663, Homework I, (2 problems)

Problem 1

Consider the $SU(N)$ Wilson loop in the small loop approximation:

$$W = \hat{P}\left\{e^{ig\oint \mathbb{A}_\mu \dot{x}^\mu ds}\right\} \simeq \mathbb{I} + \frac{ig}{2}\hat{f}^{\mu\nu}\mathbb{F}_{\mu\nu} + \dots \tag{0.1}$$

where the anti-symmetric tensor $f^{\mu\nu}$ is given by

$$\hat{f}^{\mu\nu} = \frac{1}{2}\oint (x^\mu \dot{x}^\nu - x^\nu \dot{x}^\mu) ds \tag{0.2}$$

and equals the (oriented) area of the projected Wilson loop on each plane (μ, ν). To this order, the trace gives the trivial result

$$\mathcal{W} = \frac{1}{N}\operatorname{Tr} W \simeq 1 + \dots \tag{0.3}$$

Continue the expansion and compute the first non-trivial contribution to the trace.

Problem 2

Consider a non-abelian Higgs model with gauge group $SU(2)$ and a scalar in the spin 1, vector representation. The Lagrangian is

$$\mathcal{L} = \frac{1}{2}(D_\mu\phi)^\dagger(D^\mu\phi) - V(\phi) - \frac{1}{2}\operatorname{Tr}\mathbb{F}_{\mu\nu}\mathbb{F}^{\mu\nu} \tag{0.4}$$

where ϕ is a **3-component** real vector $\phi_{a=1,2,3}$ and $(\mu^2 < 0)$

$$V(\phi) = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2 \tag{0.5}$$

$$(D_\mu\phi)_a = \partial_\mu\phi_a + g\epsilon_{abc}A^b_\mu\phi_c, \tag{0.6}$$

Expand the Lagrangian around the minimum of the potential and determine the spectrum of particles.

Problem 3

Consider a free **massive** vector field with Lagrangian (this is **not** a gauge theory)

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu \tag{0.7}$$

Write the equations of motion and show that $\partial_\mu A^\mu = 0$. Perform the usual canonical quantization by writing the solutions to the equations of motion as superposition of plane waves with all allowed polarizations and quantizing the amplitudes as creation and annihilation operators. Then compute the Feynman propagator

$$\Delta_{\mu\nu}(x - y) = \langle 0|T\{A_\mu(x)A_\nu(y)\}|0\rangle \tag{0.8}$$

Using this result, compute the propagator in momentum space $\Delta_{\mu\nu}(k)$ and determine its large momentum behavior ($k \to \infty$).