661, Spring 2016, Homework I, (2 problems)

Problem 1

A hydrogen atom in the ground state is subject to an electric field in the z-direction

$$\vec{E} = E_0\,\hat{z} \tag{0.1}$$

a) Compute the first correction to the ground state $|\psi^{(1)}\rangle$.

b) In the corrected state, compute, at first order, the mean value of the electric dipole moment $\vec{P} = -e\vec{r}$.

c) From the formula $\vec{P} = \alpha\vec{E}$, determine an expression for the polarizability α of the atom. Find upper and lower bounds for α giving explicit expressions for both.

d) (Optional) Find a function $F(\vec{r})$ such that $[F, H]|1s\rangle = z|1s\rangle$. Use this function F to evaluate the polarizability α exactly and verify that the result is within the bounds found in **c)**. **Hint:** Try $F = Arz + Bz$ and find the constants A, B.

Problem 2

Consider a particle in a potential such that the Hamiltonian is

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 + \lambda x^4 \tag{0.2}$$

a) Consider the term $V = \lambda x^4$ as a perturbation of $H_0 = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$ and find the first order correction to the energies of the first three states (ground state and two excited states).

b) Alternatively, approximate the ground state energy of H by using the variational method with a trial wave-function given by a Gaussian

$$\psi(x) = A\ e^{-\frac{1}{2}ax^2} \tag{0.3}$$

c) Finally, use the WKB approximation to find the energy eigenstates of H including the ground state.

d) Compare the results of **a)**, **b)**, and **c)** for the energy of the ground state. Should they agree? Discuss the results and the range of validity of the different approximations. You can use particular values of the parameters to make the comparison.