661, Spring 2016, Homework IV, (5 problems)

Problem 1

Consider N identical, spin $\frac{1}{2}$, non-interacting, particles in a one-dimensional harmonic oscillator potential. What is the ground state energy?. Discuss, in particular, the case when $N \gg 1$. Repeat the calculation for a 3 dimensional isotropic harmonic oscillator.

Problem 2

Two particles of spin 1 can have a total spin $s = 0, 1, 2$. However, when the particles are identical this may no longer be true. Discuss, in this case, the possible values of the total spin when the *spatial* wave function is symmetric and also when it is antisymmetric (under interchange of the two particles).

Problem 3

A one dimensional harmonic oscillator is in the ground state at $t = 0$. An external potential $V(t) = V_0\, x\, e^{-\frac{t}{\tau}}$ is then turned on.

- Use perturbation theory at first order to find the probability of the harmonic oscillator to be in any of its possible states at $t > 0$.
- Study the limit $t \to \infty$ of the probabilities and discuss briefly the result.

Problem 4

A hydrogen atom is at $t = 0$ in the ground state and then subject to an electric field $\vec{E} = E_0 e^{-\frac{t}{\tau}}\, \hat{z}$.

- Use perturbation theory at first order to find the probability of the atom to be in the $2s$ and $2p, m = \pm 1, 0$ states at a later time t.

Problem 5

Consider a system of two spin $\frac{1}{2}$ with Hamiltonian

$$H = \frac{4\Delta}{\hbar^2} \, \vec{S}_1 \cdot \vec{S}_2 \tag{0.1}$$

Assume that at $t = 0$ the system is in the state $|\uparrow\downarrow\rangle$.

- Use perturbation theory at first order to find the probability of the system to be in any of its possible states at $t > 0$.
- Solve the problem exactly and compare with the previous result. Check that they agree in the appropriate limit.