660, Fall 2017, Homework V, (4 problems)

Based on problems 3.12, 3.15, 3.18, 3.20 of Sakurai's book

Problem 1

An angular momentum eigenstate $|\ell, \ell_z = \ell\rangle$ is rotated by an infinitesimal angle $\alpha \ll 1$ about the y-axis.

a) If, in the new state, we measure $\hat{\ell}_z$, what is the probability of obtaining $\ell_z = \ell$? Find the answer up to terms of order α^2.

Note: Perform the calculation *without* using the explicit form of the $d^{(j)}_{\ell'_z,\ell_z}$ matrix.

Problem 2

The wave-function of a particle subjected to a spherically symmetric potential $V(r)$ is given by

$$\psi(\vec{r}) = (x + y + 3z)f(r) \tag{0.1}$$

a) Is ψ an eigenfunction of L^2 ? If so, what is the value of ℓ?. If not, what are the possible values of ℓ we may obtain when L^2 is measured?

b) What are the probabilities for the particle to be found in various $\hat{\ell}_z$ eigenstates?

c) Suppose it is known that $\psi(\vec{r})$ is an energy eigenfunction with eigenvalue E. Indicate how to find $V(r)$.

Problem 3

Consider an orbital angular-momentum eigenstate $|\ell = 2, \ell_z = 0\rangle$. Suppose that this state is rotated by an angle β about the y-axis.

a) If we measure $\hat{\ell}_z$, what are the possible values we can obtain and what is the probability of measuring each of them?.

Problem 4

Given two particles in angular momentum eigenstates $\ell_1 = 1$ and $\ell_2 = 1$, the total possible angular momentum is $\ell_T = 0, 1, 2$.

a) *Without* using the table, write all eigenstates of total angular momentum $|\ell_1, \ell_2; \ell_T, \ell_z\rangle$ as linear combination of the states of the basis $|\ell_1, \ell_2; \ell_{1z}\ell_{2z}\rangle$.