660, Fall 2017, Homework IV, (4 problems)

Based on problems 2.3, 2.13, 2.18, 2.15, 2.23 of Sakurai's book

Problem 1

An electron is subject to a uniform, time-independent magnetic field $\vec{B} = B\hat{z}$ in the z direction. At $t = 0$ the electron is in an eigenstate of $\vec{S}.\hat{n}$ with eigenvalue $\frac{\hbar}{2}$. Here $\hat{n}$ is an arbitrary unit vector with polar angles (θ, ϕ). (Similar to previous homework).

a) Find the state of the electron at any subsequent time t.

b) At a given time t, what are the possible results of measuring S_x, and their probabilities?. Repeat the same for S_z and S_y.

c) Compute the mean value of S_x, S_y and S_z as a function of time.

Problem 2

Consider a one-dimensional harmonic oscillator with frequency ω. Using the algebraic method (*i.e.* operators a, $a^\dagger$) compute

a) $\langle m|x|n\rangle$, $\langle m|p|n\rangle$, $\langle m|xp + px|n\rangle$, $\langle m|x^2|n\rangle$, $\langle m|x^2|n\rangle$, where $|n\rangle$, $|m\rangle$ are eigenstates of energy.

b) $\langle\alpha|x|\alpha\rangle$, $\langle\alpha|p|\alpha\rangle$, $\langle\alpha|x^2|\alpha\rangle$, $\langle\alpha|p^2|\alpha\rangle$ where $|\alpha\rangle$ is a coherent state.

c) Use the previous result to show that coherent sates have minimum uncertainty

Problem 3

Continuing from problem 2. Consider $x_H(t)$, the position operator in the Heisenberg picture.

a) Evaluate the correlation function

$$C(t) = \langle 0|x_H(t)x_H(0)|0\rangle \tag{0.1}$$

where $|0\rangle$ is the ground state.

Problem 4

A particle of mass m in one dimension is bound to a fix center by an attractive δ-function potential:

$$V(x) = -\lambda\delta(x), \qquad (\lambda > 0) \tag{0.2}$$

a) At $t = 0$ the potential is suddenly switched off, that is $V = 0$ for $t > 0$. At $t = 0$ the wave function is the one of the bound state since it cannot change instantaneously ($\partial_t \psi$ is finite for finite Hamiltonian). Compute the wave function for $t > 0$.