$$\mathcal{L}\Gamma = \left(d + n\left(1-\frac{d}{2}+\gamma\right)\right)\Gamma$$

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$$\sigma \frac{\partial \Gamma^{(n)}}{\partial \sigma} - \beta \frac{\partial \Gamma}{\partial g} - m^2\left(\gamma_{\phi^2}-2\right)\frac{\partial \Gamma}{\partial m^2} - \left(d+n\left(1-\frac{d}{2}+\gamma\right)\right)\Gamma = 0$$

$$\Gamma^{(n)} = \underbrace{\frac{1}{G^{(2)}(p_1)}}_{\Gamma^{(2)}} \cdots \underbrace{\frac{1}{G^{(2)}(p_n)}}_{\Gamma^{(2)}} G^{(n)}(p_1 \cdots p_n)$$

$$\Gamma^{(n)} = \Gamma^{(2)}(p_1)\cdots \Gamma^{(2)}(p_n)\, G^{(n)}(p_1 \cdots p_n)$$

$$\mathcal{L}\Gamma^{(n)} = \sum_j \cdots \mathcal{L}\Gamma^{(2)}(p_j) \cdots G^{(n)}(p_1 - p_n)$$

$$+ \Gamma^{(2)}(p_1)\cdots \Gamma^{(2)}(p_n)\, \mathcal{L}G^{(n)} = \left(d+n\left(1-\frac{d}{2}+\gamma\right)\right)\Gamma$$

$$= n\left(2(1+\gamma)\right)\cancel{\prod_j \Gamma^{(2)}(p_j)}\; G^{(n)} + \cancel{\Gamma^{(2)} \cdots}\; \mathcal{L}G^{(n)} =$$

$$= \cancel{\prod_j \Gamma^{(2)}}\left(d+n\left(1-\frac{d}{2}+\gamma\right)\right) G^{(n)}$$

$$\mathcal{L}G^{(n)}(\sigma p_j, \ldots) = \left(d+n\left(1-\frac{d}{2}+\gamma-2-2\gamma\right)\right) G^{(n)}$$

$$\mathcal{L}G^{(n)}(\sigma p_j, \ldots) = \left(d+n\left(-1-\frac{d}{2}-\gamma\right)\right) G^{(n)}$$

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$$G(\vec{x}, --) = \prod_j \int d^d p_j \; e^{i\sum p_j x_j} (2\pi)^d \delta^{(d)}(\Sigma p_j)\, G(p_j)$$

$$G(\frac{\bar{x}}{\sigma}, --) = \prod_j \int d^d p_j \; e^{i\sum \frac{p_j}{\sigma} x_j} (2\pi)^d \delta^{(d)}(\Sigma p_j)\, \hat{G}(p_j)$$

$$= \sigma^{nd-d} \prod_j \int d^d p_j \; e^{i\Sigma p_j x_j} \delta^{(d)}(\Sigma p_j)\, G(\sigma p_j, --)$$

$$\sigma^{-nd+d} G^{(n)}(\frac{x}{\sigma}, --) = \prod_j \int d^d p_j \; e^{i\Sigma p_j x_j} \delta^{(d)}(\Sigma p_j)\, G(p_j --)$$

$$\sigma \frac{\partial}{\partial \sigma} = (-nd+d)\, \sigma^{-nd+d} + --\sigma \frac{\partial}{\partial \sigma} G^{(n)} \; -$$

$$\mathcal{L}\left(\sigma^{-nd+d} G_x^{(n)}(\frac{x}{\sigma}, --)\right) = (-nd+d)\, \sigma^{-nd+d} G_x^{(n)} +$$

$$+ \cancel{\sigma^{-nd+d}}\, \mathcal{L} G_x^{(n)} = \cancel{\sigma^{-nd+d}}\left(d - n(1+\frac{d}{2}+\gamma)\right) G_x^{(n)}$$

$$\mathcal{L}\, \hat{G}(\frac{x}{\sigma}) = \left(\cancel{d} - n(1+\frac{d}{2}+\gamma) + nd \cancel{-d}\right) G_x^{(n)}$$

$$= n\left(-1 + \frac{d}{2} - \gamma\right) G^{(n)}(\frac{x}{\sigma})$$

$$\mathcal{L}\, G^{(n)}(\frac{x_i}{\sigma}, g, t, \mu) = n\left(\frac{d}{2} - 1 - \gamma\right) G^{(n)}(\frac{x_i}{\sigma}, --)$$

Check

at fixed point

$$\sigma \frac{\partial}{\partial \sigma} G^{(n)} = n\left(\frac{d}{2} - 1 - \gamma\right) G^{(n)}$$

$$G^{(n)}(x/\sigma) = \sigma^{n\left(\frac{d}{2}-1-\gamma_\phi\right)} G^{(n)}(x)$$

$$d_\phi = \underbrace{\frac{d}{2} - 1}_{\text{free}} - \underbrace{\gamma_\phi}_{\text{interaction}} \qquad \Big\| \quad \langle \phi\phi \rangle \sim \frac{1}{|x-y|^{d-2-2\gamma_\phi}}$$

$$\boxed{\eta = -2\gamma_\phi}$$

$$Z[j] = \left\langle e^{-\int j\phi} \right\rangle = e^{-\beta A}$$

$$= \sum_n \frac{(-)^n}{n!} \int d^dx_1 \ldots d^dx_n \; j(x_1) \ldots j(x_n) \; G^{(n)}(x_1 \ldots x_n)$$

$$Z[j(\sigma x), g, t; \mu] = \sum_n \frac{(-)^n}{n!} \int d^dx_1 \ldots d^dx_n \; j(\sigma x_1) \ldots j(\sigma x_n) \cdot G^{(n)}(x_1 \ldots x_n; g\, t\, \mu)$$

$$= \sum_n \frac{(-)^n}{n!} \int d^dx_1 \ldots d^dx_n \; j(x_1) \ldots j(x_n) \; \sigma^{-nd} G^{(n)}\left(\frac{x_i}{\sigma}; g, t, \mu\right)$$

$$\mathcal{L} Z = \sum_n \frac{(-)^n}{n!} \int d^dx_1 \ldots d^dx_n \; j(x_1) \ldots j(x_n) \left(-nd + n\left(\frac{d}{2} - 1 - \gamma\right)\right) G^{(n)}\left(\frac{x_i}{\sigma}, \ldots\right)$$

$$= \sum_n \frac{(-)^n}{n!} \int d^dx_1 \ldots d^dx_n \; j(x_1) \ldots j(x_n) \; n\left(-\frac{d}{2} - 1 - \gamma\right) G^{(n)}\left(\frac{x_i}{\sigma}, \ldots\right)$$

$$Z[A(\sigma)\, j(\sigma x)\, g\, t ; \mu] =$$

$$= \sum_n \frac{(-1)^n}{n!} \int d^d x_1 \ldots d^d x_n\, A^n(\sigma)\, j(x_1) \ldots j(x_n)\, G^{(n)}\left(\frac{x_i}{\sigma}\right) \sigma^{-nd}$$

$$\mathcal{L} Z = \sum_n \frac{(-1)^n}{n!} \int d^d x_1 \ldots d^d x_n \left(n A^{n-1} \sigma \frac{\partial A}{\partial \sigma} - n \left(\frac{d}{2} + 1 + \gamma \right) A^n \right) j(x_1) \ldots (x_n) \cdot$$

$$\cdot\, G^{(n)}\left(\frac{x}{\sigma}\right) \sigma^{-nd}$$

if $\sigma \frac{\partial A}{\partial \sigma} = A\left(\frac{d}{2} + 1 + \gamma\right) \Rightarrow \boxed{\mathcal{L} Z = 0}$

$$\mathcal{L} Z[A(\sigma)\, j(\sigma x), g, t ; \mu] = 0.$$

$$\frac{d}{d\sigma} Z[A(\sigma), j(\sigma x), g(\sigma), t(\sigma) ; \mu] = 0$$

if $\begin{cases} \sigma \frac{\partial A}{\partial \sigma} = A\left(\frac{d}{2} + 1 + \gamma\right) \\ \sigma \frac{\partial g}{\partial \sigma} = -\beta \\ \sigma \frac{\partial m^2}{\partial \sigma} = -m^2 (\gamma_{\phi^2} - 2) \end{cases}$

$$\mathcal{A}(A(\sigma)\, j(\sigma x), g(\sigma), t(\sigma) ; \mu) =$$

$$= \mathcal{A}(j(x), g, t, \mu)$$

$Z = e^{-\beta \mathcal{A}}$

indep. of σ.

$$A = \int d^d x \quad a(j(x), g, t; \mu)$$

$$A(A(\sigma) j(\sigma x)) = \int d^d x \quad a(A(\sigma) j(\sigma x), g(\sigma), t(\sigma); \mu)$$

$$= \sigma^{-d} \int d^d x \; a(A(\sigma) j(x), g(\sigma), t(\sigma); \mu)$$

indep. of σ

$$\sigma^{-d} a(A(\sigma) j(x), g(\sigma), t(\sigma); \mu) \text{ indep. of } \sigma.$$

$j(x) = H$

$$\sigma^{-d} a(A(\sigma) H, g(\sigma), t(\sigma); \mu)$$

near fixed point.

$$A = A_0 \sigma^{\frac{d}{2}+1+\gamma} \qquad g = g^* \quad t = t_0 \sigma^{2-\gamma_{\phi^2}}$$

$$\sigma^{-d} a(\sigma^{\frac{d}{2}+1+\gamma} \cancel{j(x)}, g^*, t_0 \sigma^{2-\gamma_{\phi^2}}; \mu) =$$

$$= a(\cancel{j(x)}, g^*, t_0, \mu)$$

$$\frac{M}{V} = \frac{\partial a}{\partial H} \; ; \qquad \sigma^{-d} \sigma^{\frac{d}{2}+1+\gamma} M(\sigma^{\frac{d}{2}+1+\gamma} H, g^*, t_0 \sigma^{2-\gamma_{\phi^2}}; \mu) =$$

$$= M(H, g^*, t_0, \mu)$$

$$\sigma^{-\frac{d}{2}+1+\gamma}\,\frac{M}{V}\left(\sigma^{\frac{d}{2}+1+\gamma} H,\ g^{\dagger},\ t\sigma^{2-\gamma_{\phi^2}};\ \mu\right) = \quad \sigma$$

$$= \frac{M}{V}\left(H,\ g^{\dagger},\ t_0;\ \mu\right)$$

•) $H = 0$

$$\sigma^{-\frac{d}{2}+1+\gamma}\,\frac{M}{V}\left(t_0\,\sigma^{2-\gamma_{\phi^2}}\right) = \frac{M}{V}(t_0)\,\cancel{\sigma^{-\frac{d}{2}+1+\gamma}}$$

$$\frac{M}{V} \sim t^{-\frac{\left(-\frac{d}{2}+1+\gamma\right)}{2-\gamma_{\phi^2}}} \qquad \beta = \frac{\frac{d}{2}-1-\gamma}{2-\gamma_{\phi^2}}$$

•) $H \neq 0 \quad t = 0$

$$\sigma^{-\frac{d}{2}+1+\gamma}\,\frac{M}{V}\left(\sigma^{\frac{d}{2}+1+\gamma} H\right) = \frac{M}{V}(H)$$

$$\frac{M}{V} \sim H^{\frac{\frac{d}{2}-1-\gamma}{\frac{d}{2}+1+\gamma}} \qquad \delta = \frac{\frac{d}{2}+1+\gamma}{\frac{d}{2}-1-\gamma}$$

$$\delta = \frac{d+2+2\gamma}{d-2-2\gamma} = \frac{d+2-\eta}{d-2+\eta} \quad ✓$$

$$\beta = \frac{d-2-2\gamma}{2(2-\gamma_{\phi^2})} = \frac{\nu}{2}(d-2+\eta) \quad ✓$$