Phys 661 HW 2 Solutions

Problem 1 – John Daniel

1. The 6 isotopes of Thorium that were used:
   - → Thorium-217 : Decay time of 0.241 ms
     Energy of α-decay: 9261 keV
     Note: When a number of different α decays were possible with accompanying probabilities, the mean energy of the α-decay was used.
   - → Thorium-221 : Decay time of 1.73 ms
     Energy of α-decay: 8251 keV
   - → Thorium-223 : Decay time of 0.6 s
     Energy of α-decay: 7298 keV
   - → Thorium-226 : Decay time of 30.57 minutes
     Energy of α-decay: 6300 keV
   - → Thorium-228 : Decay time of 1.9125 years
     Energy of α-decay: 5368.976 keV
   - → Thorium-230 : Decay time of 7.54×10^4 years
     Energy of α-decay: 4657.378 keV

→ The data for the time was then converted to units of years & a plot of $\ln\left(\frac{\tau_{1/2}}{\text{years}}\right)$ vs $\frac{1}{\sqrt{E_\alpha}}$, where E_α was in units of eV was created.

→ The result was a fairly linear graph with a slope of approximately 281847.13 ± 15695.3.

→ If the Thorium-217 data point is ignored, the slope, or the proportionality constant between $\ln\left(\frac{\tau_{1/2}}{\text{years}}\right)$ and $\frac{1}{\sqrt{E_\alpha}}$, is 302867 ± 74686

→ If we then compare this with the WKB Approximation, we see it is fairly consistent.

→ The WKB Approximation predicts the relationship is:

$$\ln \frac{\tau_{1/2}}{1\text{year}} \simeq 2\pi (Z-2)\alpha_e \sqrt{\frac{2M_\alpha c^2}{E}},$$

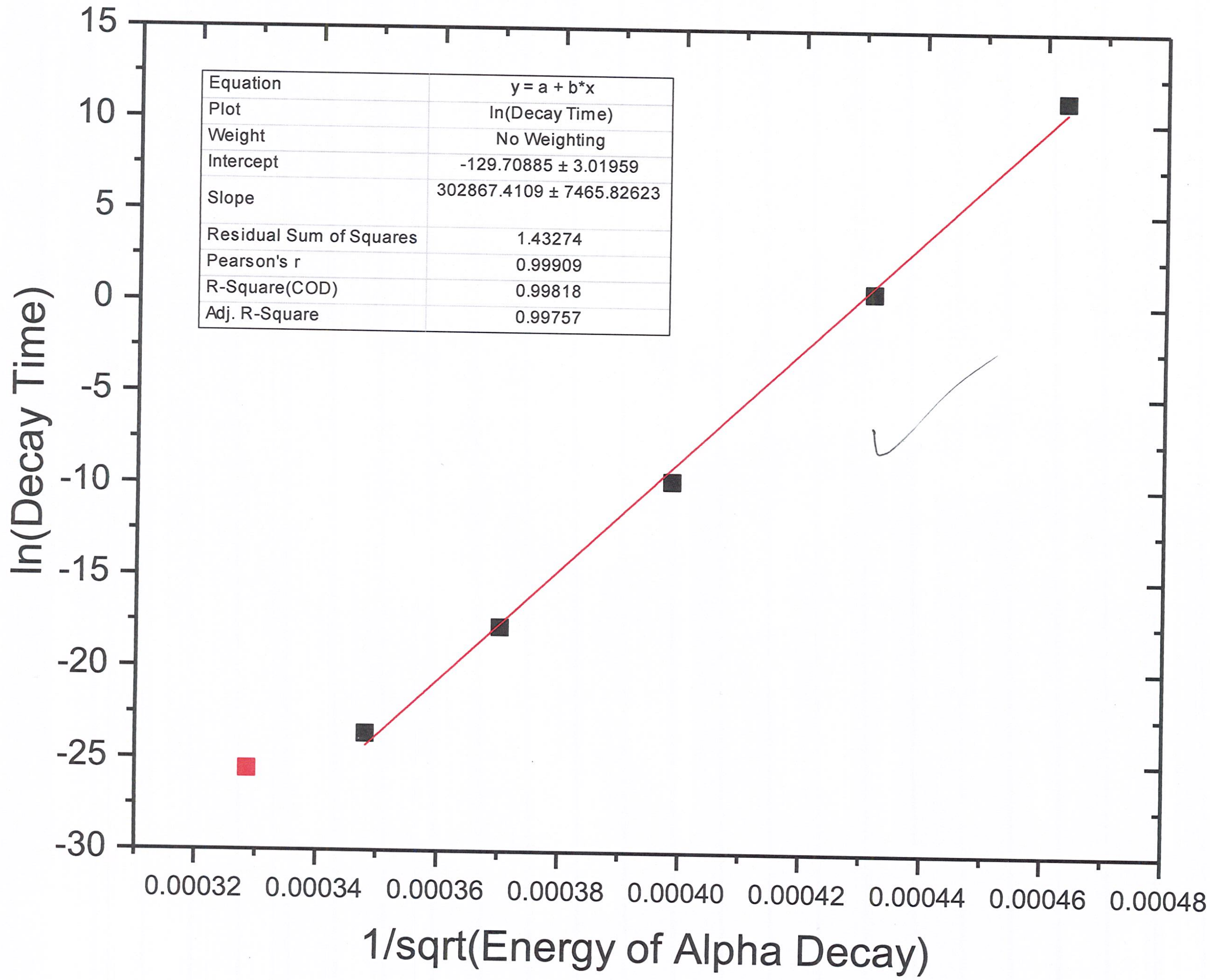
where (for Thorium), $Z = 90$, $\alpha_e = \frac{1}{137}$ (the fine structure constant) & $M_\alpha = 6.64424 \times 10^{-27}$ kg (the mass of an α particle)

→ When this is evaluated (& ensuring to divide the $M_\alpha c^2$ term by 1.6×10^{-19} to get the $M_\alpha c^2$ term in terms of eV), we find the slope ~~sh~~ is predicted to be $\simeq 348932.0728$

∴ The WKB approximation gives:

$$\ln\left(\frac{\tau_{1/2}}{1\text{year}}\right) \simeq 348932.0728 \cdot \frac{1}{\sqrt{E_\alpha}}$$, where E_α is in terms of eV

⇒ We see that this agrees broadly with what is observed (as in, it is the same order of magnitude), but there is a difference of around 40000.



Equation
y = a + b*x
Plot
ln(Decay Time)
Weight
No Weighting
Intercept
-129.70885 ± 3.01959
Slope
302867.4109 ± 7465.82623
Residual Sum of Squares
1.43274
Pearson's r
0.99909
R-Square(COD)
0.99818
Adj. R-Square
0.99757
ln(Decay Time)
15
10
5
0
-5
-10
-15
-20
-25
-30
0.00032
0.00034
0.00036
0.00038
0.00040
0.00042
0.00044
0.00046
0.00048
1/sqrt(Energy of Alpha Decay)

Problem 2 – Sean Myers

Problem 2

For Parity $\Pi |l\, l_z\rangle = (-)^l |l\, l_z\rangle$.

If you have a matrix element

$\langle\alpha|V|\beta\rangle$

then it equals $\langle\alpha|\Pi\Pi V \Pi\Pi|\beta\rangle$

$\langle\alpha|V|\beta\rangle = \varepsilon_\alpha \varepsilon_\beta \varepsilon_V \langle\alpha|V|\beta\rangle$

if $\varepsilon_\alpha \varepsilon_\beta \varepsilon_V = -1$ the element must be zero.

Also $\Pi|\vec{r}\rangle = |-\vec{r}\rangle$; so $\Pi|z\rangle = |-z\rangle$

from table: parity gives

#1 $\langle 1,0| \underset{\Pi\Pi}{\uparrow} z \underset{\Pi\Pi}{\uparrow} |2,0\rangle \Rightarrow (-)^1 (-)^2 (-) = +$ (no info gained)

#2 $\langle 1,0| \underset{\Pi\Pi}{\nearrow} zy^2 \underset{\Pi\Pi}{\nwarrow} |2,1\rangle = (-)^1(-)^2$

$(-)^1(-)^2 \langle 1,0| \underbrace{\Pi zy^2 \Pi}_{-1} |2,1\rangle \Rightarrow +$ (no info)

#3 $\langle 4,0| p_z x | 0\,0\rangle = (-)^4 (-)^0 \overset{p_z}{(-)}\overset{x}{(-)} \Rightarrow +$ (no info)

#4 $\langle 3,0| p_x^2 | 0,0\rangle = (-)^3(-)^0(+) \Rightarrow -$ (zero)

#5 $\langle 2,1| x p_x | 1,0\rangle = (-)^2(-)^1(-)(-) \Rightarrow -$ (zero)

Now for L^2 and L_z we use

$\langle l_1 m_1 | T_q^k | l_2 m_2 \rangle$ vanishes if the following is not satisfied:

for L^2 : $|l_2 - k| \leq l_1 \leq l_2 + k$

L_z : $m_1 = q + m_2$

First I determine T_q^k for each operator

#1 $\quad z = T_0^1 \quad k = 1 \quad q = 0$

$\langle 1\,0 | T_0^1 | 2\,0 \rangle \qquad m_1 = m_2 = 0$
$\qquad\qquad\qquad\qquad l_1 = 1 \; ; \; l_2 = 2$

L^2 : $|2-1| \leq 1 \leq 2+1$

$1 \leq 1 \leq 3$ true (no info)

L_z : $0 = 0 + 0$ true (no info)

#2 $\quad zy^2 \propto T_0^1 \, y^2$

$y \propto T_{+1}^1 + T_{-1}^1$

$y^2 \propto T_{+1}^1 T_{+1}^1 + T_{-1}^1 T_{-1}^1 + T_{+1}^1 T_{-1}^1 + T_{-1}^1 T_{+1}^1$

$$zy^2 = \underset{(a)}{T_0^1 T_{+1}^1 T_{+1}^1} + \underset{(b)}{T_0^1 T_{-1}^1 T_{-1}^1} + \underset{(c)}{T_0^1 T_{+1}^1 T_{-1}^1} + \underset{(d)}{T_0^1 T_{-1}^1 T_{+1}^1}$$

Checking (a) $m_1 = 0$ $m_2 = 1$ $q = 2$

$l_1 = 1$ $l_2 = 2$ $k = 3$

$T_0^1 T_{+1}^1 T_{+1}^1 \rightarrow L^2: \; |2-3| \leq 1 \leq 2+3$

$1 \leq 1 \leq 5$ true (no info)

$L_z: \; 0 = 3$ false (zero)

since L^2 can't give zero I wont check (b), (c), (d). However, L_z gives zero for (a) so I will see they all give zero.

(b) $q = -2$

$0 = -2 + 1 \neq 0$ (zero)

(c) $q = 0$:

$0 = 0 + 1 \neq 0$ (zero

(d) $q = 0$:

$\rightarrow$ zero

$$\boxed{\therefore \; L_z \rightarrow \text{(zero)}}$$

#3 $P_z X \propto T_0^1 (T_{+1}^1 - T_{-1}^1)$; $m_1 = 0$ $m_2 = 0$

$l_1 = 4$ $l_2 = 0$

$\propto T_0^1 T_{+1}^1 - T_0^1 T_{-1}^1$

$k = 2$, $q = 1$ $k = 2$, $q = -1$

X $L^2: \; |0-2| \leq 4 \leq 2$

L^2: same as other one

$L_z: \; 0 = 1 + 0$ false

$L_z: \; 0 = -1 + 0$ false

for #3 L^2, L_z give zero

#4 $p_x^2 \propto \overbrace{\left| T^1_{+1} T^1_{+1} + T^1_{-1} T^1_{-1} + T^1_{+1} T^1_{-1} + T^1_{-1} T^1_{+1} \right.}$

(a) (b) (c) (d)

$m_1 = 0$ $m_2 = 0$

$l_1 = 3$ $l_2 = 0$

(a), (b), (c), (d) have same $k = 2$

L^2: $|0-2| \le 3 \le 0+2$ X false

so $L^2 \to$ (zero)

check (a) for L_z: $q = 2$

$0 = 2 + 0 \neq 0$ (no info)

$L_z \to$ (no info) from (c) & (d)

#5 $xp_y \propto (T^1_{+1} - T^1_{-1})(T^1_{+1} + T^1_{-1})$

$\propto \left| T^1_{+1} T^1_{+1} + T^1_{+1} T^1_{-1} - T^1_{-1} T^1_{+1} - T^1_{-1} T^1_{-1} \right.$

a b c d

$m_1 = 0$ $m_2 = 0$

$l_1 = 2$ $l_2 = 0$

(a), (b), (c), (d) have same $k = 2$

L^2: $|1-2| \le 2 \le 1+3$ true

$L^2 \to$ no info

L_z: $q = m_1 - m_2 = 1$

(a), (b), (c), (d) don't have $q = 1$ $\therefore L_z \to$ (zero)

In summary

element	Π	L^2	L_z
$\langle 1,0 \| z \| 2,0 \rangle$	?	?	?
$\langle 1,0 \| z y^2 \| 2,1 \rangle$	?	?	0
$\langle 4,0 \| p_z x \| 0,0 \rangle$	?	0	0
$\langle 3,0 \| p_x^2 \| 0,0 \rangle$	0	0	?
$\langle 2,1 \| x p_x \| 1,0 \rangle$	0	?	0

Problem 3 – Dmitry Kondratyev

Problem 3

To use degenerate perturbation theory we need to know the matrix elements $\langle 2l'm'|V|2lm\rangle$. As we learned in Problem 2, operator $\hat{z}$ adds angular momentum $|LM\rangle = |10\rangle$ so with the help of selection rules we can eliminate most of matrix elements:

$$\langle 2l'm'|V|2lm\rangle = e|\vec{E}| \begin{array}{c} \begin{pmatrix} 0 & 0 & M & 0 \\ 0 & 0 & 0 & 0 \\ M^* & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \\ \begin{array}{cccc} |00\rangle & |11\rangle & |10\rangle & |1{-1}\rangle \end{array} \end{array} \quad \text{Where } M = \langle 210|z|200\rangle.$$

$$|200\rangle = \frac{1}{\sqrt{32\pi}\,a^{3/2}}\left[2-\frac{r}{a}\right]e^{-r/2a}, \quad |210\rangle = \frac{1}{\sqrt{32\pi}\,a^{3/2}}\frac{r}{a}e^{-r/2a}\cos\theta$$

$$M = \frac{1}{32\pi a^3}\underbrace{\left(\int_0^\pi \cos^2\theta\sin\theta\,d\theta\right)}_{2/3}\underbrace{\left(\int_0^{2\pi}d\varphi\right)}_{2\pi}\underbrace{\left(\int_0^\infty e^{-r/a}\cdot\frac{r}{a}\left[2-\frac{r}{a}\right]r^3dr\right)}_{-72a^4} = -3a = M^*$$

So, in the first order we have eigenstates: $|2\,1\pm1\rangle$, $\frac{1}{\sqrt{2}}\left(|200\rangle \pm |210\rangle\right)$
with energies: $E_2^{(0)}$, $E_2^{(0)} \mp 3ae|\vec{E}|$

Since the electric field is in $\hat{z}$ direction, we know that polarization will be $\vec{P} = -e\vec{r} = -e\hat{z}\cdot z$. We already know that $\langle 21\pm1|z|21\pm1\rangle = 0$, so in the eigenstates $|21\pm1\rangle$ the polarization is 0.

$$\langle\vec{P}\rangle = -e\hat{z}\cdot\frac{1}{2}\left(\langle 200|\pm\langle 210|\right)z\left(|200\rangle\pm|210\rangle\right) = -e\hat{z}\cdot\frac{1}{2}\left(\underbrace{\langle 200|z|200\rangle}_{0} + \underbrace{\langle 210|z|210\rangle}_{0} \pm\langle 210|z|200\rangle \pm \langle 200|z|210\rangle\right)$$

(equal in this case)

$$= \mp e\hat{z}\cdot\langle 200|z|210\rangle = \pm e\hat{z}\cdot 3a = \pm 3ae\hat{z} \quad \text{– for states } \frac{1}{\sqrt{2}}\left(|200\rangle \pm |210\rangle\right)$$

Polarization doesn't depend on $|\vec{E}|$, so it will be finite even when $E \to 0$

In the ground state polarization was proportional to $|\vec{E}|$ and was small for $E \to 0$