

## 663, Homework I, (1 problem)

### Problem 1

Consider the  $O(N)$  model in the broken symmetry phase and its low-energy description in terms of a non-linear  $\sigma$ -model.

a) Show that the action of the non-linear sigma model can be written as

$$S = -\frac{1}{2g^2} \int d^d x (\partial_\mu \hat{n})^2 \quad (0.1)$$

where  $\hat{n}$  is an  $N$ -dimensional unit vector.

- b) Show that the theory is renormalizable only in two dimensions ( $d = 2$ ).
- c) Compute the  $\beta$ -function for the coupling  $g$  and discuss its properties, in particular the sign.

Reference: A. Polyakov, Physics Letters **B59**, 79 (1975).