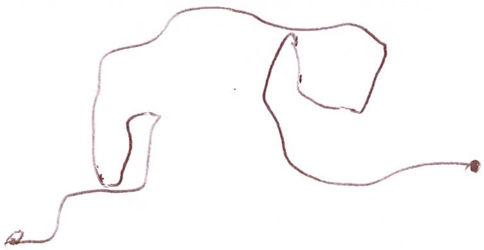
Self-avoiding polymers



u : proper length.

$\vec{r}(u)$ path.

$$S = \int_0^s du \, \frac{1}{4} (\dot{\vec{r}}(u))^2 + \frac{g}{6} \int_0^s du_1 \int_0^s du_2 \, \delta^{(d)} (\vec{r}(u_1) - \vec{r}(u_2))$$

$$G^{(2)}(\vec{k}, s) = \left\langle e^{i k . (\vec{r}(s) - r(0))} \right\rangle$$

$$Z^{(2)}(k, t) = \int_0^\infty e^{-st} \, G^{(2)}(k, s) \, ds$$

•) $g = 0$ case.

$$G^{(2)}(\vec{k}, s) = \frac{1}{\int} \int \mathcal{D}\vec{r}(u) \; e^{-\int_0^s du \frac{1}{4} \dot{r}^2 + i\vec{k}(\vec{r}(s) - \vec{r}(0))}$$

$$\int_0^s \dot{r} \, du$$

$$= \int \mathcal{D}\vec{r}(u) \; e^{-\int_0^s du \frac{1}{4} \dot{r}^2 + i \int_0^s \vec{k} . \dot{\vec{r}} \, du}$$

$$-\vec{k}^2/2$$

$$= \int \mathcal{D}\vec{r}(u) \; e^{-\int_0^s du \frac{1}{4} (\dot{r} - 2i\vec{k})^2 - \vec{k}^2 s}$$

$$= \frac{\int \mathcal{D}\vec{r}(u)\, e^{-\int_0^s du \frac{1}{4}\dot{\vec r}^2}\; e^{-\vec{k}^2 s}}{\int \mathcal{D}\vec{r}(u)\, e^{-\int_0^s du \frac{1}{4}\dot{\vec r}^2}}$$

Flory's argument

$r^2 \sim s^{2\nu} \to r \sim s^\nu$

$\dot r \sim s^{\nu-1}$

$\int^s \dot r^2 \sim s^{2\nu-1}$

$\int du \int du\, \delta^{(d)}(r-r)$

$\sim s^{-d\nu+2}$

$2-\nu d = 2\nu-1$ not

$\nu = \frac{3}{d+2}$ $d=1\ 2\ 3\ 4$ good

$d=4$ upper dim

random walk ~ surface

$$G^{(2)}(\vec k, s) = e^{-\vec k^2 s} = 1 - \vec k^2 s + \ldots$$

$$Z^{(2)}(k,t) = \int_0^\infty e^{-st-\vec k^2 s}\, ds = \frac{1}{\vec k^2 + t}$$

$$G^{(2)} = \langle 1 \rangle - \frac{1}{2} k_i k_j \langle (r_i(s) - r_i(0))(r_j(s) - r_j(0)) \rangle + \ldots$$

$A\delta_{ij}$

$Ad \cdot \langle (r-r)^2 \rangle$

$$= 1 - \frac{\vec k^2}{2d} \langle (\vec r(s) - r(0))^2 \rangle$$

$$\langle (\vec r(s) - r(0))^2 \rangle \sim s$$

$$\sqrt{\langle (\vec r(s) - \vec r(0))^2 \rangle} \sim \sqrt{s} \quad \text{random walk.}$$

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in $d=1$ self-avoidance implies $|\vec r(s) - \vec r(0)| = s$ instead of $\sqrt{s}$. we expect $\sqrt{\langle (\vec r(s) - \vec r(0))^2 \rangle} \sim s^{\nu}$

$$\boxed{\tfrac{1}{2} \le \nu \le 1}$$

$$\int \mathcal{D}\sigma(r)\; e^{-\frac{3}{2g}\int d^d r\, \sigma^2(r) + i\int_0^s du\; \sigma(\vec{r}(u))}$$

$$e^{-\frac{3}{2g}\int d^d r\, \sigma^2(r) + i\int d^d r\; \sigma(\vec{r})\oint du\; \delta^{(d)}(\vec{r}-\vec{r}(u))}$$

$$e^{-\frac{3}{2g}\int d^d r\; \left(\sigma(r) - \frac{ig}{3}\int du\; \delta^{(d)}(\vec{r}-\vec{r}(u))\right)^2} \times$$

gr

$$\times\, e^{-\frac{3}{2g}\frac{g^2}{9}\int d^d r\int du_1\int du_2\; \delta^{(d)}(\vec{r}-r(u_1))\,\delta^{(d)}(\vec{r}-r(u_2))}$$

$$e^{-\frac{g}{6}\int du_1\, du_2\; \delta^{(d)}(\vec{r}(u_1)-\vec{r}(u_2))}$$

✓

$$\int \mathcal{D}r(u)\int \mathcal{D}\sigma(r)\; e^{-\frac{3}{2g}\int d^d r\, \sigma^2(r) + i\int du\; \sigma(\vec{r}(u))}$$

$$\times\, e^{-\int_0^s du\,\frac{1}{4}\dot{r}^2 + i\vec{k}(\vec{r}(s)-\vec{r}(0))}$$

$$= \int \mathcal{D}\sigma(r)\; e^{-\frac{3}{2g}\int d^d r\, \sigma^2(r)}\int \mathcal{D}r(u)\, e^{-\int_0^s du\frac{1}{4}\dot{r}^2 + i\int du\, \sigma(\vec{r}(u))}\; e^{i\vec{k}(\vec{r}(s)-r(0))}$$

$\frac{1}{2}mv^2$

$$Z = \int \mathcal{D}\sigma(r) \; e^{-\frac{3}{2g}\int d^d r\, \sigma^2(r)} \int dr\, dr' \; \langle r' | e^{-sH} | r \rangle \; e^{ik(r'-r)}$$

$$H = p^2 + i\sigma(\vec{r})$$

$$H = -\nabla^2 - i\sigma$$

$$Z = \int_0^\infty ds \; e^{-ts} \times G$$

$$\int_0^\infty ds \; e^{-ts - sH} = \frac{1}{H+t}$$

$$Z = \int \mathcal{D}\sigma(r) \; e^{-\frac{3}{2g}\int d^d r\, \sigma^2(r)} \int dr\, dr' .$$

$$\cdot \langle r' | \frac{1}{H+t} | r \rangle \; e^{i\vec{k}(\vec{r}' - \vec{r})}$$

$$-\nabla^2 - i\sigma + t$$

$$\int \mathcal{D}\phi(r)\, \phi_1(r)\, \phi_1(r')\; e^{-\frac{1}{2}\int d^d r\, (\nabla\phi_a)^2 + (t+i\sigma)\phi_a^2}$$

$$= \det{}^{-N/2}\left(-\nabla^2 + t + i\sigma\right) \cdot \langle r' | (-\nabla^2 + t + i\sigma)^{-1} | r \rangle$$

$N \to 0$

$$Z = \int \mathcal{D}\phi\; \phi_1(r)\, \phi_1(r')\, e^{ik(r'-r)} \int \mathcal{D}\sigma(r)\; e^{-\frac{3}{2g}\int d^d r\, \sigma^2(r)}$$

$$e^{-\frac{1}{2}\int d^d r\, (\nabla\phi_a)^2 + (t - i\sigma)\phi_a^2}$$

$$-\frac{3}{2g}\sigma^2 + \frac{i}{2}\sigma\phi_a^2$$

$$-\frac{3}{2g}\left(\sigma - \frac{g}{6}i\phi_a^2\right) - \frac{3g}{2\times 36}\phi_a^4$$

$$= \int \mathcal{D}\phi\;\; \phi_1(K)\, \phi_1(-K)\; e^{-\frac{1}{2}\int d^d r\left(\frac{(\nabla\phi_a)^2}{2} + \frac{t}{2}\phi_a^2 + \frac{g}{4!}(\phi_a^2)^2\right)}$$

$N \to 0$!!

$$Z^{(2)}(\vec{k},t) = \int \mathcal{D}\phi \; \phi_1(k)\,\phi_1(-k) \;\times$$

$$\times\; e^{-\int d^d r \left(\frac{1}{2}(\nabla\phi_a)^2 + \frac{1}{2} t\,\phi_a^2 + \frac{g}{4!}(\phi_a^2)^2 \right)}$$

$$= \langle \phi_1(k)\,\phi_1(-k) \rangle_{N \to 0}$$

$$\int d^d x \; e^{i\vec{k}\vec{x}} \underbrace{\langle \phi_1(\vec{x})\,\phi_1(0) \rangle}_{\frac{e^{-r/\xi}}{r^{d-2+\eta}}} \sim \xi^{\cancel{+d-d}+2-\eta} f(k\xi)$$

$$t \to t_c \qquad Z^{(2)} \sim (t-t_c)^{-\nu(2-\eta)} f\left(k(t-t_c)^{-\nu}\right)$$

$$f(0) + k^2 (t-t_c)^{-2\nu}$$

$$Z^{(2)} = \int_0^\infty ds\; e^{-st}\, G^{(2)}(s)$$

$$e^{st_c}$$

$$e^{-s(t-t_c)}\, s^{\nu(2-\eta)} + e^{-s(t-t_c)}\, k^2\, s^{\nu(2-\eta)+2\nu}$$

normalizing

$$1 + a\vec{k}^2 s^{2\nu} + \dots \qquad \langle (\vec{r}(s) - \vec{r}(0))^2 \rangle \simeq s^{2\nu}$$

$$\sqrt{(\vec{r}(s) - \vec{r}(0))^2} \sim s^{\nu} \qquad \nu = 0.588$$