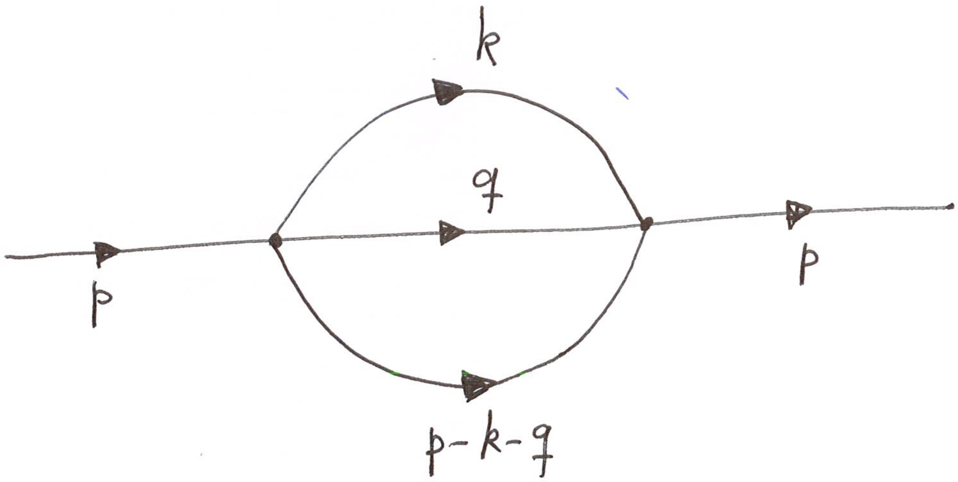


$$\frac{(-i\lambda)^2}{6} \int \frac{d^dk}{(2\pi)^d} \int \frac{d^dq}{(2\pi)^d} \frac{i}{k^2-m^2+i\varepsilon} \frac{i}{q^2-m^2+i\varepsilon} \frac{i}{(p-k-q)^2-m^2+i\varepsilon}$$

$$\frac{i}{k^2-m^2+i\varepsilon} = \int_0^\infty d\alpha \; e^{i(k^2-m^2+i\varepsilon)}$$

$$\frac{(-i\lambda)^2}{6} \int_0^\infty d\alpha_1 d\alpha_2 d\alpha_3 \int \frac{d^dk}{(2\pi)^d} \int \frac{d^dq}{(2\pi)^d} \, e^{i\sum \alpha_j(-m^2+i\varepsilon) + i\alpha_1 k^2 + i\alpha_2 q^2 + i\alpha_3(p^2+k^2+q^2)}$$

$$\times e^{i\alpha_3(-2pk-2pq+2kq)}$$

$$\int d^dk \; e^{i\alpha k^2 - 2ipk} = \left(\frac{\pi}{\alpha}\right)^{d/2} e^{i\frac{\pi}{4}(2-d)} \; e^{-ip^2/\alpha}$$

$$\frac{(-i\lambda)^2}{6}\int_0^\infty \frac{d\alpha_1 d\alpha_2 d\alpha_3}{(4\pi^2)^d}\, e^{i\sum\alpha_j(-m^2+i\epsilon)} \times$$

$$\times \int d^d q \; e^{i(\alpha_2+\alpha_3)q^2}\left(\frac{\pi}{\alpha_1+\alpha_3}\right)^{d/2} e^{i\frac{\pi}{4}(2-d)} \times$$

$$\times\, e^{-i\frac{\alpha_3^2(p-q)^2}{\alpha_1+\alpha_3}+i\alpha_3 p^2-2i\alpha_3 pq}$$

we have :

$$\underbrace{i(\alpha_2+\alpha_3)q^2}\;\underbrace{-\frac{i\alpha_3^2p^2}{\alpha_1+\alpha_3}}\;\underbrace{-\frac{i\alpha_3^2q^2}{\alpha_1+\alpha_3}}+\frac{2i\alpha_3^2pq}{\alpha_1+\alpha_3}\;\underbrace{+i\alpha_3p^2}-2i\alpha_3pq$$

$$=\frac{iq^2}{(\alpha_1+\alpha_3)}\left(\alpha_1\alpha_2+\alpha_2\alpha_3+\alpha_1\alpha_3+\cancel{\alpha_3^2}-\cancel{\alpha_3^2}\right)+\frac{ip^2}{\alpha_1+\alpha_3}\left(-\cancel{\alpha_3^2}+\alpha_1\alpha_3+\cancel{\alpha_3^2}\right)+$$

$$+\frac{2i\,pq}{\alpha_1+\alpha_3}\left(\cancel{\alpha_3^2}-\alpha_1\alpha_3+\cancel{\alpha_3^2}\right)$$

$$\frac{(-i\lambda)^2}{6}\frac{1}{(4\pi^2)^d}\pi^{d/2}e^{\frac{i\pi}{4}(2-d)}\int_0^\infty d\alpha_j\; e^{i\sum\alpha_j(-m^2+i\epsilon)}\times$$

$$\times\int d^dq\,\frac{1}{(\alpha_1+\alpha_3)^{d/2}}\; e^{iq^2\frac{(\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3)}{\alpha_1+\alpha_3}-\frac{2i\alpha_1\alpha_3}{\alpha_1+\alpha_3}pq+\frac{i\alpha_1\alpha_3}{\alpha_1+\alpha_3}p^2}$$

$$\frac{(-i\lambda)^2}{6} \frac{\pi^d}{(4\pi^2)^d} e^{i\frac{\pi}{2}(2-d)} \int_0^\infty d\alpha_j \, e^{i\sum\alpha_j(-m^2+i\epsilon)} \times$$ ③

$$\times \frac{1}{\cancel{(\alpha_1+\alpha_3)^{d/2}}} \left(\frac{\cancel{\alpha_1+\alpha_3}}{\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3} \right)^{d/2} e^{-i \frac{\alpha_1^2\alpha_3^2 p^2}{(\alpha_1+\alpha_3)^{\cancel{2}}} \frac{\cancel{\alpha_1+\alpha_3}}{\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3}}$$

$$\times e^{i\frac{\alpha_1\alpha_3}{\alpha_1+\alpha_3} p^2}$$

$$\frac{(-i\lambda)^2}{6} \frac{1}{(4\pi)^d} e^{i\frac{\pi}{2}(2-d)} \int_0^\infty d\alpha_j \frac{e^{i\sum\alpha_j(-m^2+i\epsilon)}}{(\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3)^{d/2}}$$

$$\times e^{i\frac{(\cancel{-\alpha_1^2\alpha_3^2}+\alpha_1^2\alpha_2\alpha_3+\cancel{\alpha_1^2\alpha_3^2}+\alpha_1\alpha_2\alpha_3^2)}{(\alpha_1+\alpha_3)(\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3)} p^2}$$

$\leftarrow (\alpha_1+\alpha_3)\,\alpha_1\alpha_2\alpha_3$

Finally

$$\frac{(-i\lambda)^2}{6} \frac{e^{i\frac{\pi}{2}(2-d)}}{(4\pi)^d} \int_0^\infty d\alpha_j \frac{e^{i\sum\alpha_j(-m^2+i\epsilon)}}{(\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3)^{d/2}} \times$$

$$\times e^{i\frac{\alpha_1\alpha_2\alpha_3}{\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3} p^2}$$

Nice symmetry under interchange $\alpha_1, \alpha_2\, \alpha_3$.

$$\frac{(-i\lambda)^2}{6}\frac{e^{i\frac{\pi}{2}(2-d)}}{(4\pi)^d}\int_0^\infty d\rho\int_0^\infty d\alpha_j\,\delta(\rho-\Sigma\alpha_j)\frac{e^{i\rho(-m^2+i\varepsilon)}}{(\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3)^{d/2}}\times$$

$$\times\, e^{i\frac{\alpha_1\alpha_2\alpha_3}{\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3}p^2}$$

$\alpha_i \to \rho\alpha_i$

$$\frac{(-i\lambda)^2}{6}\frac{e^{i\frac{\pi}{2}(2-d)}}{(4\pi)^d}\int_0^\infty d\rho\int_0^\infty d\alpha_j\,\frac{\rho^3}{\rho}\,\delta(1-\Sigma\alpha_j)\times \qquad \rho^{3-d-1}$$

$$\times\frac{e^{i\rho(-m^2+i\varepsilon)}}{\rho^d\,(\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3)^{d/2}}\,e^{i\rho\frac{\alpha_1\alpha_2\alpha_3}{\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3}p^2}$$

$$=\frac{(-i\lambda)^2}{6}\frac{e^{i\frac{\pi}{2}(2-d)}}{(4\pi)^d}\Gamma(3-d)\int_0^\infty d\alpha_j\,\delta(1-\Sigma\alpha_j)\times\frac{1}{(\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3)^{d/2}}$$

$$\times\left(e^{\frac{i\pi}{2}(3-d)}\right)\left(-m^2+\frac{\alpha_1\alpha_2\alpha_3}{\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3}p^2+i\varepsilon\right)^{d-3}$$

$$\int_0^\infty d\rho\,\rho^{a-1}e^{i(b+i\varepsilon)\rho}=\int_0^\infty d\rho\,\rho^{a-1}e^{-(\varepsilon-ib)\rho}=\Gamma(a)(\varepsilon-ib)^{-a}=\Gamma(a)(-i)^{-a}\underbrace{(b+i\varepsilon)^{-a}}$$

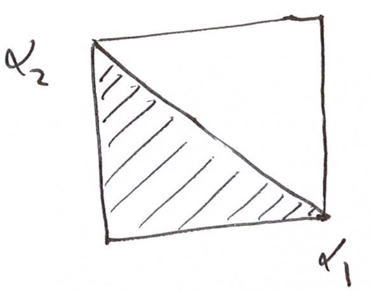
$$=\Gamma(a)\,e^{-i\frac{\pi}{2}a}(-b-i\varepsilon)^{-a}\,e^{i\frac{\pi}{2}a}$$

$$= \frac{(-i\lambda)^2}{6} \, \frac{e^{i\frac{\pi}{2}(5-2d)}}{(4\pi)^d} \, \Gamma(3-d) \int_0^\infty d\alpha_j \; \delta(1-\Sigma\alpha_j) \times$$

$$\times \frac{1}{(\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3)^{d/2}} \left(-m^2 + \frac{\alpha_1\alpha_2\alpha_3}{\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_2\alpha_3}\, p^2 + i\epsilon\right)^{d-3}$$

$\alpha_3 \to 1-\alpha_1-\alpha_2$ $\qquad \displaystyle\int_0^1 d\alpha_1 \int_0^1 d\alpha_2 \; \theta(1-\alpha_1-\alpha_2)$

$\uparrow$ because $\alpha_3 > 0$



$\alpha = \alpha_1 + \alpha_2$

$\alpha : 0 \to 1 \qquad 0 \le \alpha_1 \le \alpha \qquad (\alpha_2 > 0)$

$$\int_0^1 d\alpha \int_0^\alpha d\alpha_1 = \int_0^1 d\alpha \int_0^1 d\beta \times \alpha$$

$\beta = \alpha_1/\alpha \qquad \alpha_1 = \alpha\beta$

$\alpha_1 = \alpha\beta \qquad \alpha_2 = \alpha - \alpha_1 = \alpha(1-\beta) \qquad \alpha_3 = 1-\alpha_1-\alpha_2 = 1-\alpha$

$$= \frac{(-i\lambda)^2}{6} \frac{e^{\frac{i\pi}{2}(5-2d)}}{(4\pi)^d} \Gamma(3-d) \times \int_0^1 d\alpha \int_0^1 d\beta \; \alpha \; \times$$

$$\times \frac{1}{\left(\alpha\beta\,\alpha(1-\beta) + \alpha\beta(1-\alpha) + \alpha(1-\alpha)(1-\beta)\right)^{d/2}} \left(-m^2 + \frac{\alpha^2\beta(1-\beta)(1-\alpha)\; p^2}{(\qquad)}\right)^{d-3}$$

$$\cancel{\alpha^2\beta} - \alpha^2\beta^2 + \cancel{\alpha\beta} - \cancel{\alpha^2\beta} + \alpha - \cancel{\alpha\beta} - \alpha^2 + \alpha^2\beta$$

$$\alpha(1-\alpha) + \alpha^2\beta(1-\beta) = \alpha\left(1-\alpha+\alpha\beta(1-\beta)\right)$$

$$= \frac{(-i\lambda)^2}{6} \frac{e^{\frac{i\pi}{2}(5-2d)}}{(4\pi)^d} \Gamma(3-d) \int_0^1 d\alpha \int_0^1 d\beta \; \alpha^{1-d/2} \; \times$$

$$\times \frac{1}{\left(1-\alpha+\alpha\beta(1-\beta)\right)^{d/2}} \left(-m^2 + \frac{\alpha\beta(1-\beta)(1-\alpha)\; p^2}{(1-\alpha+\alpha\beta(1-\beta))}\right)^{d-3}$$

$$= \frac{(-i\lambda)^2}{6} \frac{e^{\frac{i\pi}{2}(5-2d)}\,\Gamma(3-d)\,(-m^2)^{d-3}}{(4\pi)^d} \int_0^1 d\alpha \int_0^1 d\beta \; \alpha^{1-d/2} \; \times$$

$$\times \frac{1}{\left(1-\alpha+\alpha\beta(1-\beta)\right)^{d/2}} \left(1 - \frac{\alpha\beta(1-\beta)(1-\alpha)}{(1-\alpha+\alpha\beta(1-\beta))}\, p^2/m^2\right)^{d-3}$$

$$f(p^2) = \underset{\infty}{A} + B\frac{p^2}{m^2} + C\frac{p^4}{m^4} + \cdots$$

$\underline{p^2 \to 0}$

$$\frac{(-i\lambda)^2}{6} \frac{e^{i\frac{\pi}{2}(5-2d)} \Gamma(3-d)(-m^2)^{d-3}}{(4\pi)^d} \int_0^1 d\alpha \int_0^1 d\beta \; \alpha^{1-d/2} \times$$

$$\times \frac{1}{(1-\alpha+\alpha\beta(1-\beta))^{d/2}} = A$$

$$\frac{(-i\lambda)^2}{6} \frac{e^{i\frac{\pi}{2}(5-2d)}}{(4\pi)^d} \Gamma(3-d)(-m^2)^{d-3} \int_0^1 d\alpha \int_0^1 d\beta \; \alpha^{1-d/2}$$

$$(-(d-3)) \frac{\alpha\beta(1-\beta)(1-\alpha)}{(1-\alpha+\alpha\beta(1-\beta))^{d/2+1}} = B$$

Next one will have

$$\Gamma(3-d)\frac{(d-3)(d-4)}{2} = \frac{1}{2} \underbrace{\Gamma(3-d)(3-d)}_{\Gamma(4-d)}(4-d) = \frac{1}{2} \underline{\Gamma(5-d)}$$

$d \to 4$ $\Gamma(1)$ finite

all others are finite!

$$B = \frac{(-i\lambda)^2}{6} \frac{e^{i\frac{\pi}{2}(5-2d)}}{(4\pi)^d} \Gamma(4-d)\,(-m^2)^{d-3} \times$$

$$\times \int_0^1 d\alpha \int_0^1 d\beta \; \alpha^{1-d/2} \frac{\alpha\beta(1-\beta)(1-\alpha)}{(1-\alpha+\alpha\beta(1-\beta))^{d/2+1}}$$

We can replace $d=4$ except in $\Gamma(4-d)$ since the rest is finite

$$B = \frac{(-i\lambda)^2}{6} \frac{e^{-\frac{3i\pi}{2}}}{(4\pi)^4} \Gamma(4-d)\,(-m^2) \int_0^1 d\alpha \int_0^1 d\beta \frac{\beta(1-\beta)(1-\alpha)}{(1-\alpha+\alpha\beta(1-\beta))^3}$$

$$\int_0^1 d\alpha \frac{1-\alpha}{(1-\alpha+\alpha\beta(1-\beta))^3}$$

$$\int_0^1 d\alpha \frac{1-\alpha}{(1+\alpha b)^3} = -\frac{1+b}{2b^2} \left.\frac{1}{(1+\alpha b)^2}\right|_0^1 + \frac{1}{b^2} \left.\frac{1}{1+\alpha b}\right|_0^1 =$$

$$= -\frac{1+b}{2b^2}\left(\frac{1}{(1+b)^2} - 1\right) + \frac{1}{b^2}\left(\frac{1}{1+b} - 1\right)$$

$$= -\frac{1}{2b^2(1+b)} + \frac{1+b}{2b^2} + \frac{1}{(1+b)b^2} - \frac{1}{b^2} = \frac{1}{2b^2(1+b)} + \frac{1}{2b^2} + \frac{1}{2b} = \frac{\cancel{1} - \cancel{1} - \cancel{b} + \cancel{b} + b^2}{2\cancel{b^2}(1+b)}$$

$$= \frac{1}{2}\frac{1}{1+b}$$

$$\int_0^1 d\alpha \frac{1-\alpha}{\left(1+\left(\beta(1-\beta)-1\right)\alpha\right)} = \frac{1}{2\left(\cancel{1}+\beta(1-\beta)\cancel{-1}\right)}$$

$$\int_0^1 d\beta \frac{\cancel{\beta(1-\beta)}}{2\cancel{\beta(1-\beta)}} = \frac{1}{2}\int_0^1 d\beta = \frac{1}{2}$$

$$\int_0^1 d\alpha \int_0^1 d\beta \frac{\beta(1-\beta)(1-\alpha)}{\left(1-\alpha+\alpha\beta(1-\beta)\right)^3} = \frac{1}{2}$$

$$B = \frac{(-i\lambda)^2}{2\times 6\,(4\pi)^4}\, e^{-\frac{3i\pi}{2}} (-m^2)\, \Gamma(4-d)$$

$d = 4-\varepsilon$ $\qquad \Gamma(4-d) = \Gamma(\varepsilon) = \frac{1}{\varepsilon}\Gamma(1+\varepsilon) = \frac{1}{\varepsilon} + \cdots$

$$B = \frac{(-i\lambda)^2}{12}\, e^{-\frac{3i\pi}{2}} (-m^2)\, \frac{1}{\varepsilon} + \text{finite}$$

$$B = \frac{i\lambda^2 m^2}{12(4\pi)^4}\frac{1}{\varepsilon} + \cdots$$

$$C = \int_0^1 d\alpha \int_0^1 d\gamma \; \frac{\alpha^{1-d/2}}{\left(1-\alpha+\alpha\,\gamma(1-\gamma)\right)^{d/2}}$$

$\gamma = \beta + \frac{1}{2}$ $\qquad \beta : -\frac{1}{2} \to \frac{1}{2}$

$1-\gamma = \frac{1}{2} - \beta$ $\qquad \gamma(1-\gamma) = \frac{1}{4} - \beta^2$

$$C = \int_0^1 d\alpha \int_{-1/2}^{+1/2} d\beta \; \frac{\alpha^{1-d/2}}{\left(1-\alpha+\frac{\alpha}{4}-\frac{\alpha\beta^2}{\cancel{4}}\right)^{d/2}}$$

$$= 2\int_0^1 d\alpha \int_0^{1/2} d\beta \; \frac{\alpha^{1-d/2}}{\left(1-\alpha+\frac{\alpha}{4}-\frac{\alpha\beta^2}{\cancel{4}}\right)^{d/2}}$$

$\beta = \tilde{\beta}/2$

$$= \frac{2}{2}\int_0^1 d\alpha \int_0^1 d\tilde{\beta} \; \frac{\alpha^{1-d/2}}{\left(1-\alpha+\frac{\alpha}{4}-\frac{\alpha}{4}\tilde{\beta}^2\right)^{d/2}}$$

$\gamma = \tilde{\beta}^2$ $\qquad d\gamma = 2\tilde{\beta}\, d\tilde{\beta}$

$$= \int_0^1 d\alpha \int_0^1 \frac{d\gamma}{2\sqrt{\gamma}} \; \frac{\alpha^{1-d/2}}{\left(1-\alpha+\frac{\alpha}{4}-\frac{\alpha}{4}\gamma\right)^{d/2}}$$

$\gamma \to 1-\gamma$

$$= \frac{1}{2}\int_0^1 d\alpha \int_0^1 \frac{d\gamma}{\sqrt{1-\gamma}} \; \frac{\alpha^{1-d/2}}{\left(1-\alpha+\frac{\alpha}{4}\gamma\right)^{d/2}}$$

$$\gamma = \frac{1}{1+\rho} \qquad d\gamma = -\frac{1}{(1+\rho)^2}\, d\rho$$

$$C = \frac{1}{2} \int_0^1 d\alpha \int_0^\infty \frac{d\rho}{(1+\rho)^2} \frac{1}{\sqrt{1-\frac{1}{1+\rho}}} \frac{\alpha^{1-d/2}}{\left(1-\alpha+\frac{\alpha}{4}\frac{1}{1+\rho}\right)^{d/2}}$$

$$= \frac{1}{2} \int_0^1 d\alpha \int_0^\infty \frac{d\rho}{\sqrt{\rho}} \frac{(1+\rho)^{d/2}}{(1+\rho)^{3/2}} \frac{\alpha^{1-d/2}}{\left(1-\alpha+\frac{\alpha}{4}+\rho(1-\alpha)\right)^{d/2}}$$

$$= \frac{1}{2} \int_0^1 d\alpha \int_0^\infty \frac{d\rho}{\sqrt{\rho}} (1+\rho)^{\frac{d}{2}-\frac{3}{2}} \frac{\alpha^{1-\frac{d}{2}}}{\Big(1-\frac{3\alpha}{4}+\underbrace{\rho(1-\alpha)}_{\tilde\rho}\Big)^{d/2}}$$

$$\tilde\rho = \rho\underbrace{(1-\alpha)}_{\geq 0}$$

$$= \frac{1}{2} \int_0^1 d\alpha \int_0^\infty \frac{d\tilde\rho}{\sqrt{\tilde\rho}} (1-\alpha)^{-\frac{1}{2}} \left(1+\frac{\tilde\rho}{1-\alpha}\right)^{\frac{d}{2}-\frac{3}{2}} \frac{\alpha^{1-\frac{d}{2}}}{\left(1-\frac{3\alpha}{4}+\tilde\rho\right)^{d/2}}$$

$\tilde\rho \to \rho$

$$= \frac{1}{2} \int_0^1 d\alpha \int_0^\infty \frac{d\rho}{\sqrt{\rho}} (1-\alpha)^{-\frac{1}{2}+\frac{3}{2}-\frac{d}{2}} \alpha^{1-\frac{d}{2}} \frac{(\rho+1-\alpha)^{\frac{d}{2}-\frac{3}{2}}}{\left(\rho+1-\frac{3\alpha}{4}\right)^{d/2}}$$

$$= \frac{1}{2} \int_0^1 d\alpha\, \underbrace{\alpha^{1-\frac{d}{2}} (1-\alpha)^{1-\frac{d}{2}}}_{\substack{\text{symmetric} \\ \alpha \to 1-\alpha}} \underbrace{\int_0^\infty \frac{d\rho}{\sqrt{\rho}} \frac{(\rho+1-\alpha)^{\frac{d}{2}-\frac{3}{2}}}{\left(\rho+1-\frac{3\alpha}{4}\right)^{d/2}}}_{\text{finite when } d \to 4}$$

$$\alpha = \beta + 1/2 \qquad \beta : -\tfrac{1}{2} \to 1/2$$

$$\alpha(1-\alpha) = \frac{1}{4} - \beta^2$$

$$= \frac{1}{2} \int_{-1/2}^{1/2} d\beta \left(\frac{1}{4} - \beta^2\right)^{1-\frac{d}{2}} \int_0^\infty \frac{d\rho}{\sqrt{\rho}} \frac{\left(\rho + \frac{1}{2} - \cancel{\beta}\right)^{\frac{d}{2}-\frac{3}{2}}}{\left(\rho + 1 - \frac{3\beta}{4} - \frac{3}{8}\right)^{d/2}}$$

$\beta = \tilde{\beta}/2$

$$= \frac{1}{2} \int_{-1}^{1} \frac{d\tilde{\beta}}{2} \left(\frac{1}{4} - \frac{\tilde{\beta}^2}{4}\right)^{1-\frac{d}{2}} \int_0^\infty \frac{d\rho}{\sqrt{\rho}} \frac{\left(\rho + \frac{1}{2} - \frac{\tilde{\beta}}{2}\right)^{\frac{d}{2}-\frac{3}{2}}}{\left(\rho + 1 - \frac{3\tilde{\beta}}{8} - \frac{3}{8}\right)^{d/2}}$$

$$= \frac{1}{4} \frac{1}{4^{1-d/2}} \int_0^1 d\beta \left(1-\beta^2\right)^{1-d/2} \int_0^\infty \frac{d\rho}{\sqrt{\rho}} \left[\frac{\left(\rho + \frac{1}{2} - \frac{\beta}{2}\right)^{\frac{d}{2}-\frac{3}{2}}}{\left(\rho + \frac{5}{8} - \frac{3\beta}{8}\right)^{d/2}} + \frac{\left(\rho + \frac{1}{2} + \frac{\beta}{2}\right)^{\frac{d}{2}-\frac{3}{2}}}{\left(\rho + \frac{5}{8} + \frac{3\beta}{8}\right)^{d/2}} \right]$$

$$= \frac{1}{2^{4-d}} \int_0^1 d\beta \left(1-\beta^2\right)^{1-d/2} f(\beta)$$

$$f(\beta) = \int_0^\infty \frac{d\rho}{\sqrt{\rho}} \left[\frac{\left(\rho + \frac{1}{2} - \frac{\beta}{2}\right)^{\frac{d}{2}-\frac{3}{2}}}{\left(\rho + \frac{5}{8} - \frac{3\beta}{8}\right)^{d/2}} + \frac{\left(\rho + \frac{1}{2} + \frac{\beta}{2}\right)^{\frac{d}{2}-\frac{3}{2}}}{\left(\rho + \frac{5}{8} + \frac{3\beta}{8}\right)^{d/2}} \right]$$

$$C = \frac{1}{2^{4-d}} \underbrace{\int_0^1 d\beta\, (1-\beta^2)^{1-d/2} \left[f(\beta) - f(1)\right]}_{\text{finite} \to \text{we put } d=4} +$$

$$+ \frac{1}{2^{4-d}} \int_0^1 d\beta\, (1-\beta^2)^{1-\frac{d}{2}} f(1)$$

$$f(1) = 2\,\frac{d+2}{d-2} \qquad f_{d=4}(1) = 6$$

maple

$$\int_0^1 d\beta\, (1-\beta^2)^{-1} \left[f_{d=4}(\beta) - 6\right] = 1 \quad \text{(maple numeric)}$$

$$C = 1 + 2^{-\varepsilon} \times 2\,\frac{6-\varepsilon}{2-\varepsilon} \underbrace{\int_0^1 d\beta (1-\beta^2)^{1-\frac{d}{2}}}_{\frac{1}{2}\sqrt{\pi}\,\frac{\Gamma(2-d/2)}{\Gamma\left(\frac{5}{2}-\frac{d}{2}\right)}}$$

$$\frac{1}{2}\sqrt{\pi}\,\frac{\Gamma(\varepsilon/2)}{\Gamma\left(\frac{1}{2}+\varepsilon/2\right)} = \frac{1}{\varepsilon} + \ln 2 + \mathcal{O}(\varepsilon) \qquad \frac{1}{2} - \frac{1}{6} = \frac{3-1}{6} = \frac{1}{3}$$

$$C = 1 + (1-\varepsilon\ln 2)\,\frac{\cancel{2}\times 6}{\cancel{2}}\left(1 - \frac{\varepsilon}{6} + \frac{\varepsilon}{2}\right)\left(\frac{1}{\varepsilon} + \ln 2\right)$$

$$= 1 + \frac{6}{\varepsilon}\left(1 \cancel{- \varepsilon\ln 2} \cancel{+ \varepsilon\ln 2} + \frac{\varepsilon}{2} - \frac{\varepsilon}{6}\right) = 1 + \frac{6}{\varepsilon} + 2 = \frac{6}{\varepsilon} + 3 + \mathcal{O}(\varepsilon)$$

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$$\boxed{C = \frac{6}{\varepsilon} + 3 + \mathcal{O}(\varepsilon)}$$