] &(0.1)\\ &= \int d^d x \left[\frac{1+\delta_Z}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2}(m^2 + \delta_m)\phi^2 - \mu^\epsilon \frac{\lambda + \delta_\lambda}{4!}\phi^4 \right] &(0.2)\end{aligned}$$

a) Compute the counter–terms δ_Z, δ_m, δ_λ to one loop in perturbation theory.

b) Compute the two loop diagrams assuming zero external momenta and get the counter–terms at two loops.

c) Rewrite the relation between bare and renormalized parameters as

$$\begin{aligned} \lambda_0 &= \mu^\epsilon \left(1 + \frac{a_1(\lambda)}{\epsilon} + \frac{a_2(\lambda)}{\epsilon^2} + \dots \right) &(0.3)\\ m_0^2 &= m^2 \left(1 + \frac{b_1(\lambda)}{\epsilon} + \frac{b_2(\lambda)}{\epsilon^2} + \dots \right) &(0.4)\\ \phi_0 &= \left(1 + \frac{c_1(\lambda)}{\epsilon} + \frac{c_2(\lambda)}{\epsilon^2} + \dots \right) &(0.5)\end{aligned}$$

namely compute the coefficients a_j, b_j, c_j to second order in perturbation theory.