663, Homework II, (2 problems)

Problem 1

Consider an $SU(N)$ gauge theory with a fermion in the fundamental. In the BRST approach one introduces extra fields c, $\bar{c}$ and B. We say, by definition, that c has ghost number 1 and $\bar{c}$ has ghost number -1.

a) Use the gauge fixing function $\partial^\mu A_\mu$ (Lorentz gauge) and compute the dimensions of the different fields (including B).

b) Consider the Lagrangian density to be a BRST, Lorentz, and global $SU(N)$ invariant polynomial in the fields. Show that a BRST variation preserves the number $N_B + N_{\bar{c}}$, where N_B is the number of fields B in a given term, and the same for $N_{\bar{c}}$. Notice that the operator that counts this number can be written as

$$N_B + N_{\bar{c}} = B\frac{\delta}{\delta B} + \bar{c}\frac{\delta}{\delta \bar{c}} \tag{0.1}$$

c) Show that, if a function $\mathcal{L}$ of the fields is BRST invariant then

$$\delta_{BRST}\left[\left(\bar{c}\frac{\delta}{\delta B}\right)\mathcal{L}\right] = (N_B + N_{\bar{c}})\mathcal{L} \tag{0.2}$$

Hint: Use that the BRST variation can be written as $\delta_{BRST} = B\frac{\delta}{\delta \bar{c}} + \tilde{\delta}$ where $\tilde{\delta}$ does not involve B or $\bar{c}$.

d) Conclude that any BRST invariant Lagrangian of ghost number zero can be written as the usual Yang-Mills Lagrangian plus the BRST variation of another function Ψ

e) Using that the dimension of all the vertices in the Lagrangian is 4 or less, write the most general function Ψ and the corresponding Lagrangian.

Problem 2

Consider an $SU(N)$ gauge theory with N_f fermionic and N_b scalar fields ψ, ϕ in the fundamental representation and $\bar{N}_f$ fermions Ψ and $\bar{N}_b$ bosons Φ in the adjoint. That is under a gauge transformation $U(x)$ the fields transform as

$$\psi \to U\psi, \quad \phi \to U\phi, \quad \Psi \to U\Psi U^\dagger \quad \Phi \to U\Phi U^\dagger, \tag{0.3}$$

a) Compute the one-loop correction to the gauge field propagator due to the matter fields. Use dimensional regularization. Check that the self-energy you computed is transverse.

b) Extract the divergent part of the self-energy and find the corresponding counter-term for the gauge field due to the matter fields.

c) Compute the correction to the one-loop beta function of the gauge coupling.

d) Find values of N_f, N_b, $\tilde{N}_f$, $\tilde{N}_b$ such that the one-loop beta function vanishes.