1-loop

$$\delta_z = 0 \qquad \delta_m = -\frac{\lambda}{2} I_1 \qquad \delta_\lambda = -\frac{3}{2}\lambda^2 \frac{\partial I_1}{\partial m^2}$$

2-loops

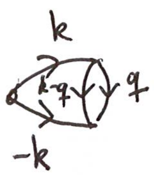
$$\delta_z = -\frac{i\lambda^2}{6}\frac{\partial H}{\partial p^2}(p^2=0) \qquad \delta_m = -i\lambda^2\left( i\, I_1 \frac{\partial I_1}{\partial m^2} - \frac{\cancel{\lambda}}{4} I_1^2 - \frac{\cancel{\lambda}}{6} H(p^2=0)\right)$$

$\delta_\lambda =$

---

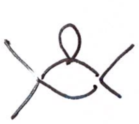
$$= 3\times \frac{(-i\lambda)^3}{4}\left(i\frac{\partial I_1}{\partial m^2}\right)^2 = \frac{3i}{4}\lambda^3\left(i\frac{\partial I_1}{\partial m^2}\right)^2$$

$$= 6\times \frac{(-i\lambda)^3}{2}\int\frac{d^dk}{(2\pi)^d}\left(\frac{i}{k^2-m^2+i\epsilon}\right)^2 \frac{i}{(q^2-m^2+i\epsilon)}\frac{i}{(k-q)^2 - m^2+i\epsilon}$$




$$= \frac{\cancel{3}i}{\cancel{2}}\lambda^3 \frac{i}{\cancel{3}}\frac{\partial}{\partial m^2}\int\frac{d^dk}{(2\pi)^d}\frac{i}{k^2-m^2+i\epsilon}\frac{i}{(q^2-m^2+i\epsilon)}\frac{i}{(k-q)^2-m^2+i\epsilon}$$

$$= \frac{i}{\cancel{2}}\lambda^3\, i\,\frac{\partial}{\partial m^2} H(p^2=0)$$



$$\frac{(-i\lambda)^2}{2} I_1 \int\frac{d^dk}{(2\pi)^d}\left(\frac{i}{k^2-m^2+i\epsilon}\right)^3 = +\frac{\lambda^2}{2} I_1 \int\frac{d^dk}{(2\pi)^d}\frac{i}{(k^2-m^2+i\epsilon)^3}$$

$$= \frac{\lambda^2}{2} I_1 \frac{1}{2}\frac{\partial^2}{(\partial m^2)^2} I_1$$

 $\rightarrow \quad I_1 = \int \frac{d^d k}{(2\pi)^d} \frac{i}{k^2-m^2+i\epsilon} =$

$$= \int_0^\infty d\alpha \int e^{i\alpha(k^2-m^2+i\epsilon)} \frac{d^d k}{(2\pi)^d}$$

$$= \int_0^\infty \frac{d\alpha}{(2\pi)^d} \left(\frac{\pi}{\alpha}\right)^{d/2} e^{-i\frac{\pi}{4}(d-2)} e^{i\alpha(-m^2+i\epsilon)}$$

$$= \frac{1}{(4\pi)^{d/2}} e^{-i\frac{\pi}{4}(d-2)} \int_0^\infty d\alpha \, \alpha^{1-d/2-1} e^{i\alpha(-m^2+i\epsilon)}$$

$$e^{-\alpha(\epsilon+im^2)}$$

$$= \frac{1}{(4\pi)^{d/2}} \cancel{e^{-i\frac{\pi}{4}(d-2)}} \Gamma(1-d/2) \left(\epsilon+im^2\right)^{d/2-1}$$

$$\cancel{e^{\frac{i\pi}{2}\frac{(d-2)}{2}}}$$

$$= \frac{\Gamma(1-d/2)}{(4\pi)^{d/2}} \left(m^2-i\epsilon\right)^{\frac{d-2}{2}}$$

$(-1+\epsilon/2)\frac{\epsilon}{2}\Gamma(-1+\frac{\epsilon}{2}) = \frac{\epsilon}{2}\Gamma(\frac{\epsilon}{2}) =$
$= \Gamma(1+\epsilon/2) =$
$= 1 - \frac{\epsilon}{2}\gamma \ldots$

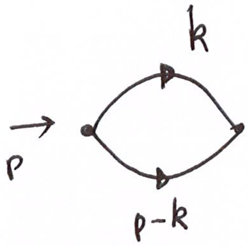
$d=4-\epsilon$

$$I_1 = \frac{\Gamma(-1+\epsilon/2)}{(4\pi)^{2-\epsilon/2}} \left(m^2-i\epsilon\right)^{1-\epsilon/2}$$

$\Gamma(-1+\epsilon/2) = -\frac{2}{\epsilon} + \ldots$

$$I_1^{div} = -\frac{2}{\epsilon}\frac{m^2}{16\pi^2} = -\frac{m^2}{8\pi^2\epsilon} + \ldots$$

2




$$B(p) = \int \frac{d^d k}{(2\pi)^d} \frac{i}{k^2 - m^2 + i\varepsilon} \frac{i}{(p-k)^2 - m^2 + i\varepsilon}$$

$$B(p) = \int_0^\infty d\alpha_1 d\alpha_2 \int \frac{d^d k}{(2\pi)^d} e^{i\alpha_1 (k^2 - m^2 + i\varepsilon) + i\alpha_2 ((p-k)^2 - m^2 + i\varepsilon)}$$

$$= \int_0^\infty d\alpha_1 \, d\alpha_2 \int \frac{d^d k}{(2\pi)^d} e^{i(\alpha_1+\alpha_2) k^2 - 2i\alpha_2 pk + i\alpha_2 p^2 + i(\alpha_1+\alpha_2)(-m^2 + i\varepsilon)}$$

$$= \int_0^\infty \frac{d\alpha_1 \, d\alpha_2}{(2\pi)^d} \left(\frac{\pi}{\alpha_1+\alpha_2}\right)^{d/2} e^{-i\frac{\pi}{4}(d-2)} e^{\cancel{+} i \frac{\alpha_2^2 p^2}{\alpha_1+\alpha_2} + i\alpha_2 p^2 + i(\alpha_1+\alpha_2)(-m^2+i\varepsilon)}$$

$$e^{+i \frac{(-\alpha_2^2 + \alpha_1\alpha_2 \cancel{+\alpha_2^2})}{\alpha_1+\alpha_2} p^2}$$

$$= \frac{e^{-i\frac{\pi}{4}(d-2)}}{(4\pi)^{d/2}} \int_0^\infty d\alpha_1 d\alpha_2 \frac{e^{i\frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2} p^2 + i(\alpha_1+\alpha_2)(-m^2+i\varepsilon)}}{(\alpha_1+\alpha_2)^{d/2}}$$

$$\int_0^\infty d\rho \; \delta(\rho - \alpha_1 - \alpha_2) \int_0^\infty d\alpha_1 d\alpha_2 \frac{e^{i\frac{\alpha_1\alpha_2}{\rho} p^2 + i\rho(-m^2+i\varepsilon)}}{\rho^{d/2}}$$

$$= \frac{e^{-i\frac{\pi}{4}(d-2)}}{(4\pi)^{d/2}} \int_0^1 d\alpha_1 d\alpha_2 \; \delta(1-\alpha_1-\alpha_2) \int_0^\infty d\rho \; \rho^{1-d/2} e^{i\rho(\alpha_1\alpha_2 p^2 - m^2 + i\varepsilon)}$$

$$= \frac{e^{-i\pi/4 (d-2)}}{(4\pi)^{d/2}} \int_0^1 d\alpha d\alpha_2 \; \delta(1-\alpha_1-\alpha_2) \, \Gamma(2-d/2) \, e^{-\frac{i\pi}{4}(4-d/0)} \, (m^2 - \alpha_1\alpha_2 p^2 - i\varepsilon)^{d/2-2}$$

$$= \frac{e^{-i\frac{\pi}{4}(d-2)+i\frac{\pi}{4}(d-4)}}{(4\pi)^{d/2}} \Gamma(2-d/2) \int_0^1 d\alpha \, (m^2-\alpha(1-\alpha)p^2-i\varepsilon)^{d/2-2}$$ 3

$$= -\frac{i}{(4\pi)^{d/2}} \Gamma(2-d/2) \int_0^1 d\alpha \, (m^2-\alpha(1-\alpha)p^2-i\varepsilon)^{d/2-2}$$

$$B(p) = -\frac{i}{(4\pi)^{d/2}} \Gamma(2-d/2) \int_0^1 d\alpha \, (m^2-\alpha(1-\alpha)p^2-i\varepsilon)^{d/2-2}$$

$\varepsilon/2\,\Gamma(\varepsilon/2) = \Gamma(1+\varepsilon/2) = 1-\gamma\frac{\varepsilon}{2}$

$d = 4-\varepsilon$

$$B = -\frac{i}{16\pi^2}(4\pi)^{\varepsilon/2}\,\Gamma(\varepsilon/2) \int_0^1 d\alpha \, (m^2-\alpha(1-\alpha)p^2-i0^+)^{-\varepsilon/2}$$

$$= -\frac{i}{16\pi^2}\frac{2}{\varepsilon}(1-\gamma\varepsilon/2)\left(1+\frac{\varepsilon}{2}\ln 4\pi\right)\left(1-\frac{\varepsilon}{2}\int_0^1 d\alpha \ln(m^2-\alpha(1-\alpha)p^2-i0^+\right)$$

$$= -\frac{i}{8\pi^2}\left(\frac{1}{\varepsilon}-\frac{\gamma}{2}+\frac{1}{2}\ln 4\pi-\frac{1}{2}\int_0^1 d\alpha \ln(m^2-\alpha(1-\alpha)p^2-i0^+)\right)$$

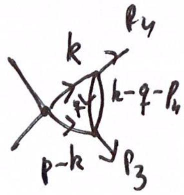
2-loops

(a)

$$= \frac{(-i\lambda)^3}{4} B^2(s)$$

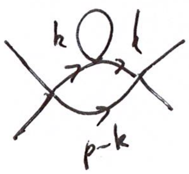
 +  

$$= \frac{(-i\lambda)^3}{4} \left( B^2(s) + B^2(t) + B^2(u) \right)$$




$$\frac{(-i\lambda)^3}{2} \int \frac{d^dk}{(2\pi)^d} \int \frac{d^dq}{(2\pi)^d} \frac{i}{(k^2-m^2+i\varepsilon)} \frac{i}{(p-k)^2-m^2+i\varepsilon} \frac{i}{q^2-m^2+i\varepsilon} \frac{i}{(k-q-p_4)^2-m^2+i\varepsilon}$$

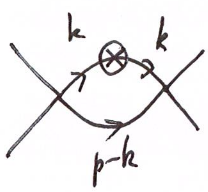
$$\frac{(-i\lambda)^3}{2} \int \frac{d^dk}{(2\pi)^d} \frac{i}{(k^2-m^2+i\varepsilon)} \frac{i}{(p-k)^2-m^2+i\varepsilon} B(\cancel{k}+p_4)$$




$$\frac{(-i\lambda)^3}{2} \int \frac{d^dk}{(2\pi)^d} \left( \frac{i}{k^2-m^2+i\varepsilon} \right)^2 \frac{i}{(p-k)^2-m^2+i\varepsilon} \quad I_1$$

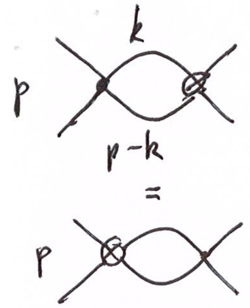
$$\frac{(-i\lambda)^3}{2} \frac{I_1\, i}{2} \frac{\partial}{\partial m^2} \int \frac{d^dk}{(2\pi)^d} \frac{i}{k^2-m^2+i\varepsilon} \frac{i}{(p-k)^2-m^2+i\varepsilon}$$

$$\frac{(-i\lambda)^3}{4}\, i I_1 \frac{\partial}{\partial m^2} B(p^2)$$




$$= (-i\lambda)^2 \int \frac{d^dk}{(2\pi)^d} \left( \frac{i}{(k^2-m^2+i\varepsilon)} \right)^2 \frac{i}{(p-k)^2-m^2+i\varepsilon}\, i\left( \delta_Z^{(1)} k^2 - \delta_m^{(1)} \right)$$

$$= (-i) \frac{(-i\lambda)^2}{2} \delta_m^{(1)}\, i \frac{\partial B(p^2)}{\partial m^2}$$




$$\cancel{2\times}\,\frac{(-i\lambda)(-i\delta_\lambda)}{\cancel{2}}\;B(p^2)$$

2-loops

$$\frac{(-i\lambda)^3}{4}\left(B^2(s)+B^2(t)+B^2(u)\right)+\frac{(-i\lambda)^3}{4}\,iI_1\frac{\partial}{\partial m^2}\left(B(s)+B(t)+B(u)\right)$$

$$+\frac{1}{2}(-i\lambda)^2\delta_m^{(1)}\frac{\partial}{\partial m^2}\left(B(s)+B(t)+B(u)\right)-\lambda\delta_\lambda\left(B(s)+B(t)+B(u)\right)$$

$$+\frac{1}{2}(-i\lambda)^3\int\frac{d^dk}{(2\pi)^d}\frac{i}{k^2-m^2+i\epsilon}\frac{i}{(\underbrace{p}_{p_1+p_2}-k)^2-m^2+i\epsilon}B((k-p_4)^2)$$

+6 permutations.

---

$$\frac{(-i\lambda)^2}{2}\left(\frac{(-i\lambda)\,iI_1}{2}+\underbrace{\delta_m^{(1)}}_{-\frac{\lambda}{2}I_1^{div}}\right)\frac{\partial}{\partial m^2}\left(B(s)+B(t)+B(u)\right)\to\text{finite}$$

$$\frac{\lambda I_1}{2}$$

---

$$3\frac{(-i\lambda)^3}{4}\left(B^{div}\right)^2+\frac{(-i\lambda)^3}{4}2B^{div}B^{finite}_{s+t+u}+\frac{(-i\lambda)^3}{4}\left(B^f_{s+t+u}\right)^2$$

$$-\lambda\delta_\lambda 3B^{div}-\lambda\delta_\lambda\left(B^f_s+B^f_t+B^f_u\right)$$

$(-i)^3 = (-i)(-i)(-i) = i$ (c)

$$3\frac{(-i\lambda)^3}{4} B_{div}^2 \; -3\lambda\delta_\lambda B_{div}.$$

$$i\frac{\lambda^3}{2} B_{div}\left(B_s^f + B_t^f + B_u^f\right) - \lambda\delta_\lambda\left(B_s^f + B_t^f + B_u^f\right)$$

$$\lambda\left(\frac{i\lambda^2}{2} B_{div} - \delta_\lambda\right)\left(B_s^f + B_t^f + B_u^f\right)$$

$$\delta_\lambda = \frac{3\lambda^2}{16\pi^2\varepsilon} \qquad B^{div} = -\frac{i}{8\pi^2\varepsilon}$$

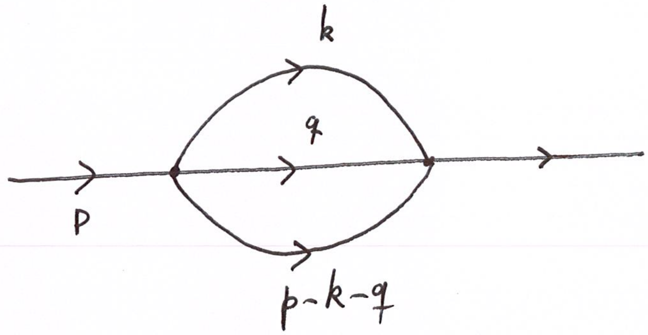
$$\lambda\left(\frac{\lambda^2}{16\pi^2\varepsilon} - \frac{3\lambda^2}{16\pi^2\varepsilon}\right)\left(B_s^f + B_t^f + B_u^f\right)$$

$$\lambda\left(-\frac{\lambda^2}{8\pi^2\varepsilon}\right)\left(B_s^f + B_t^f + B_u^f\right)$$

$$\left.\begin{array}{ll} -\dfrac{\lambda^3}{8\pi^2\varepsilon}\left(B_s^f + B_t^f + B_u^f\right) & \text{I} \\ & \\ +\dfrac{\lambda^3}{8\pi^2\varepsilon}\left(B_s^f + B_t^f + B_u^f\right) & \text{II} \end{array}\right\} 0.$$

↑ from 

$1 - \alpha + \alpha - \alpha\beta$




$$\frac{(-i\lambda)^2}{6}\underbrace{\int\frac{d^dk}{(2\pi)^d}\int\frac{d^dq}{(2\pi)^d}\frac{i}{k^2-m^2+i\varepsilon}\,\frac{i}{q^2-m^2+i\varepsilon}\,\frac{i}{(p-k-q)^2-m^2+i\varepsilon}}_{H}$$

$$H=\int\frac{d^dk}{(2\pi)^d}\int\frac{d^dq}{(2\pi)^2}\int_0^\infty d\alpha_1 d\alpha_2 d\alpha_3\; e^{i\alpha_1(k^2-m^2+i\varepsilon)+i\alpha_2(q^2-m^2+i\varepsilon)}\;\times$$

$$\times\; e^{i\alpha_3((p-k-q)^2-m^2+i\varepsilon)}$$

$$\int d^dk\; e^{i\alpha k^2+2ipk}=\int_{-\infty}^{+\infty}dk_0\; e^{-(-i\alpha k_0^2)+2ip_0k_0}\int_{-\infty}^{+\infty}d^{d-1}\vec{k}\; e^{-i\alpha\vec{k}^2-2i\vec{p}\vec{k}}$$

$$=\sqrt{\frac{\pi}{-i\alpha}}\left(\frac{\pi}{i\alpha}\right)^{\frac{d-1}{2}}e^{+\frac{4p_0^2}{4i\alpha}-\frac{4\vec{p}^2}{4i\alpha}}=\left(\frac{\pi}{\alpha}\right)^{d/2}e^{-i\frac{\pi}{4}(d-2)}\,e^{-ip^2/\alpha}$$

$$e^{i\frac{\pi}{4}-i\frac{\pi}{4}(d-1)}$$

$$H=\int\frac{d^dk}{(2\pi)^d}\int\frac{d^dq}{(2\pi)^d}\int_0^\infty d\alpha_1 d\alpha_2 d\alpha_3\; e^{i\Sigma\alpha_j(-m^2+i\varepsilon)}\;\times$$

$$\times\; e^{i\alpha_1k^2+i\alpha_2q^2+i\alpha_3k^2+i\alpha_3q^2+i\alpha_3p^2-2i\alpha_3pk-2i\alpha_3pq}$$

$$\times\; e^{2i\alpha_3kq}$$

$$= \frac{1}{(4\pi^2)^d} \int_0^\infty d\alpha_j \; e^{i\sum \alpha_j(-m^2+i\epsilon)} \int d^dk \left(\frac{\pi}{\alpha_2+\alpha_3}\right)^{d/2} \times$$

$$\times\, e^{-\frac{i\pi}{4}(d-2)} \; e^{i(\alpha_1+\alpha_3)k^2 + i\alpha_3 p^2 - 2i\alpha_3 pk - \frac{i\alpha_3^2}{\alpha_2+\alpha_3}(k-p)^2}$$

$$i(\alpha_1+\alpha_3)k^2 + i\alpha_3 p^2 - 2i\alpha_3 pk - \frac{i\alpha_3^2 k^2}{\alpha_2+\alpha_3} - \frac{i\alpha_3^2 p^2}{\alpha_2+\alpha_3} + \frac{2i\alpha_3^2 kp}{\alpha_2+\alpha_3}$$

$$\frac{ik^2}{\alpha_2+\alpha_3}\left(\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3+\cancel{\alpha_3^2}-\cancel{\alpha_3^2}\right) + \frac{ip^2}{\alpha_2+\alpha_3}\left(\alpha_2\alpha_3+\cancel{\alpha_3^2}-\cancel{\alpha_3^2}\right) +$$

$$+ \frac{2ikp}{\alpha_2+\alpha_3}\left(\cancel{\alpha_3^2}-\alpha_2\alpha_3-\cancel{\alpha_3^2}\right)$$

$$= \frac{1}{(4\pi^2)^d}\,\pi^{d/2} \int_0^\infty d\alpha_j \; e^{i\sum\alpha_j(-m^2+i\epsilon)} \; \cancel{\frac{1}{(\alpha_2+\alpha_3)^{d/2}}} \; e^{-\frac{i\pi}{4}(d-2)}$$

$$\left(\frac{\pi\,\cancel{(\alpha_2+\alpha_3)}}{\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3}\right)^{d/2} e^{-i\frac{\pi}{4}(d-2)} \; e^{-i\frac{\cancel{(\alpha_2+\alpha_3)}}{(\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3)}\frac{\alpha_2^2\alpha_3^2\,p^2}{(\alpha_2+\alpha_3)^{\cancel{2}}}} \times$$

$$\times\, e^{ip^2\frac{\alpha_2\alpha_3}{\alpha_2+\alpha_3}}$$

$$= \frac{1}{(4\pi)^d} e^{-\frac{i\pi}{2}(d-2)} \int_0^\infty d\alpha_j \, e^{i\Sigma\alpha_j(-m^2+i\varepsilon)} \frac{1}{(\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3)^{d/2}}$$

$$e^{\frac{-ip^2}{(\alpha_2+\alpha_3)(\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3)}\left(\cancel{\alpha_2^2\alpha_3^2} - \alpha_1\alpha_2^2\alpha_3 - \alpha_1\alpha_3^2\alpha_2 - \cancel{\alpha_2^2\alpha_3^2}\right)}$$

$$- \alpha_1\alpha_2\alpha_3(\alpha_2+\alpha_3)$$

$$H = \frac{e^{-\frac{i\pi}{2}(d-2)}}{(4\pi)^d} \int_0^\infty d\alpha_j \frac{e^{i\Sigma\alpha_j(-m^2+i\varepsilon)}}{(\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3)^{d/2}} e^{\frac{i\,\alpha_1\alpha_2\alpha_3\,p^2}{(\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3)}}$$

$$H(p^2=0) = \frac{e^{-\frac{i\pi}{2}(d-2)}}{(4\pi)^d} \int_0^\infty d\alpha_j \frac{e^{i\Sigma\alpha_j(-m^2+i\varepsilon)}}{(\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3)^{d/2}}$$

$$\frac{\partial H}{\partial p^2}(p^2=0) = i\frac{e^{-i\frac{\pi}{2}(d-2)}}{(4\pi)^d} \int_0^\infty d\alpha_j \frac{\alpha_1\alpha_2\alpha_3 \, e^{i\Sigma\alpha_j(-m^2+i\varepsilon)}}{(\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3)^{d/2+1}}$$

$$H = \frac{e^{-i\frac{\pi}{2}(d-2)}}{(4\pi)^d} \int_0^\infty d\rho \int_0^\infty d\alpha_j \; \delta(\rho - \Sigma\alpha_j) \frac{e^{i\rho(-m^2+i\varepsilon)}}{(\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3)^{d/2}} \, e^{\frac{i\,\alpha_1\alpha_2\alpha_3 p^2}{(\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3)}}$$

$\alpha_j \to \rho\,\alpha_j$

$$= \frac{e^{-i\frac{\pi}{2}(d-2)}}{(4\pi)^d} \int_0^\infty d\rho \; \rho^{3-1-d} \int_0^1 \frac{d\alpha_j \; \delta(1-\Sigma\alpha_j)}{(\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3)^{d/2}} \, e^{i\rho(-m^2+i\varepsilon)} \times$$

$$\times \, e^{i\rho \frac{\alpha_1\alpha_2\alpha_3}{\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3} p^2}$$

$$= \frac{e^{-i\frac{\pi}{2}(d-2)}}{(4\pi)^d} \, \Gamma(3-d) \int_0^1 \frac{d\alpha_j \; \delta(1-\Sigma\alpha_j)}{(\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3)^{d/2}} \left(m^2 - \frac{\alpha_1\alpha_2\alpha_3}{\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3} p^2 - i\varepsilon\right)^{d-3}$$

$d-2-1$

$$\cdot \, e^{i\frac{\pi}{2}(d-3)}$$

$$= \frac{e^{-i\frac{\pi}{2}}}{(4\pi)^d} \, \Gamma(3-d) \int_0^1 \frac{d\alpha_j \; \delta(1-\Sigma\alpha_j)}{(\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3)^{d/2}} \left(m^2 - \frac{\alpha_1\alpha_2\alpha_3}{\alpha_1\alpha_2+\alpha_3\alpha_1+\alpha_2\alpha_3} p^2 - i\varepsilon\right)^{d-3}$$

$$H(p^2=0) = \frac{e^{-\frac{i\pi}{2}}}{(4\pi)^d}\,\Gamma(3-d)\,(m^2-i\varepsilon)^{d-3}\int_0^1 d\alpha_j\,\frac{\delta(1-\Sigma\alpha_j)}{(\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3)^{d/2}}$$

$$\frac{\partial H}{\partial p^2}(p^2=0) = +\frac{e^{-\frac{i\pi}{2}}}{(4\pi)^d}\,\Gamma(4-d)\,\underbrace{\int_0^1 d\alpha_j\,\frac{\delta(1-\Sigma\alpha_j)\,\alpha_1\alpha_2\alpha_3}{(\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3)^{d/2+1}}}\,(m^2)^{d-4}$$

$$\int_0^1 d\alpha_j\,\frac{\delta(1-\Sigma\alpha_j)\,\alpha_1\alpha_2\alpha_3}{(\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3)^{d/2+1}} =$$

$$\beta = (1-\alpha)\gamma$$

$$= \int_0^1 d\alpha\int_0^{1-\alpha} d\beta\,\frac{\alpha\beta(1-\alpha-\beta)}{(\alpha\beta+\alpha(1-\alpha-\beta)+\beta(1-\alpha-\beta))^{d/2+1}}$$

$$= \int_0^1 d\alpha\int_0^1 d\gamma\,\frac{\alpha(1-\alpha)^2\gamma\,(1-\alpha)(1-\gamma)}{(\alpha(1-\alpha)\gamma+\alpha(1-\alpha)(1-\gamma)+(1-\alpha)\gamma(1-\alpha)(1-\gamma))^{d/2+1}}$$

$d=4$

$$= \int_0^1 d\alpha\int_0^1 d\gamma\,\frac{\alpha\cancel{(1-\alpha)^3}\gamma(1-\gamma)}{\cancel{(1-\alpha)^3}(\alpha\gamma+\alpha(1-\gamma)+\gamma(1-\alpha)(1-\gamma))^3}$$

$$= \int_0^1 d\gamma \int_0^1 d\alpha \frac{\alpha \gamma (1-\gamma)}{\left(\alpha(1-\gamma(1-\gamma)) + \gamma(1-\gamma)\right)^3}$$

$$\int_0^1 d\alpha \frac{\alpha}{(\alpha a+b)^3} = \frac{1}{a} \int_0^1 d\alpha \frac{\alpha a + b - b}{(\alpha a+b)^3} =$$

$$= \frac{1}{a} \int_0^1 d\alpha \frac{1}{(\alpha a+b)^2} - \frac{b}{a} \int_0^1 d\alpha \frac{1}{(\alpha a+b)^3} =$$

$$= \frac{1}{a^2} \left. \frac{-1}{\alpha a+b} \right|_0^1 - \frac{b}{a^2(-2)} \left. \frac{1}{(\alpha a+b)^2} \right|_0^1$$

$$= -\frac{1}{a^2}\left(\frac{1}{a+b} - \frac{1}{b}\right) + \frac{b}{2a^2}\left(\frac{1}{(a+b)^2} - \frac{1}{b^2}\right)$$

$$= -\frac{1}{a^2} \frac{1}{a+b} + \frac{1}{a^2 b} + \frac{b}{2a^2(a+b)^2} - \frac{1}{2a^2 b}$$

$$= \frac{1}{2a^2 b} + \frac{1}{2a^2(a+b)^2}(b - 2a - 2b)$$

$$= \frac{1}{2a^2 b} - \frac{2a+b}{2a^2(a+b)^2} = \frac{1}{2a^2}\left(\frac{1}{b} - \frac{2a+b}{(a+b)^2}\right)$$

$$= \frac{1}{2a^2} \frac{a^2 \cancel{+2ab} \cancel{+b^2} \cancel{-2ab} \cancel{-b^2}}{b(a+b)^2} = \frac{1}{2b(a+b)^2}$$

$$= \int_0^1 d\gamma \, \frac{\cancel{\gamma(1-\gamma)}}{2\cancel{\gamma(1-\gamma)}\,(1-\cancel{\gamma(1-\gamma)}+\cancel{\gamma(1-\gamma)})^2} = \frac{1}{2}$$

$d=4-\varepsilon$ div. piece.

$$\frac{\partial H}{\partial p^2}(p^2=0) \overset{6}{=} \frac{e^{-\frac{i\pi}{2}}}{(4\pi)^4}\,\Gamma(\varepsilon)\,\frac{1}{2} = \frac{-i}{2^9\pi^4}\,\frac{1}{\varepsilon}$$

$$H(p^2=0) = \frac{e^{-\frac{i\pi}{2}}}{(4\pi)^d}\,\Gamma(3-d)\,(m^2-i\varepsilon)^{d-3}\int_0^1 d\alpha\int_0^{1-\alpha} d\beta\,\frac{1}{(\alpha\beta+\alpha(1-\alpha-\beta)+\beta(1-\alpha-\beta))^{d/2}}$$

$$= \frac{-i}{(4\pi)^d}\,\Gamma(3-d)\,(m^2-i\varepsilon)^{d-3}\int_0^1 d\alpha\int_0^1 d\gamma\,\frac{(1-\alpha)}{(\alpha(1-\alpha)\gamma+\alpha(1-\alpha)(1-\gamma)+\gamma(1-\alpha)^2(1-\gamma))^{d/2}}$$

$$= \frac{-i}{(4\pi)^d}\,\Gamma(3-d)\,(m^2-i\varepsilon)^{d-3}\int_0^1 d\alpha\int_0^1 d\gamma\,\frac{(1-\alpha)^{1-d/2}}{(\cancel{\alpha\gamma}+\alpha-\cancel{\alpha\gamma}+\gamma(1-\alpha)(1-\gamma))^{d/2}}$$

$$= -\frac{i}{(4\pi)^d}\,\Gamma(3-d)\,(m^2-i\varepsilon)^{d-3}\underbrace{\int_0^1 d\alpha\int_0^1 d\gamma\,\frac{(1-\alpha)^{1-d/2}}{(\alpha+(1-\alpha)\gamma(1-\gamma))^{d/2}}}_{\frac{6}{\varepsilon}+3+\mathcal{O}(\varepsilon)}$$

$d = 4 - \varepsilon$

$$H(p^2=0) = -\frac{i}{(4\pi)^{4-\varepsilon}} \, \Gamma(-1+\varepsilon) \, (m^2 - i\varepsilon)^{1-\varepsilon} \left( \frac{6}{\varepsilon} + 3 + \mathcal{O}(\varepsilon) \right)$$

$$\varepsilon \, \Gamma(-1+\varepsilon)(-1+\varepsilon) = \varepsilon \Gamma(\varepsilon) = \Gamma(1+\varepsilon) = 1 - \gamma\varepsilon$$

$$\Gamma(-1+\varepsilon) = -\frac{1}{\varepsilon(1-\varepsilon)} (1 - \gamma\varepsilon) = -\frac{1}{\varepsilon} (1 - \gamma\varepsilon + \varepsilon)$$

$$H(p^2=0) = \cancel{-} \frac{i m^2}{(4\pi)^4} \left( 1 + \varepsilon \ln 4\pi - \gamma\varepsilon + \varepsilon - \varepsilon \ln(m^2 - i\varepsilon) \right) \left( +\frac{1}{\varepsilon} \right) \left( \frac{6}{\varepsilon} + 3 \right)$$

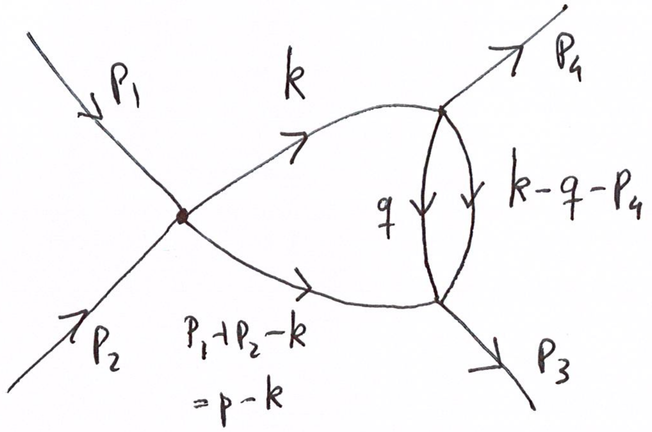
$$\frac{6}{\varepsilon}\left(1 + \frac{\varepsilon}{2}\right)$$

$$= \frac{i m^2}{(4\pi)^4} \frac{6}{\varepsilon^2} \left( 1 + \frac{\varepsilon}{2} + \varepsilon \ln 4\pi - \gamma\varepsilon + \varepsilon - \varepsilon \ln(m^2 - i\varepsilon) \right)$$

$$\underline{H_{div}}(p^2=0) = \frac{6 i m^2}{(4\pi)^4} \frac{1}{\varepsilon^2} \left( 1 + \frac{3\varepsilon}{2} + \varepsilon \ln 4\pi - \gamma\varepsilon - \varepsilon \ln(m^2 - i\varepsilon) \right)$$

$$= \frac{3 i m^2}{2^7 \pi^4 \varepsilon^2} + \frac{3 i m^2}{2^7 \pi^4 \varepsilon} \cancel{\varepsilon} \left( +\frac{1}{2} + I_1^f \frac{16\pi^2}{m^2} \right)$$

$$= \frac{3 i m^2}{2^7 \pi^4 \varepsilon^2} + \frac{3 i m^2}{2^8 \pi^4 \varepsilon} + \frac{3 i}{8 \pi^2 \varepsilon} I_1^f$$




$$\frac{(-i\lambda)^3}{2}\int\frac{d^dk}{(2\pi)^d}\,\frac{i}{k^2-m^2+i\varepsilon}\,\frac{i}{(p-k)^2-m^2+i\varepsilon}\int\frac{d^dq}{(2\pi)^d}\,\frac{i}{q^2-m^2+i\varepsilon}\,\frac{i}{(k-q-p_4)^2-m^2+i\varepsilon}$$

$$\frac{(-i\lambda)^3}{2}\int d\alpha_1 d\alpha_2\int\frac{d^dk}{(2\pi)^d}\,e^{i\alpha_1(k^2-m^2+i\varepsilon)+i\alpha_2((p-k)^2-m^2+i\varepsilon)}\;\times$$

$$\times\;\frac{e^{-i\frac{\pi}{4}(d-2)}}{(4\pi)^{d/2}}\int\frac{d\alpha_3\,d\alpha_4}{(\alpha_3+\alpha_4)^{d/2}}\,e^{i\frac{\alpha_3\alpha_4}{\alpha_3+\alpha_4}(k-p_4)^2+i(\alpha_3+\alpha_4)(-m^2+i\varepsilon)}$$

$$i\left(\alpha_1+\alpha_2+\frac{\alpha_3\alpha_4}{\alpha_3+\alpha_4}\right)k^2-2i\alpha_2\,pk+i\alpha_2 p^2+\frac{i\alpha_3\alpha_4}{\alpha_3+\alpha_4}p_4^2-\frac{2i\alpha_3\alpha_4}{\alpha_3+\alpha_4}k\cdot p_4$$

---

$$\frac{(-i\lambda)^3}{2}\,\frac{e^{-i\frac{\pi}{2}(d-2)}}{(4\pi)^d}\int d\alpha_1\ldots d\alpha_4\;\frac{e^{i\sum\alpha_j(-m^2+i\varepsilon)+A}}{\cancel{(\alpha_3+\alpha_4)^{d/2}}}\;\frac{\cancel{(\alpha_3+\alpha_4)^{d/2}}}{\left((\alpha_1+\alpha_2)(\alpha_3+\alpha_4)+\alpha_3\alpha_4\right)^{d/2}}$$

$$A=i\alpha_2 p^2+\frac{i\alpha_3\alpha_4}{\alpha_3+\alpha_4}p_4^2-\frac{i(\alpha_3+\alpha_4)}{(\alpha_1+\alpha_2)(\alpha_3+\alpha_4)+\alpha_3\alpha_4}\left(\alpha_2 p+\frac{\alpha_3\alpha_4}{\alpha_3+\alpha_4}p_4\right)^2$$

$$= i p^2 \left( \frac{\alpha_1 \alpha_2 (\alpha_3 + \alpha_4) + \alpha_3 \alpha_4 \alpha_2}{(\alpha_1+\alpha_2)(\alpha_3+\alpha_4) + \alpha_3 \alpha_4} \right)$$

$$+ i p_4^2 \, \frac{\alpha_3 \alpha_4 \cancel{(\alpha_3+\alpha_4)} \left( (\alpha_1+\alpha_2) \cancel{(\alpha_3+\alpha_4)} + \cancel{\alpha_3 \alpha_4} \right) - \cancel{(\alpha_3+\alpha_4)} \cancel{\alpha_3^2 \alpha_4^2}}{\left( (\alpha_1+\alpha_2)(\alpha_3+\alpha_4) + \alpha_3 \alpha_4 \right) \cancel{(\alpha_3+\alpha_4)^2}}$$

$$- \frac{2i \cancel{(\alpha_3+\alpha_4)} \; \alpha_2 \alpha_3 \alpha_4}{(\alpha_1+\alpha_2)(\alpha_3+\alpha_4) + \alpha_3 \alpha_4 \; \cancel{(\alpha_3+\alpha_4)}} \, p \cdot p_4$$

$$= \frac{i}{(\alpha_1+\alpha_2)(\alpha_3+\alpha_4) + \alpha_3 \alpha_4} \Big[ p^2 \left( \alpha_1 \alpha_2 (\alpha_3+\alpha_4) + \alpha_2 \alpha_3 \alpha_4 \right) + p_4^2 \left( \alpha_1 \alpha_3 \alpha_4 + \alpha_2 \alpha_3 \alpha_4 \right)$$
$$- 2 \alpha_2 \alpha_3 \alpha_4 \, p \cdot p_4 \Big]$$

$$A = \frac{i}{(\alpha_1+\alpha_2)(\alpha_3+\alpha_4) + \alpha_3 \alpha_4} \left[ p^2 \alpha_1 \alpha_2 (\alpha_3+\alpha_4) + p_4^2 \alpha_1 \alpha_3 \alpha_4 + p_3^2 \alpha_2 \alpha_3 \alpha_4 \right]$$

---

$$\frac{(-i\lambda)^3}{2} \, \frac{e^{-i\frac{\pi}{2}(d-2)}}{(4\pi)^d} \int d\alpha_1 \ldots d\alpha_4 \, \frac{e^{i \sum \alpha_j (-m^2 + i\varepsilon) + A}}{\left( (\alpha_1+\alpha_2)(\alpha_3+\alpha_4) + \alpha_3 \alpha_4 \right)^{d/2}}$$

---

$$\frac{(-i\lambda)^3}{2} \, \frac{e^{-i\frac{\pi}{2}(d-2)}}{(4\pi)^d} \int \frac{d\alpha_1 \ldots d\alpha_4 \; \delta(1 - \sum \alpha_j)}{\left( (\alpha_1+\alpha_2)(\alpha_3+\alpha_4) + \alpha_3 \alpha_4 \right)^{d/2}} \int_0^\infty d\rho \; \rho^{4-1-d} \; \times$$

$$\times \, e^{i\rho \overbrace{\left( \sum \alpha_j \right)}^{=1} (-m^2 + i\varepsilon) + \frac{i\rho}{(\alpha_1+\alpha_2)(\alpha_3+\alpha_4) + \alpha_3 \alpha_4} \left[ p^2 \alpha_1 \alpha_2 (\alpha_3+\alpha_4) + p_4^2 \alpha_1 \alpha_3 \alpha_4 + p_3^2 \alpha_2 \alpha_3 \alpha_4 \right]}$$

(3)

$$\frac{(-i\lambda)^3}{2}\,\frac{e^{-i\frac{\pi}{2}(d-2)}}{(4\pi)^d}\,\Gamma(4-d)\int\frac{d\alpha_1\ldots d\alpha_4\;\delta(1-\Sigma\alpha_j)}{\left[(\alpha_1+\alpha_2)(\alpha_3+\alpha_4)+\alpha_3\alpha_4\right]^{d/2}}\;\times$$

$$\times\; e^{-i\frac{\pi}{2}(4-d)}\left(m^2-i\varepsilon-\frac{p^2\alpha_1\alpha_2(\alpha_3+\alpha_4)+p_4^2\alpha_1\alpha_3\alpha_4+p_3^2\alpha_2\alpha_3\alpha_4}{(\alpha_1+\alpha_2)(\alpha_3+\alpha_4)+\alpha_3\alpha_4}\right)^{d-4}$$

---

$$\int_0^1 d\alpha_3\int_0^{1-\alpha_3}d\alpha_4\int_0^{1-\alpha_3-\alpha_4}d\alpha_2\;\rightarrow\;\int_0^1 d\alpha_3\int_0^{1-\alpha_3}d\alpha_4\int_0^1 d\gamma\,(1-\alpha_3-\alpha_4)$$

$$\int_0^1 d\alpha\int_0^1 d\beta\int_0^1 d\gamma\;(1-\alpha)^2(1-\beta)$$

$\alpha_2=\gamma(1-\alpha_3-\alpha_4)$

$\alpha_4=(1-\alpha_3)\beta$

$\alpha_3=\alpha$

$\alpha_1=1-\alpha_2-\alpha_3-\alpha_4=(1-\alpha_3-\alpha_4)(1-\gamma)$

$\qquad=(1-\alpha)(1-\beta)(1-\gamma)$

$1-\alpha_3-\alpha_4=(1-\alpha)(1-\beta)$

$\alpha_2=\gamma(1-\alpha)(1-\beta)$

$\alpha_3+\alpha_4=1-(1-\alpha)(1-\beta)$

---

$$-\frac{(-i\lambda)^3}{2}\,\frac{\Gamma(4-d)}{(4\pi)^d}\int_0^1 d\alpha\,d\beta\,d\gamma\;\frac{(1-\alpha)^2(1-\beta)}{\left[(1-\alpha)(1-\beta)\left(1-(1-\alpha)(1-\beta)\right)+\alpha(1-\alpha)\beta\right]^{d/2}}$$

$1-(1-\alpha)(1-\beta)$

$$\left(m^2-i\varepsilon-\frac{p^2\gamma(1-\gamma)(1-\alpha)^2(1-\beta)^2+p_4^2(1-\alpha)^2(1-\beta)(1-\gamma)\alpha\beta+p_3^2\gamma(1-\alpha)^2(1-\beta)\alpha\beta}{(1-\alpha)(1-\beta)\left(1-(1-\alpha)(1-\beta)\right)+\alpha(1-\alpha)\beta}\right)^{d-4}$$

$\alpha \to 1-\alpha \qquad \beta \to 1-\beta$

$$-\frac{(-i\lambda)^3}{2}\frac{\Gamma(4-d)}{(4\pi)^d}\int_0^1 d\alpha\, d\beta\, d\gamma\, \frac{\alpha^2\beta}{\left[\alpha\beta(1-\alpha\beta)+\alpha(1-\alpha)(1-\beta)\right]^{d/2}}$$

$$\left(m^2 - i\varepsilon - \frac{p^2\gamma(1-\gamma)\alpha^{\cancel{2}}\beta^{\cancel{2}\,(1-\alpha\beta)} + p_4^2\alpha^{\cancel{2}}(1-\alpha)\beta(1-\beta)\gamma(1-\gamma) + p_3^2\gamma\alpha^{\cancel{2}}\beta(1-\beta)(1-\alpha)}{\cancel{\alpha}\beta(1-\alpha\beta)+\cancel{\alpha}(1-\alpha)(1-\beta)}\right)^{4-d}$$

---

$$-\frac{(-i\lambda)^3}{2}\frac{\Gamma(4-d)}{(4\pi)^d}\int_0^1 d\alpha\, d\beta\, d\gamma\, \frac{\alpha^{2-d/2}\,\beta}{\left[\beta(1-\alpha\beta)+(1-\alpha)(1-\beta)\right]^{d/2}}$$

$$\left(m^2 - i\varepsilon - \alpha\frac{p^2\beta^2\gamma(1-\gamma)^{1-\alpha\beta} + \beta(1-\beta)(1-\alpha)\left((1-\gamma)p_4^2+\gamma p_3^2\right)}{\underbrace{\beta(1-\alpha\beta)+(1-\alpha)(1-\beta)}_{\cancel{\beta}-\alpha\beta^2+(1-\alpha)\cancel{-\beta}+\alpha\beta}}\right)^{4-d}$$

$$(1-\alpha)+\alpha\beta(1-\beta)$$

---

$\alpha \to 0$ <u>ok</u>

$\alpha \to 1$ problem in $\beta$ integral.

$$\int_0^1 d\alpha\, F(\alpha,\beta) = \int_0^1 d\alpha\,\left(F(\alpha,\beta)-F(1,\beta)\right) + \underbrace{F(1,\beta)}_{\uparrow}$$

$$\int d\beta\, d\gamma\, \frac{\beta}{\beta^{d/2}(1-\beta)^{d/2}}\left(m^2 - i\varepsilon - \frac{p^2\beta^{\cancel{2}}\gamma(1-\gamma)\cancel{(1-\beta)}}{\beta\cancel{(1-\beta)}}\right)^{4-d}$$

$$-\frac{(-i\lambda)^3}{2}\frac{\Gamma(4-d)}{(4\pi)^d}\int_0^1 d\alpha\, d\beta\, d\gamma \frac{\alpha^{2-d/2}\beta}{\left[(1-\alpha)+\alpha\beta(1-\beta)\right]^{d/2}}$$

$$\left(m^2-i\varepsilon-\alpha\beta\frac{p^2\beta(1-\alpha\beta)\gamma(1-\gamma)+\cancel{\beta}(1-\beta)(1-\alpha)\left(\gamma p_3^2+(1-\gamma)p_4^2\right)}{(1-\alpha)+\alpha\beta(1-\beta)}\right)^{4-d}$$

---

$\beta\to 1$ $\qquad F(\alpha,\beta)-F(\alpha,1)$

$$-\frac{(-i\lambda)^3}{2}\frac{\Gamma(4-d)}{(4\pi)^d}\int_0^1 d\alpha\int_0^1 d\gamma\;\alpha^{2-d/2}(1-\alpha)^{-d/2}\times$$

$$\times\left(m^2-i\varepsilon-\frac{\alpha p^2\cancel{\beta(1-\alpha)}\gamma(1-\gamma)}{\cancel{1-\alpha}}\right)^{4-d}$$

$\alpha\to 1-\alpha$

$$-\frac{(-i\lambda)^3}{2}\frac{\Gamma(4-d)}{(4\pi)^d}\int_0^1 d\gamma\int_0^1 d\alpha\;(1-\alpha)^{2-d/2}\alpha^{-d/2}\times$$

$$\times\left(m^2-i\varepsilon-(1-\alpha)p^2\gamma(1-\gamma)\right)^{4-d}$$

Alternative.



$$\frac{(-i\lambda)^3}{2}\frac{e^{-i\frac{\pi}{2}(d-2)}}{(4\pi)^d}\int \frac{d\alpha_1 \ldots d\alpha_4 \; e^{i\sum\alpha_j(-m^2+i\varepsilon)+i\frac{p^2(\alpha_1\alpha_2)(\alpha_3+\alpha_4)+p_4^2\alpha_1\alpha_3\alpha_4+p_3^2\alpha_2\alpha_3\alpha_4}{(\alpha_1+\alpha_2)(\alpha_3+\alpha_4)+\alpha_3\alpha_4}}}{((\alpha_1+\alpha_2)(\alpha_3+\alpha_4)+\alpha_3\alpha_4)^{d/2}}$$

$$\int_0^\infty d\rho_1 \int_0^\infty d\rho_2 \quad \delta(\rho_1-\alpha_1-\alpha_2)\,\delta(\rho_2-\alpha_3-\alpha_4)$$

$$\alpha_1 \to \rho_1\alpha_1 \qquad \alpha_3 \to \rho_2\alpha_3$$
$$\alpha_2 \to \rho_1\alpha_2 \qquad \alpha_4 \to \rho_2\alpha_4$$

---

$$\frac{(-i\lambda)^3}{2}\frac{e^{-i\frac{\pi}{2}(d-2)}}{(4\pi)^d}\int_0^\infty d\rho_1\int_0^\infty d\rho_2\int d\alpha_1\ldots d\alpha_4 \; \rho_1^{2-1}\rho_2^{2-1} \times \frac{\delta(1-\alpha_1-\alpha_2)\,\delta(1-\alpha_3-\alpha_4)}{(\rho_1\rho_2+\rho_2^2\alpha_3\alpha_4)^{d/2}}$$

$$\times \; e^{i(\rho_1+\rho_2)(-m^2+i\varepsilon)+i\frac{p^2\rho_1^2\rho_2\,\alpha_1\alpha_2+p_4^2\rho_1\rho_2^2\alpha_1\alpha_3\alpha_4+p_3^2\rho_1\rho_2^2\alpha_2\alpha_3\alpha_4}{\rho_1\rho_2+\rho_2^2\alpha_3\alpha_4}}$$

$$\delta(\rho-\rho_1-\rho_2)\,.$$

---

$$\frac{(-i\lambda)^3}{2}\frac{e^{-i\frac{\pi}{2}(d-2)}}{(4\pi)^d}\int_0^\infty d\rho\int d\rho_1\,d\rho_2\,d\alpha_1\ldots d\alpha_4 \; \delta(1-\alpha_1-\alpha_2)\,\delta(1-\alpha_3-\alpha_4)\,\delta(1-\rho_1-\rho_2)$$

$$\rho^{4-1-d}\frac{\rho_1\rho_2}{(\rho_1\rho_2+\rho_2^2\alpha_3\alpha_4)^{d/2}} \; e^{i\rho(-m^2+i\varepsilon)+i\rho\frac{p^2\rho_1^2\cancel{\rho_2}\,\alpha_1\alpha_2+p_4^2\rho_1\rho_2^{\cancel{2}}\alpha_1\alpha_3\alpha_4+p_3^2\rho_1\rho_2^{\cancel{2}}\alpha_2\alpha_3\alpha_4}{\rho_1\cancel{\rho_2}+\rho_2^{\cancel{2}}\alpha_3\alpha_4}}$$

$$\frac{(-i\lambda)^3}{2}\frac{e^{-i\frac{\pi}{2}(d-2)}}{(4\pi)^d}\int d\rho_1 d\rho_2\, d\alpha_1 \ldots d\alpha_4 \frac{\delta(1-\alpha_1-\alpha_2)\,\delta(1-\alpha_3-\alpha_4)\,\delta(1-\rho_1-\rho_2)}{(\rho_1\rho_2+\rho_2^2\alpha_3\alpha_4)^{d/2}}\rho_1\rho_2$$

$$\Gamma(4-d)\, e^{-i\frac{\pi}{2}(4-d)}\left(m^2-i\varepsilon-\frac{p^2\rho_1^2\alpha_1\alpha_2+P_4^2\rho_1\rho_2\,\alpha_1\alpha_3\alpha_4+P_3^2\rho_1\rho_2\,\alpha_2\alpha_3\alpha_4}{\rho_1+\rho_2\alpha_3\alpha_4}\right)^{d-4}$$

---

$$\frac{(-i\lambda)^3}{2}\frac{e^{-i\frac{\pi}{2}(\cancel{d}-2)}}{(4\pi)^d}\Gamma(4-d)\, e^{i\frac{\pi}{2}(\cancel{d}-4)}\int_0^1 d\alpha\, d\beta\, d\gamma \frac{\gamma(1-\gamma)^{1-d/2}}{(\gamma+(1-\gamma)\beta(1-\beta))^{d/2}}$$

$$\left(m^2-i\varepsilon-\frac{p^2\gamma^2\alpha(1-\alpha)+\gamma(1-\gamma)\beta(1-\beta)\left(\alpha P_4^2+(1-\alpha)P_3^2\right)}{\gamma+(1-\gamma)\beta(1-\beta)}\right)^{d-4}$$

---

$$-\frac{i\lambda^3}{2}\frac{\Gamma(4-d)}{(4\pi)^d}\int_0^1 d\alpha\, d\beta\, d\gamma\, \frac{\gamma(1-\gamma)^{1-d/2}}{(\gamma+(1-\gamma)\beta(1-\beta))^{d/2}}\times$$

$$\times\left(m^2-\frac{p^2\gamma^2\alpha(1-\alpha)+\gamma(1-\gamma)\beta(1-\beta)\left(\alpha P_4^2+(1-\alpha)P_3^2\right)}{\gamma+(1-\gamma)\beta(1-\beta)}-i\varepsilon\right)^{d-4}$$

---

$\gamma\to 1$

$$-\frac{i\lambda^3}{2}\frac{\Gamma(4-d)}{(4\pi)^d}\int_0^1 d\alpha\, d\beta\, d\gamma\,(1-\gamma)^{1-d/2}\left(m^2-p^2\alpha(1-\alpha)-i\varepsilon\right)^{d-4}$$

$$\int_0^1 d\gamma\,(1-\gamma)^{1-d/2}=\int_0^1 d\gamma\,(1-\gamma)^{-1+\varepsilon/2}=-\left.\frac{(1-\gamma)^{\varepsilon/2}}{\varepsilon/2}\right|_0^1=+\frac{2}{\varepsilon}$$

$d=4-\varepsilon$

$$-\frac{i\lambda^3}{\cancel{2}}\frac{\overset{1/\varepsilon}{\Gamma(\varepsilon)}}{(4\pi)^{4-\varepsilon}}\frac{\cancel{2}}{\varepsilon}\int_0^1 d\alpha\,(m^2-p^2\alpha(1-\alpha)-i\varepsilon)^{-\varepsilon}$$

$$\left(1-\varepsilon\int_0^1 d\alpha\,\ln(m^2-\alpha(1-\alpha)s-i\varepsilon)\right)$$

$$+\frac{i\lambda^3}{(4\pi)^4}\frac{1}{\varepsilon^{\cancel{2}}}\cancel{\varepsilon}\int_0^1 d\alpha\ln(m^2-\alpha(1-\alpha)s-i\varepsilon)$$

$$\frac{i\lambda^3}{(4\pi)^4}\frac{1}{\varepsilon}\int_0^1 d\alpha\,\ln(m^2-\alpha(1-\alpha)s-i\varepsilon)=\frac{i\lambda}{16\pi^2\varepsilon}\frac{1}{16\pi^2}\int\cdots$$

$$=\frac{\lambda}{16\pi^2\varepsilon}B_f(s)$$

$$B_f=\frac{i}{16\pi^2}\int_0^1 d\alpha\,\ln(m^2-\alpha(1-\alpha)p^2-i0^+)$$

$\times 2$ $\delta J$

$$\frac{\lambda^3}{8\pi^2\varepsilon}\left(B_f(s)+B_f(t)+B_f(u)\right)\qquad \text{II}$$

$$\text{I}+\text{II}=0\ \checkmark$$