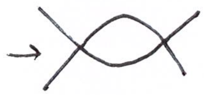
 $-i\lambda$

 $\dfrac{(-i\lambda)}{2}\, I_1$



$p=0$

$$\frac{(-i\lambda)^2}{2}\int\frac{d^dk}{(2\pi)^d}\left(\frac{i}{k^2-m^2+i\varepsilon}\right)^2 = -\frac{\lambda^2}{2}\, i\, \frac{\partial I_1}{\partial m^2}$$

$$\to\ -\frac{3\lambda^2}{2}\, i\, \frac{\partial I_1}{\partial m^2}$$

 $i\left(\delta_Z^{(1)} p^2 - \delta_m^{(1)}\right)$

 $-i\,\delta_\lambda^{(1)}$

1-loop

 $= -\dfrac{i\lambda}{2}\, I_1 - i\,\delta_m^{(1)} \quad \Rightarrow \quad \delta_m^{(1)} = -\dfrac{\lambda}{2}\, I_1^{div}$

$$\delta_Z^{(1)} = 0$$

 $\Big|_{div} = -\dfrac{3\lambda^2}{2}\, i\left(\dfrac{\partial I_1}{\partial m^2}\right)^{div} - i\,\delta_\lambda^{(1)}$

$$\delta_\lambda^{(1)} = -\frac{3\lambda^2}{2}\left(\frac{\partial I_1}{\partial m^2}\right)^{div}$$

$$= \frac{(-i\lambda)^2}{4} \int \frac{d^dk}{(2\pi)^d} \left(\frac{i}{k^2-m^2+i\varepsilon}\right)^2 I_1 = -\frac{\lambda^2}{4}\, i\, I_1 \frac{\partial I_1}{\partial m^2}$$

$$= \frac{(-i\lambda)^2}{6} H \Rightarrow \frac{(-i\lambda)^2}{6}\left(H^{div}(p^2=0) + p^2 \left(\frac{\partial H}{\partial p^2}\right)^{div}(p^2=0)\right).$$

$$= \frac{(-i\delta_\lambda^{(1)})}{2} I_1$$

$$= \frac{(-i\lambda)}{2} \int \frac{d^dk}{(2\pi)^d} \left(\frac{i}{k^2-m^2+i\varepsilon}\right)^2 i\,(\delta_Z^{(1)} p^2 - \delta_m^{(1)})$$

$$= -\frac{\lambda}{2}\, \delta_m^{(1)}\, i\, \frac{\partial I_1}{\partial m^2}$$

$$= i\,(p^2 \delta_Z^{(2)} - \cancel{m^2}\, \delta_m^{(2)})$$

$$-\frac{i\lambda^2}{4} I_1 \frac{\partial I_1}{\partial m^2} - \frac{\lambda^2}{6} H_0^{div} - \frac{i\delta_\lambda^{(1)}}{2} I_1 - \frac{\lambda}{2}\, \delta_m^{(1)}\, i\, \frac{\partial I_1}{\partial m^2} - i\,\delta_m^{(2)} = 0$$

$$\delta_m^{(2)} = -\frac{\lambda^2}{4} I_1 \frac{\partial I_1}{\partial m^2} + \frac{i\lambda^2}{6} H_0^{div} - \frac{1}{2}\delta_\lambda^{(1)} I_1 - \frac{\lambda}{2}\, \delta_m^{(1)} \frac{\partial I_1}{\partial m^2}$$

$$-\frac{\lambda^2}{6} H_0' + i\,\delta_Z^{(2)} = 0 \Rightarrow \delta_Z^{(2)} = -\frac{i\lambda^2}{6} H_0'$$

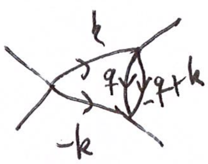
 $$= \frac{(-i\lambda)^3}{4}\, \cancel{i}\, \frac{\partial I_1}{\partial m^2}\, i\, \frac{\partial I_1}{\partial m^2} = -\frac{i\lambda^3}{4}\left(\frac{\partial I_1}{\partial m^2}\right)^2$$

$$\boxed{-\frac{3i}{4}\lambda^3\left(\frac{\partial I_1}{\partial m^2}\right)^2}$$

 $$= \frac{(-i\lambda)^3}{2}\int\frac{d^dk}{(2\pi)^d}\left(\frac{i}{k^2-m^2+i\varepsilon}\right)^3 I_1$$

$$= -\frac{i\lambda^3}{2} I_1 \int\frac{d^dk}{(2\pi)^d}\frac{i}{(k^2-m^2+i\varepsilon)^3}$$

$$= -\frac{i\lambda^3}{4} I_1 \frac{\partial^2 I_1}{(\partial m^2)^2} \rightarrow \boxed{-\frac{3i\lambda^3}{4} I_1 \frac{\partial^2 I_1}{(\partial m^2)^2}}$$



$$= \frac{(-i\lambda)^3}{2}\int\frac{d^dk}{(2\pi)^d}\int\frac{d^dq}{(2\pi)^d}\left(\frac{i}{k^2-m^2+i\varepsilon}\right)^2\frac{i}{q^2-m^2+i\varepsilon}\frac{i}{(k-q)^2-m^2+i\varepsilon}$$

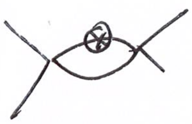
$$= -\frac{\lambda^3}{2}\int\frac{d^dk}{(2\pi)^d}\int\frac{d^dq}{(2\pi)^d}\frac{i}{(k^2-m^2+i\varepsilon)^2}\frac{i}{q^2-m^2+i\varepsilon}\frac{i}{(k-q)^2-m^2+i\varepsilon}$$

$$= -\frac{\lambda^3}{6}\frac{\partial H_0}{\partial m^2}$$

$(+i)(-i)(-i)i$

×6 diags

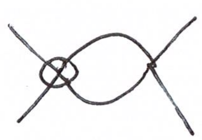
$$\rightarrow \boxed{-\lambda^3\frac{\partial H_0}{\partial m^2}}$$



$$= (-i\lambda)^2 \int \frac{d^dk}{(2\pi)^d} \left(\frac{i}{k^2-m^2+i\varepsilon}\right)^3 i(\delta_Z^{(1)} p^2 - \delta_m^{(1)})$$

$$= -i\lambda^2 \delta_m^{(1)} \int \frac{d^dk}{(2\pi)^d} \frac{i}{(k^2-m^2+i\varepsilon)^3} = -\frac{i\lambda^2}{2} \delta_m^{(1)} \frac{\partial^2 I_1}{(\partial m^2)^2}$$

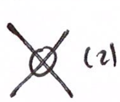
$$\boxed{-\frac{3i\lambda^2}{2} \delta_m^{(1)} \frac{\partial^2 I_1}{(\partial m^2)^2}}$$



$$= (+i\lambda)\frac{(+i\delta_\lambda)}{2}\, i \frac{\partial I_1}{\partial m^2}$$

$$= -\frac{i\lambda\delta_\lambda}{2} \frac{\partial I_1}{\partial m^2}$$

$$\rightarrow \boxed{-3i\lambda\delta_\lambda \frac{\partial I_1}{\partial m^2}}$$



$$(2) \rightarrow -i\delta_\lambda^{(2)}$$

$$-\cancel{i}\delta_\lambda^{(2)} - \frac{3i}{4}\lambda^3\left(\frac{\partial I_1}{\partial m^2}\right)^2 - \frac{3i\lambda^3}{4} I_1 \frac{\partial^2 I_1}{(\partial m^2)^2} - \frac{\cancel{A}\lambda^3}{\cancel{B}} \frac{\partial H}{\partial m^2}\frac{1}{i}$$

$$-\frac{3\cancel{i}\lambda^2}{2}\delta_m^{(1)} \frac{\partial^2 I_1}{(\partial m^2)^2} - 3\cancel{i}\lambda\delta_\lambda \frac{\partial I_1}{\partial m^2} = 0$$

$$\delta_\lambda^{(2)} = -\frac{3}{4}\lambda^3\left(\frac{\partial I_1}{\partial m^2}\right)^2 - \frac{3\lambda^3}{4} I_1 \frac{\partial^2 I_1}{(\partial m^2)^2} + i\lambda^3 \frac{\partial H}{\partial m^2}$$

$$-\frac{3}{2}\lambda^2 \delta_m^{(1)} \frac{\partial^2 I_1}{(\partial m^2)^2} - 3\lambda\,\delta_\lambda \frac{\partial I_1}{\partial m^2}$$

$$= -\frac{3}{4}\lambda^3\left(\frac{\partial I_1}{\partial m^2}\right)^2 - \frac{3\lambda^3}{4} I_1 \frac{\partial^2 I_1}{(\partial m^2)^2} + i\lambda^3 \frac{\partial H}{\partial m^2}$$

$$+\frac{3}{4}\lambda^3 I_1^{div} \frac{\partial^2 I_1}{(\partial m^2)^2} + \frac{9\lambda^3}{2}\left(\frac{\partial I_1}{\partial m^2}\right)^{div}\left(\frac{\partial I_1}{\partial m^2}\right)$$

---

finite

$$= -\frac{3}{4}\lambda^3\left(\frac{\partial I_1}{\partial m^2}\right)^2 - \frac{3}{4}\lambda^3 \underbrace{\left(I_1 - I_1^{div}\right)}_{\text{finite}} \frac{\partial^2 I_1}{(\partial m^2)^2}$$

$$+\frac{9}{2}\lambda^3\left(\frac{\partial I_1}{\partial m^2}\right)^{div}\left(\frac{\partial I_1}{\partial m^2}\right) + i\lambda^3\frac{\partial H}{\partial m^2}$$

6

$$\delta_m^{(2)} = -\frac{\lambda^2}{4} I_1 \frac{\partial I_1}{\partial m^2} + \frac{i\lambda^2}{6} H_0^{div} + \frac{3\lambda^2}{4}\left(\frac{\partial I_1}{\partial m^2}\right)^{div} I_1 + \frac{\lambda^2}{4} I_1^{div} \frac{\partial I_1}{\partial m^2}$$

$$= -\frac{\lambda^2}{4}\left(I_1 - I_1^{div}\right)\frac{\partial I_1}{\partial m^2} + \frac{3\lambda^2}{4} I_1 \left(\frac{\partial I_1}{\partial m^2}\right)^{div} + \frac{i\lambda^2}{6} H_0^{div}$$

$$= +\frac{\lambda^2}{4} I_1^f \frac{1}{8\pi^2\varepsilon} + \frac{3\lambda^2}{4}\left(-\frac{m^2}{8\pi^2\varepsilon} + I_1^f\right)\left(-\frac{1}{8\pi^2\varepsilon}\right) +$$

$$+ \frac{i\lambda^2}{6} H_0^{div}$$

$$\delta_m^{(2)} = \frac{\lambda^2 I_1^f}{32\pi^2\varepsilon} + \frac{3m^2\lambda^2}{2^8\pi^4\varepsilon^2} - \frac{3\lambda^2 I_1^f}{32\pi^2\varepsilon} + \frac{i\lambda^2}{6} H_0^{div.}$$

$$= -\frac{\lambda^2 I_1^f}{16\pi^2\varepsilon} + \frac{3m^2\lambda^2}{2^8\pi^4\varepsilon^2} + \frac{i\lambda^2}{6} H_0^{div}$$

$$= -\frac{\lambda^2 I_1^f}{16\pi^2\varepsilon} + \cancel{\frac{3m^2\lambda^2}{2^8\pi^4\varepsilon^2}} \cancel{+ \frac{9m^2\lambda^2}{3\times 2^9\pi^4\varepsilon}} + \frac{\lambda^2}{6}\cancel{\frac{3\cdot 2}{8\pi^2\varepsilon}} I_1^f$$

$$\boxed{\delta_m^{(2)} \cancel{= -\frac{3m^2\lambda^2}{2^9\pi^4\varepsilon^2}}} \quad \cancel{+ \frac{m^2\lambda^2}{32\pi^2\varepsilon} I_1^f} \qquad \boxed{\delta_m^{(2)} = -\cancel{\frac{\lambda^2 m^2}{2^9\pi^4\varepsilon}}}$$

$$-\cancel{\frac{m^2\lambda^2}{2^8\pi^4\varepsilon^2}} - \cancel{\frac{\lambda^2 m^2}{2^9\pi^4\varepsilon}}$$

$$\delta_m^{(2)} = \frac{3m^2\lambda^2}{2^8\pi^4\varepsilon^2} - \frac{\lambda^2 I_1^f}{16\pi^2\varepsilon} + \frac{i\lambda^2}{6}\left(\frac{3im^2}{2^7\pi^4\varepsilon^2} + \frac{3im^2}{2^8\pi^4\varepsilon} - \frac{3i I_1^f}{8\pi^2\varepsilon}\right)$$

$$= \frac{3m^2\lambda^2}{2^8\pi^4\varepsilon^2} - \frac{\lambda^2 I_1^f}{16\pi^2\varepsilon} - \frac{\cancel{3}\lambda^2 m^2}{\cancel{6}_{2}\times 2^7\pi^4\varepsilon^2} - \frac{3m^2\lambda^2}{6\times 2^8\pi^4\varepsilon} + \frac{3\lambda^2 I_1^f}{6\times 8\pi^2\varepsilon}$$

$$= \frac{m^2\lambda^2}{2^7\pi^4\varepsilon^2} - \frac{m^2\lambda^2}{2^9\pi^4\varepsilon} - \cancel{\frac{\lambda^2 I_1^f}{16\pi^2\varepsilon}} + \cancel{\frac{\lambda^2 I_1^f}{16\pi^2\varepsilon}}$$

$$= \frac{m^2\lambda^2}{2^9\pi^4}\left(\frac{1}{\varepsilon^2} - \frac{1}{4\varepsilon}\right)$$

$$\delta_Z^{(2)} = -\frac{i\lambda^2}{6} H_0' = +\frac{i\lambda^2}{6}\left(+\frac{i}{2^9\pi^4}\right)\frac{1}{\varepsilon} = -\frac{\lambda^2}{3\times 2^{10}\pi^4}\frac{1}{\varepsilon}$$

$$\delta_\lambda^{(2)} = -\frac{3}{4}\lambda^3\left(\frac{\partial I_1}{\partial m^2}\right)^2 + \frac{9}{2}\lambda^3\left(\frac{\partial I_1}{\partial m^2}\right)^{div}\frac{\partial I_1}{\partial m^2} + i\lambda^3\frac{\partial H}{\partial m^2}$$

$$\cancel{\frac{\partial H}{\partial m^2} = \frac{3i}{2^7\pi^4\varepsilon^2} + \frac{3i}{2^8\pi^4\varepsilon} - \frac{3i}{8\pi^2\varepsilon}\frac{I_1^f}{m^2}}$$

$$\cancel{\frac{\partial I_1}{\partial m^2} = -\frac{1}{8\pi^2\varepsilon}\left(1 - \frac{8\pi^2\varepsilon I_1^f}{m^2} - 8/2\right) = \frac{1}{16\pi^2\varepsilon} + \frac{I_1^f}{m^2\varepsilon}}$$

$$H(p^2=0) = \frac{3im^2}{2^7\pi^4\varepsilon^2} + \frac{3im^2}{2^8\pi^4\varepsilon} - \frac{3i}{8\pi^2\varepsilon} I_1^f$$

$$\frac{\partial H}{\partial m^2}(p^2=0) = \frac{3i}{2^7\pi^4\varepsilon^2} + \frac{3i}{2^8\pi^4\varepsilon} - \frac{3i}{8\pi^2\varepsilon}\frac{\partial I_1^f}{\partial m^2}$$

$$\delta_\lambda^{(2)} = -\frac{3}{4}\lambda^3\left(\frac{\partial I_1^{div}}{\partial m^2}\right)^2 - \frac{3}{2}\lambda^3\frac{\partial I_1^{div}}{\partial m^2}\frac{\partial I_1^f}{\partial m^2} + \frac{9}{2}\lambda^3\left(\frac{\partial I_1^{div}}{\partial m^2}\right)\left(\frac{\partial I_1^{div}}{\partial m^2}\right)$$

$$+\frac{9}{2}\lambda^3\left(\frac{\partial I_1^{div}}{\partial m^2}\right)\frac{\partial I_1^f}{\partial m^2} - \frac{3\lambda^3}{2^7\pi^4\varepsilon^2} - \frac{3\lambda^3}{2^8\pi^4\varepsilon} + \frac{3\lambda^3}{8\pi^2\varepsilon}\frac{\partial I_1^f}{\partial m^2}$$

$$-3\frac{\partial I_1^{div}}{\partial m^2}\frac{\partial I_1^f}{\partial m^2}$$

$$I_1^{div} = -\frac{m^2}{8\pi^2\varepsilon} \qquad \frac{\partial I_1^{div}}{\partial m^2} = -\frac{1}{8\pi^2\varepsilon}$$

$$\frac{18-3}{4}$$

$$\delta_\lambda^{(2)} = \left(\frac{9}{2}-\frac{3}{4}\right)\lambda^3\left(\frac{\partial I_1^{div}}{\partial m^2}\right)^2 + \lambda^3\left(\frac{\partial I_1^{div}}{\partial m^2}\right)\left(\frac{\partial I_1^f}{\partial m^2}\right)\cancel{\left(-\frac{3}{2}+\frac{9}{2}-3\right)}$$

$$-6/2$$

$$-\frac{3\lambda^3}{2^7\pi^4\varepsilon^2} - \frac{3\lambda^3}{2^8\pi^4\varepsilon} = \frac{15}{4}\lambda^3\frac{1}{2^6\pi^4\varepsilon^2} - \frac{3\lambda^3}{2^7\pi^4\varepsilon^2} - \frac{3\lambda^3}{2^8\pi^4\varepsilon}$$

$$= \frac{15-6}{2^8\pi^4\varepsilon^2}\lambda^3 - \frac{3\lambda^3}{2^8\pi^4\varepsilon} = \frac{9}{2^8\pi^4\varepsilon^2}\lambda^3 - \frac{3\lambda^3}{2^8\pi^4\varepsilon}$$

Summary

$$\delta_Z^{(1)} = 0 \quad ; \quad \delta_m^{(1)} = \frac{\lambda m^2}{16\pi^2\varepsilon} \quad ; \quad \delta_\lambda^{(1)} = \frac{3\lambda^2}{16\pi^2\varepsilon}$$

$$\delta_Z^{(2)} = -\frac{\lambda^2}{3\times 2^{10}\pi^4\varepsilon} \quad ; \quad \delta_m^{(2)} = \frac{m^2\lambda^2}{2^7\pi^4}\left(\frac{1}{\varepsilon^2} - \frac{1}{4\varepsilon}\right) ; \quad \delta_\lambda^{(2)} = \frac{9\lambda^3}{2^8\pi^4\varepsilon^2} - \frac{3\lambda^3}{2^8\pi^4\varepsilon}$$

$$\mu^{-\varepsilon}\lambda_0 = \frac{\lambda + \delta_\lambda}{(1+\delta_Z)^2} = (\lambda + \delta_\lambda)(1 - \delta_Z + \delta_Z^2 + \cdots)^2 = (\lambda + \delta_\lambda)(1 - 2\delta_Z + \cdots)$$

$$= \lambda - 2\delta_Z\lambda + \delta_\lambda + \cdots$$

$$= \lambda - 2\delta_Z^{(2)}\lambda + \delta_\lambda^{(1)} + \delta_\lambda^{(2)}$$

$$= \lambda + \frac{3\lambda^2}{16\pi^2\varepsilon} + \frac{2\lambda^3}{3\times 2^{10}\pi^4\varepsilon} + \frac{9\lambda^3}{2^8\pi^4\varepsilon^2} - \frac{3\lambda^3}{2^8\pi^4\varepsilon}$$

2^6

Define $g = \frac{\lambda}{8\pi^2}$

$$g_0 = \mu^\varepsilon\left(g + \frac{3}{2}\frac{g^2}{\varepsilon} + \frac{g^3}{3\times 8\varepsilon} + \frac{9g^3}{4\varepsilon^2} - \frac{3g^3}{4\varepsilon}\right)$$

$$= \mu^\varepsilon\left(g + \frac{3}{2}\frac{g^2}{\varepsilon} + \frac{9g^3}{4\varepsilon^2} - \frac{1-18}{24\varepsilon}g^3 + \cdots\right)$$

$$g_0 = \mu^{\varepsilon}\left(g + \frac{3}{2}\frac{g^2}{\varepsilon} - \frac{17}{24\varepsilon}g^3 + \frac{9g^3}{4\varepsilon^2} + \cdots\right)$$

$$\phi_0 = (1+\delta_Z)^{1/2}\phi_R = \phi_R\left(1+\frac{1}{2}\delta_Z + \cdots\right)$$

$$\phi_0 = \phi_R\left(1 - \frac{\lambda^2}{3\times 2^{11}\pi^4\varepsilon} + \cdots\right) = \phi_R\left(1 - \frac{g^2}{3\times 2^5\varepsilon} + \cdots\right)$$

$$m_0^2 = (m^2+\delta_m)(1-\delta_Z+\delta_Z^2 + \cdots)$$

$$= m^2 - m^2\delta_Z^{(2)} + \delta_m^{(1)} + \delta_m^{(2)} + \cdots$$

$$= m^2\left(1 + \frac{\lambda^2}{3\times 2^{10}\pi^4\varepsilon} + \frac{\lambda}{16\pi^2\varepsilon} + \frac{\lambda^2}{2^{?}\pi^4}\left(\frac{1}{\varepsilon^2} - \frac{1}{4\varepsilon}\right)\right)$$

$$= m^2\left(1 + \frac{g}{2\varepsilon} + \frac{g^2}{2\varepsilon^2} + \frac{g^2}{3\times 2^4\varepsilon} - \frac{g^2}{8\varepsilon}\right)$$

$$\frac{1}{3\times 2^4} - \frac{1}{8} = \frac{1-6}{3\times 2^4} = -\frac{5}{3\times 2^4}$$

$$m_0^2 = m^2\left(1 + \frac{g}{2\varepsilon} - \frac{5}{48\varepsilon}g^2 + \frac{g^2}{2\varepsilon^2} + \cdots\right)$$

<u>Summary</u>

$$g_c = \mu^\varepsilon \left(g + \frac{3}{2}\frac{g^2}{\varepsilon} - \frac{17}{24\varepsilon} g^3 + \frac{9}{4\varepsilon^2} g^3 + \cdots \right)$$

$$\phi_c = \phi_R \left(1 - \frac{g^2}{96\varepsilon} + \cdots \right)$$

$$m_c^2 = m^2 \left(1 + \frac{g}{2\varepsilon} - \frac{5}{48\varepsilon} g^2 + \frac{g^2}{2\varepsilon^2} + \cdots \right)$$

$$a_1 = \frac{3}{2} g^2 - \frac{17}{24} g^3 \qquad a_2 = \frac{9}{4} g^3$$

$$b_1 = \frac{g}{2} - \frac{5}{48} g^2 \qquad b_2 = g^2/2$$

$$c_1 = - g^2/96 \qquad c_2 = 0 .$$

$$\beta = -a_1 + g \frac{\partial a_1}{\partial g} = -\frac{3}{2} g^2 + \frac{17}{24} g^3 + 3g^2 - \frac{17}{8} g^3$$

$$= \frac{3}{2} g^2 + \frac{17}{24}(1-3) g^3 = \frac{3}{2} g^2 - \frac{17}{12} g^3 + \cdots$$

$$\gamma = g \frac{\partial c_1}{\partial g} = - g^2/48$$

$$\gamma_m = g \frac{\partial b_1}{\partial g} = \frac{g}{2} - \frac{5}{24} g^2$$

$$g_0 = \mu^\epsilon \left(g + \sum_{j=1}^{\infty} \frac{a_j(g)}{\epsilon^j} \right)$$

$$m_0^2 = m^2 \left(1 + \sum_{j=1}^{\infty} \frac{b_j(g)}{\epsilon^j} \right)$$

$$\phi_0 = \phi_R \left(1 + \sum_{j=1}^{\infty} \frac{c_j(g)}{\epsilon^j} \right)$$

$\mu \to \mu + \delta\mu \qquad g \to g + \delta g \qquad m^2 \to m^2 + \delta m^2 \qquad \phi_R \to \phi_R + \delta\phi_R$

so that g_0, m_0^2, ϕ_0 are fixed.

$$0 = \delta g_0 = \epsilon \frac{\delta\mu}{\mu} \cancel{\mu^\epsilon} \left(g + \sum_{j=1}^{\infty} \frac{a_j(g)}{\epsilon^j} \right) + \cancel{\mu^\epsilon} \left(\delta g + \sum_{j=1}^{\infty} \frac{a_j'(g)\,\delta g}{\epsilon^j} \right)$$

$$\epsilon \frac{\delta\mu}{\mu} g + \epsilon \frac{\delta\mu}{\mu} \sum_{j=1}^{\infty} \frac{a_j(g)}{\epsilon^j} = -\delta g - \delta g \sum_{j=1}^{\infty} \frac{a_j'(g)}{\epsilon^j}$$

δg finite $\qquad \delta g = \delta g_0 + \epsilon\,\delta g_1 \qquad$ (highest order on LHS $\to 1$)

$$\epsilon \frac{\delta\mu}{\mu} g + \frac{\delta\mu}{\mu} \sum_{j=1}^{\infty} a_j(g)\, \epsilon^{-j+1} = -\delta g_0 - \epsilon\,\delta g_1 - \delta g_0 \sum_{j=1}^{\infty} \frac{a_j'(g)}{\epsilon^j} - \epsilon\,\delta g_1 \sum_{j=1}^{\infty} \frac{a_j'(g)}{\epsilon^j}$$

$$\epsilon \frac{\delta\mu}{\mu} g + \frac{\delta\mu}{\mu} \sum_{j=0}^{\infty} a_{j+1}\, \epsilon^{-j} = -\delta g_0 - \epsilon\,\delta g_1 - \delta g_0 \sum_{j=1}^{\infty} a_j'\, \epsilon^{-j} - \delta g_1 \sum_{j=0}^{\infty} a_{j+1}'\, \epsilon^{-j}$$

$$\frac{\delta\mu}{\mu} g = -\delta g_1 \quad ; \quad \frac{\delta\mu}{\mu} a_1 = -\delta g_0 \cancel{-\delta g_0 a_0} - \delta g_1 a'_1$$

$$\frac{\delta\mu}{\mu} a_{j+1} = -\delta g_0 a'_j - \delta g_1 a'_{j+1}$$

$$\delta g_1 = -g \frac{\delta\mu}{\mu} \quad ; \qquad \delta g_0 = -\frac{\delta\mu}{\mu} a_1 + g a'_1 \frac{\delta\mu}{\mu}$$

$$\cancel{\frac{\delta\mu}{\mu}} a_{j+1} = \cancel{\frac{\delta\mu}{\mu}} a_1 a'_j - g a'_1 \cancel{\frac{\delta\mu}{\mu}} a'_j + g \cancel{\frac{\delta\mu}{\mu}} a'_{j+1}$$

$$\left[\delta g_1 = -g \frac{\delta\mu}{\mu} \qquad \delta g_0 = (-a_1 + g a'_1) \frac{\delta\mu}{\mu} \right.$$

$$\left[a_{j+1} - g a'_{j+1} = a_1 a'_j - g a'_1 a'_j \right.$$

$$\delta g = (-\epsilon g + g a'_1 - a_1) \frac{\delta\mu}{\mu}$$

$$\boxed{\mu \frac{\partial g}{\partial\mu} = -\epsilon g + g a'_1 - a_1}$$ β-function $\beta = \mu \frac{\partial g}{\partial\mu}$.

Also $a_{j+1} - g a'_{j+1} = a_1 a'_j - g a'_1 a'_j$

Here : $a_1 = \frac{3}{2} g^2 - \frac{17}{24} g^3 + \cdots$

$a_2 = \frac{9}{4} g^3 + \cdots$

$$\boxed{\begin{aligned}\mu \frac{\partial g}{\partial\mu} &= -\epsilon g + 3g^2 - \frac{17}{24} 3g^3 - \\ &\quad - \frac{3}{2} g^2 + \frac{17}{24} g^3 + \cdots \\ &= -\epsilon g + \frac{3}{2} g^2 - \frac{17}{12} g^3 + \cdots\end{aligned}}$$

$$\beta = -\epsilon g + \frac{3}{2} g^2 - \frac{17}{12} g^3 + \cdots$$

$$a_2 - g\, a_2' = a_1 a_1' - g\, {a_1'}^2$$

$$\frac{9}{4} g^3 - \frac{3 \times 9}{4} g^3 = -\frac{9}{2} g^3 \quad \Big\| \quad \frac{3}{2} g^2 (3g) - g\,(3g)^2$$

$$\frac{9}{2} g^3 - 9 g^3 = -\frac{9}{2} g^3$$

✓ agree.

$$0 = \delta m^2 \Big(1 + \sum_{j=1}^{\infty} b_j\, \epsilon^{-j}\Big) + m^2 \sum_{j=1}^{\infty} (b_j'\, \delta g)\, \epsilon^{-j}$$

$$\frac{\delta m^2}{m^2} \Big(1 + \sum_{j=1}^{\infty} b_j\, \epsilon^{-j}\Big) = -\sum_{j=1}^{\infty} \delta g_0\, b_j'\, \epsilon^{-j} - \sum_{j=1}^{\infty} b_j'\, \delta g_1\, \epsilon^{1-j}$$

$$= -\delta g_0 \sum_{j=1}^{\infty} b_j'\, \epsilon^{-j} - \sum_{j=0}^{\infty} b_{j+1}'\, \delta g_1\, \epsilon^{-j}$$

$\dfrac{\delta m^2}{m^2} = \alpha_0 +$ ~~α~~ no ϵ term

$$\frac{\delta m^2}{m^2} = -b_1'\, \delta g_1 = g \frac{\delta \mu}{\mu} b_1' \quad ; \quad \delta m^2 = m^2 g\, b_1' \frac{\delta \mu}{\mu}$$

$$\frac{\delta m^2}{m^2} b_j = -\delta g_0\, b_j' - b_{j+1}'\, \delta g_1 = \Big((a_1 - g\, a_1')\, b_j' + g\, b_{j+1}'\Big) \frac{\delta \mu}{\mu}$$

$$g \cancel{\frac{\delta\mu}{\mu}} b'_1 b_j = \cancel{\frac{\delta\mu}{\mu}} \left(g b'_{j+1} + (a_1 - g a'_1) b'_j \right)$$

$$g b'_{j+1} = g b'_1 b_j - (a_1 - g a'_1) b'_j$$

Here: $b_1 = g/2 - \frac{5}{48} g^2 + \ldots \quad b_2 = g^2/2 + \ldots$

$$\delta m^2 = m^2 g \left(\frac{1}{2} - \frac{5}{24} g + \ldots \right) \frac{\delta\mu}{\mu}$$

$$\frac{\delta m^2}{m^2} = \left(\frac{g}{2} - \frac{5}{24} g^2 + \ldots \right) \frac{\delta\mu}{\mu}$$

$$g b'_2 = g b'_1 b_1 - (a_1 - g a'_1) b'_1$$

$$\left. \begin{aligned} & g b'_2 = g^2 \\ & g b'_1 b_1 - (a_1 - g a'_1) b'_1 = b'_1 (g b_1 - a_1 + g a'_1) \\ & \qquad = \left(\frac{1}{2}\right)\left(\frac{g^2}{2} - \frac{3}{2} g^2 + 3g^2 \right) = \frac{1}{2}\left(2g^2\right) = g^2 \end{aligned} \right\} \checkmark \ \underline{\text{agree.}}$$

m^2 and ϕ_R eqns. are completely equivalent $\quad b_j \to c_j$

$$\Rightarrow \quad \delta\phi_R = \phi_R \, g \, c_1' \frac{\delta\mu}{\mu}$$

$$g c_{j+1}' = g c_1' c_j - (a_1 - g a_1') c_j'$$

Here: $\quad c_1 = -g^2/96 \qquad c_2 = 0 \; (O(g^2)) \qquad c_2 \sim g^3$

$$\delta\phi_R = \phi_R \, g \left(-\frac{g}{48}\right) \frac{\delta\mu}{\mu} = -\frac{g^2}{48} \phi_R \frac{\delta\mu}{\mu}$$

$$\underbrace{g c_2'}_{O(g^3)} = \underbrace{g c_1' c_1}_{O(g^4)} - (a_1 - g a_1') c_1'$$

$$\downarrow$$

$$\frac{3}{2} g^2 - g 3 g$$

$$+ \frac{3}{2} g^2 \frac{g}{48}$$

$$g c_2' = \frac{3}{96} g^3$$

$$c_2 = \frac{1}{96} g^3 \quad \text{(prediction).}$$

$$S = \int d^d x \left(\frac{1}{2} \partial_\mu \phi_0 \partial^\mu \phi_0 - \frac{1}{2} m_0^2 \phi_0^2 - \frac{\lambda_0}{4!} \phi_0^4 \right)$$

$$= \int d^d x \left(\frac{1+\delta_Z}{2} \partial_\mu \phi_R \partial^\mu \phi_R - \frac{1}{2}(m^2 + \delta_{m^2}) \phi_R^2 - \frac{\mu^\varepsilon}{4!} (\lambda + \delta_\lambda) \phi_R^4 \right)$$

$$\Gamma_0^{(n)} = \frac{1}{\langle \phi_0 \phi_0 \rangle} \cdots \frac{1}{\langle \phi_0 \phi_0 \rangle} \langle \underbrace{\phi_0 \cdots \phi_0}_{n} \rangle$$

$$\phi_0 = \sqrt{1+\delta_Z}\, \phi_R = Z_\phi^{1/2} \phi_R$$

$$\Gamma_0^{(n)} = Z_{\phi_R}^{-n/2} \Gamma_R^{(n)}$$

$$Z_{\phi_R}^{-n/2} \Gamma_R^{(n)} (p_j; \lambda, m^2, \mu) = Z_{\phi_R + \delta\phi}^{-n/2} \Gamma_R^{(n)} (p_j, \lambda + \delta\lambda, m^2 + \delta m^2, \mu + \delta\mu)$$

$$-\frac{n}{2} \mu \frac{\partial \ln Z_\phi}{\partial \mu} \Gamma_R^{(n)} + \mu \frac{\partial \lambda}{\partial \mu} \frac{\partial \Gamma^{(n)}}{\partial \lambda} + \mu \frac{\partial m^2}{\partial \mu} \frac{\partial \Gamma^{(n)}}{\partial m^2} + \mu \frac{\partial \Gamma_R^{(n)}}{\partial \mu} = 0$$

$$0 = \delta \phi_0 = \frac{1}{2} Z_\phi^{-1/2} \delta Z_\phi \, \phi_R + Z_\phi^{1/2} \delta \phi_R = 0$$

$$\frac{1}{2} \frac{\delta Z_\phi}{Z_\phi} = -\frac{\delta \phi_R}{\phi_R} \qquad \mu \partial_\mu \ln Z_\phi = -2 \mu \frac{\partial_\mu \phi_R}{\phi_R}$$

We only need coeff of $\boxed{1/\epsilon}$

Define

$$\gamma = -\frac{1}{2}\mu \frac{\partial \ln Z_\phi}{\partial \mu} = \mu \frac{\partial_\mu \phi_R}{\phi_R}$$

$$\gamma = g c_1'$$

$$\gamma_{\phi^2} = \frac{1}{m^2}\mu \frac{\partial m^2}{\partial \mu}$$

$$\gamma_{\phi^2} = g b_1'$$

$$\beta = \mu \frac{\partial g}{\partial \mu}$$

$$\beta = -\epsilon g + g a_1' - a_1$$

$$\boxed{\mu \frac{\partial \Gamma_R^{(n)}}{\partial \mu} + m^2 \gamma_{\phi^2} \frac{\partial \Gamma^{(n)}}{\partial m^2} + n\gamma \Gamma_R^{(n)} + \beta \frac{\partial \Gamma^{(n)}}{\partial g} = 0}$$

<u>Here</u>

$$\beta = -\epsilon g + \frac{3}{2}g^2 - \frac{17}{12}g^3 + \cdots$$

$$\gamma_{\phi^2} = \frac{g}{2} - \frac{5}{24}g^2 + \cdots$$

$$\gamma = -g^2/48 + \cdots$$

fixed point

$\underline{m^2=0}$ $\qquad\qquad$ $\beta=0$

$$-\epsilon\cancel{g}+\frac{3}{2}g^{\cancel{2}}=0 \quad \begin{cases}\nearrow & g=0 \text{ free theory.}\\ \searrow & g=\frac{2}{3}\epsilon\end{cases}$$

$$g=\frac{2}{3}\epsilon+\delta$$

$$-\epsilon+\frac{3}{2}g-\frac{17}{12}g^2=0$$

$$-\cancel{\epsilon}+\cancel{\epsilon}+\frac{3}{2}\delta-\frac{17}{12}\frac{4}{9}\epsilon^2=0$$

$$\delta=\frac{\cancel{2}}{3}\frac{17}{\cancel{12}}\frac{\cancel{4}^{\,2}}{9}\epsilon^2=\frac{34}{81}\epsilon^2$$

$$g^*=\frac{2}{3}\epsilon+\frac{34}{81}\epsilon^2+\cdots$$

$$\mu\frac{\partial\Gamma_R^{(n)}}{\partial\mu}+n\gamma^*\Gamma_R^{(n)}=0$$

$$\Gamma_R^{(n)}\sim A\mu^{-n\gamma^*} \qquad \Gamma_R^{(n)} \text{ units}$$

$$\int d^dx\,\frac{1}{2}\underset{\substack{\downarrow\\ M^{-1+d/2}}}{\partial_\mu\phi}\,\partial_\mu\phi$$

$$\cancel{[\Gamma_R^{(n)}]=[\phi^n]=M^{(\frac{d}{2}-1)n}}$$

$$\cancel{\Gamma_R^{(n)}=A\,p^{n\gamma^*+n(\frac{d}{2}-1)}\mu^{-n\gamma^*}}$$

$$G^{(n)}(x_1 \cdots x_n) = \langle \phi(x_1) \cdots \phi(x_n) \rangle \sim M^{n\left(\frac{d}{2}-1\right)}$$

$$\int d^d x_1 \cdots d^d x_n \; e^{ip_1x_1 + \cdots + ip_nx_n} \; G^{(n)}(x_1 \cdots x_n) = (2\pi)^d \, \tilde{G}^{(n)}(p_1 \cdots p_n) \, \delta^{(d)}(\Sigma p_i)$$

$$M^{-nd + n\frac{d}{2} - n} = \tilde{G} \; M^{-d}$$

$$\tilde{G}^{(n)} \sim M^{-n\frac{d}{2} - n + d} \qquad G^{(2)} \sim M^{-2} \checkmark . \quad (1/p^2)$$

$$\tilde{G}^{(n)} = \underbrace{G^{(2)}(p_1) \cdots G^{(2)}(p_n)}^{M^{-2n}} \; \Gamma^{(n)}(p_1 \cdots p_n)$$

$$\Gamma^{(n)}(p_1 \cdots p_n) = M^{-n\frac{d}{2} - n + d + 2n} = M^{n + d - n\frac{d}{2}}$$

$$\Gamma^{(n)}(p_1 \cdots p_n) \sim |\vec{p}|^{\,n + d - \frac{nd}{2} + n\gamma^*} \; \mu^{-n\gamma^*} \; F(\hat{p}_j)$$

$$\Gamma^{(2)} = |p|^{2 + \cancel{d} - \cancel{d} + 2\gamma^*} \; \mu^{-2\gamma^*} \sim |p|^{2-\eta} \, \mu^{-2\gamma^*}$$

$$G^{(2)} = |p|^{-2-2\gamma^*} \; \mu^{2\gamma^*} \sim \frac{1}{p^{2-\eta}}$$

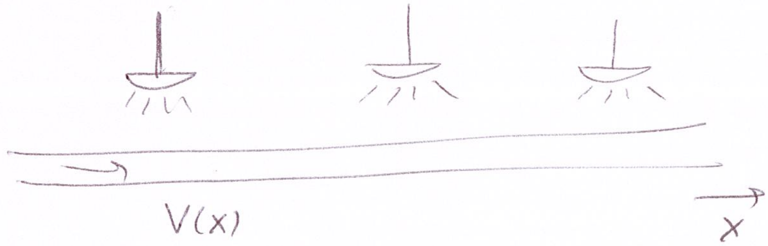
$$\Delta_{(2)}(x) = \int d^d p \; e^{ipx} \, G^{(2)}(p) \sim X^{-d+2+2\gamma^*} \; \mu^{2\gamma^*}$$

$$= \frac{\mu^{2\gamma^*}}{|\vec{X}|^{d-2-2\gamma^*}} = \frac{1}{|x|^{d-2+\eta}}$$

$$\eta = -2\gamma^* = \frac{g_*^2}{24} = \epsilon^2/54 \quad \overset{0.018}{}$$

$$\mu\frac{\partial\Gamma}{\partial\mu}+\beta(g)\frac{\partial\Gamma}{\partial g}+m^2\gamma_{\phi^2}(g)\frac{\partial\Gamma}{\partial m^2}+n\gamma\Gamma=0$$

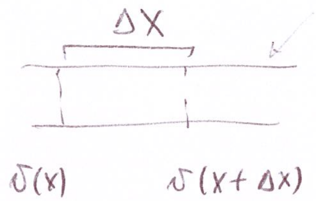
Bacterial analogy



 $\rho(t)$ $\quad \frac{d\rho}{dt}=\alpha\rho$

in the pipe

cross section $S(x)$ variable so that $v \sim 1/S(x)$



$$\Delta\rho\,\Delta x=[\rho(x,t+\Delta t)-\rho(x,t)]\,\Delta x=\frac{\partial\rho}{\partial t}\Delta t\,\Delta x$$

$$=\cancel{\rho(V(x)\Delta t-v(x+\Delta x)\Delta t)+\alpha\rho\Delta x\Delta t}$$

$$=\cancel{-\frac{\partial v}{\partial x}\rho\Delta x\Delta t+\alpha\rho\Delta x\Delta t}$$

$$=(\rho(x)-\rho(x+\Delta x))v(x)\Delta t+\alpha\rho\Delta x\Delta t$$

$$=-v(x)\frac{\partial\rho}{\partial x}\Delta x\Delta t+\alpha\rho\Delta x\Delta t$$

$$\boxed{\frac{\partial\rho}{\partial t}+V(x)\frac{\partial\rho}{\partial x}=\alpha(x)\,\rho}$$

Initial condition

$$\rho(x,t_0)=\rho_0(x)$$

We can solve it as follows:

we find a trajectory

$$X(t, x_0) \;/\; X(0, x_0) = x_0$$

$$\frac{\partial X}{\partial t}(t, x_0) = V(X(t, x_0))$$

then take

$$\rho(X(t, x_0), t) \;/\; \rho(X(0, x_0), t_0) = \rho_0(x_0)$$

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + \frac{\partial \rho}{\partial x}\frac{\partial x}{\partial t} = \frac{\partial \rho}{\partial t} + V(x)\frac{\partial \rho}{\partial x} \stackrel{?}{=} \alpha(x)\rho$$

$$\rho(X(t, x_0), t) = \rho_0(x_0)\, e^{\int_{t_0}^{t} \alpha(X(t', x_0))\, dt'}$$

$$\frac{d\rho}{dt} = \alpha(x)\rho \quad \checkmark$$

Solve.

$$g(\mu) \;/\; \mu\frac{\partial g}{\partial \mu} = \beta(g) \qquad g(\mu_0) = g_0$$

$$m^2(\mu) \;/\; \mu\frac{\partial \ln m^2}{\partial \mu} = \gamma_{\phi^2}(g(\mu))$$

$$\Gamma = \Gamma(p_j, g_1, m_1^2, \mu_1) \qquad \text{initial cond.}$$

$$\Gamma(p_j, g(\mu), m^2(\mu), \mu) = \Gamma(p_j, g_1, m_1^2, \mu_1) \cdot e^{-n \int_{\mu_1}^{\mu} \gamma(g(\mu')) \frac{d\mu'}{\mu'}}$$

$$\mu \frac{d\Gamma}{d\mu} = \mu \frac{\partial g}{\partial \mu} \frac{\partial \Gamma}{\partial g} + \mu \frac{\partial m^2}{\partial \mu} \frac{\partial \Gamma}{\partial m^2} + \mu \frac{\partial \Gamma}{\partial \mu} = -n\gamma(g(\mu))\Gamma$$

$$\beta(g(\mu)) \frac{\partial \Gamma^{(n)}}{\partial g} + m^2 \gamma_{\phi^2} \frac{\partial \Gamma^{(n)}}{\partial m^2} + \mu \frac{\partial \Gamma^{(n)}}{\partial \mu} + n\gamma\Gamma^{(n)} = 0$$

$$\Gamma = \mu^{n+d-\frac{nd}{2}} \bar{\Gamma}(p_i/\mu, g, m^2/\mu^2)$$

$$\Gamma(\sigma p_j, g, m^2, \mu) = \mu^{n+d-\frac{nd}{2}} \bar{\Gamma}(\sigma p_i/\mu, g, m^2/\mu^2)$$

$$= \left(\frac{\mu}{\sigma}\right)^{n+d-nd/2} \sigma^{n+d-nd/2} \bar{\Gamma}\left(\sigma p_i/\mu, g, \frac{m^2/\sigma^2}{\mu^2/\sigma^2}\right)$$

$$= \sigma^{n+d-nd/2} \bar{\Gamma}(p_j, g, m^2/\sigma^2, \mu/\sigma)$$

$$= \cancel{\sigma^{n+d-nd/2} \Gamma(p_j, g,}$$

$$\frac{\partial \Gamma(\sigma p_i, g, m^2, \mu)}{\partial \sigma} = \left(n+d-\frac{nd}{2}\right)\sigma^{n+d-nd/2-1}\,\bar{\Gamma}(p_i, g, m^2/\sigma^2, \mu/\sigma)$$

$$+\sigma^{n+d-nd/2}\,\frac{\partial \Gamma}{\partial m^2}\left(-\frac{2m^2}{\sigma^3}\right) + \frac{\mu}{\sigma^2}\frac{\partial \Gamma}{\partial \mu}$$

$$=$$

$$\sigma\frac{\partial \Gamma}{\partial \sigma} = \left(n+d-\frac{nd}{2}\right)\Gamma - \frac{2m^2}{\sigma^2}\,\sigma^{n+d-n\frac{d}{2}}\frac{\partial \bar{\Gamma}}{\partial m^2} - \frac{\mu}{\sigma}\frac{\partial \bar{\Gamma}}{\partial \mu}$$

$$= \left(n+d-\frac{nd}{2}\right)\Gamma - 2m^2\frac{\partial \Gamma}{\partial m^2} - \mu\frac{\partial \Gamma}{\partial \mu}$$

$$= \left(n+d-\frac{nd}{2}\right)\Gamma - 2m^2\frac{\partial \Gamma}{\partial m^2} + \beta\frac{\partial \Gamma}{\partial g} + m^2\gamma_{\phi^2}\frac{\partial \Gamma}{\partial m^2} +$$

$$+ n\gamma\Gamma$$

$$\sigma\frac{\partial \Gamma}{\partial \sigma} = \beta\frac{\partial \Gamma}{\partial g} - m^2(\gamma_{\phi^2}+2)\frac{\partial \Gamma}{\partial m^2} - \left(n(1+\gamma)+d-\frac{nd}{2}\right)\Gamma = 0$$

$$-n - n\gamma + 2n - \cancel{d}$$

$$n - n - n\gamma$$

$$\sigma \frac{\partial \Gamma}{\partial \sigma} - \beta \frac{\partial \Gamma}{\partial g} - m^2 (\gamma_{\phi^2} - 2) \frac{\partial \Gamma}{\partial m^2} - \left(d + n\left(1 - \frac{d}{2} + \gamma\right)\right) \Gamma = 0$$

solve $\quad \sigma \dfrac{\partial g}{\partial \sigma} = -\beta$

$$\sigma \frac{\partial m^2}{\partial \sigma} = -m^2 (\gamma_{\phi^2} - 2)$$

$$\Gamma(\sigma) = \Gamma(\sigma_0)\, e^{\int_{\sigma_0}^{\sigma} \left(d + n\left(1 - \frac{d}{2} + \gamma\right)\right) \frac{d\sigma'}{\sigma'}}$$

$$\Gamma(\sigma p_i, g_\sigma, m^2_\sigma, \mu) = \Gamma(p_i, g, m^2, \mu) \times$$

$$\times\, e^{\int_1^{\sigma} \left(d + n\left(1 - \frac{d}{2}\right)\right) \frac{d\sigma'}{\sigma'} + n \int_1^{\sigma} \gamma \frac{d\sigma'}{\sigma'}}$$

$d \ln \sigma'$

$\ln \sigma'$

$$= \sigma^{d + n\left(1 - \frac{d}{2}\right)}\, e^{n \int_1^{\sigma} \gamma \frac{d\sigma'}{\sigma'}}\, \Gamma(p_i, g, m^2, \mu)$$

$\cancel{\text{if } m^2_\sigma \to 0 \quad g_\sigma \to g^* \quad \gamma \to \gamma^* = \gamma(g^*)}$

$\cancel{\Gamma(\sigma p_i, g^*, 0, \mu) = \sigma^{d + n\left(1 - \frac{d}{2}\right) + n\gamma^*}\, \Gamma(p_i, g, m^2, \mu)}$

example

$$\beta(g) = -b_1 g^3$$

$$\Gamma(\sigma p_i, g, m^2, \mu)$$

$$\sigma \frac{\partial g}{\partial \sigma} = b_1 g^3$$

$$\int \frac{\partial g}{g^3} = b_1 \int \frac{d\sigma}{\sigma}$$

$$-\frac{1}{2g^2}\Big|_{g_1}^{g} = b_1 \ln \sigma$$

$$-\frac{1}{2}\left(\frac{1}{g^2} - \frac{1}{g_1^2}\right) = b_1 \ln \sigma$$

$$\frac{1}{g^2} = \frac{1}{g_1^2} - 2b_1 \ln\sigma \Rightarrow g^2 = \frac{1}{\frac{1}{g_1^2} - 2b_1\ln\sigma} = \frac{g_1^2}{1-2b_1 g_1^2 \ln\sigma}$$

$$\frac{1}{g_1^2} = \frac{1}{g^2} + 2b_1\ln\sigma \Rightarrow g_1^2 = \frac{1}{\frac{1}{g^2} + 2b_1\ln\sigma} = \frac{g^2}{1+2bg^2\ln\sigma}$$

$$\sigma \to \infty \qquad g_1 \to 0$$

Notice

$$\sigma \frac{\partial m^2}{\partial \sigma} = -m^2 \gamma_{\phi^2} + 2m^2$$

$$\simeq 2m^2$$

$$\int_{m_1^2}^{m^2} \frac{dm^2}{m^2} = 2\int_1^{\sigma} \frac{d\sigma}{\sigma}$$

$$\ln \frac{m^2}{m_1^2} = 2\ln\sigma = \ln\sigma^2$$

$$m^2 = m_1^2 \sigma^2 \Rightarrow$$

$$\Rightarrow m_1^2 = \frac{m^2}{\sigma^2} \underset{\sigma\to\infty}{\to} 0$$

We have to solve

$$\sigma \frac{\partial g}{\partial \sigma} = -\beta(g)$$

with $g(\sigma) = g$ and find $g(1)$

$$\Gamma(\sigma p_j, g, m^2, \mu) = \sigma^{d+n(1-\frac{d}{2})} e^{n\int_1^\sigma \gamma \frac{d\sigma'}{\sigma'}}.$$

$$\cdot \Gamma(p_j, g_1, m_1^2, \mu)$$

if $m_1^2 \to 0$ $g_1 \to g_1^*$ U.V. fixed point.

$$\Gamma(\sigma p_j, g, m^2, \mu) \simeq \sigma^{d+n(1-\frac{d}{2})}$$

$$e^{n\int_1^\sigma \gamma(g(\sigma)) \frac{d\sigma'}{\sigma'}} \qquad \Gamma(p_j, g_1^*, 0, \mu)$$

$$e^{n\gamma^* \ln\sigma}$$

$$\sigma^{n\gamma^*}$$

$$\Gamma^{(4)} = \frac{i\lambda^2}{16\pi^2}\mu^{\epsilon}\left(-\frac{3}{2}\ln 4\pi - \frac{3}{2}\gamma + \frac{3i\pi}{2} - \frac{1}{2}\int_0^1 d\alpha\left[\ln\left(\alpha(1-\alpha)\frac{s\sigma^2}{\mu^2} - \frac{m^2}{\mu^2} + i\epsilon\right)\right.\right.$$

$$-\frac{1}{2}\int_0^1 d\alpha\left[\ln\left(\alpha(1-\alpha)\frac{t\sigma^2}{\mu^2} - \frac{m^2}{\mu^2}\right) + i\epsilon\right] -$$

$$-\frac{1}{2}\int_0^1 d\alpha\left[\ln\left(\alpha(1-\alpha)\frac{u\sigma^2}{\mu^2} - \frac{m^2}{\mu^2}\right) + i\epsilon\right]$$

$$\sigma\frac{\partial\Gamma^{(4)}}{\partial\sigma} = -\frac{i\lambda^2}{32\pi^2}\mu^{\epsilon}\left(\int_0^1 d\alpha\,\frac{\alpha(1-\alpha)\,\frac{2\sigma^2 s}{\mu^2}}{\alpha(1-\alpha)\frac{s\sigma^2}{\mu^2} - \frac{m^2}{\mu^2} + i\epsilon} + \right.$$

$$+\int \qquad - \qquad t$$

$$\left. + \int \qquad - \qquad u \right)$$

$$m^2\frac{\partial\Gamma}{\partial m^2} = -\frac{i\lambda^2}{32\pi^2}\mu^{\epsilon}\left(\int_0^1 d\alpha\,\frac{-m^2/\mu^2}{\alpha(1-\alpha)\frac{s\sigma^2}{\mu^2} - \frac{m^2}{\mu^2} + i\epsilon} + \right.$$

$$+ \qquad$$

$$\left. + \qquad \right)$$

$$\sigma \frac{\partial \Gamma^{(4)}}{\partial \sigma} - \left(\frac{g}{2} - 2\right) m^2 \frac{\partial \Gamma}{\partial m^2}$$

highest ord

$$\boxed{\Gamma = \frac{4ig^2}{\epsilon} n^2}$$

$$\sigma \frac{\partial \Gamma^{(n)}}{\partial \sigma} + 2m^2 \frac{\partial \Gamma}{\partial m^2}$$

$$= -\frac{ig^2}{\epsilon} 2n^2 \, 2 \int_0^1 d\alpha \; \frac{\cancel{\frac{\sigma^2 s}{\mu^2}\alpha(1-\alpha) - 2m^2/\mu^2}}{\cancel{\alpha(1-\alpha)\frac{s\sigma^2}{\mu^2} - m^2/\mu^2}}$$

$$= -4n^2 ig^2 (3) = -12n^2 ig^2$$

$$-\beta \frac{\partial \Gamma}{\partial g} = -8ign^2 \frac{3}{2} g^2 \int_0^1 - -$$

$$\cancel{= -12ig^3 n^2}$$

$$\cancel{-\beta \frac{\partial \Gamma}{\partial g} (-i\lambda)} = \qquad \Gamma = -i8n^2 g$$

cancel ✓.

$$-\beta \frac{\partial \Gamma}{\partial g} = 8in^2 \frac{3}{2} g^2 = 12in^2 g^2 \checkmark$$

satisfied

$$-(4(1+\gamma) + 4 - 8)\Gamma = -\gamma\Gamma \longrightarrow 0$$