Hyperfine transition

Problem

In the hydrogen atom, the spin of the proton couples to the magnetic field produced by the electron. When the electron is in the $1s$ state, the relevant interaction is

$$V_{hf} = -\frac{8\pi}{3}\vec{\mu}_e \cdot \vec{\mu}_p \; \delta^{(3)}(\vec{x}) \tag{0.1}$$

where

$$\vec{\mu}_e = -\frac{e}{m_e\, c}\vec{S}_e \tag{0.2}$$

is the magnetic moment of the electron and

$$\vec{\mu}_p = g_p \frac{e}{2\, m_p\, c}\vec{S}_p \tag{0.3}$$

that of the proton. Here $g = 5.59$ is the gyromagnetic radius of the proton and $m_e = 0.511 MeV/c^2$, $m_p = 938 MeV/c^2$ are the masses of the electron and proton respectively. If we ignore the perturbation, we find four degenerate ground states (electron and proton spin up or down). The matrix of the perturbation in those states corresponds to:

$$V_{hf} = \frac{4\pi}{3}\frac{e^2}{m_e m_p c^2} g_p |\psi_{1s}(0)|^2 \frac{1}{2}((\vec{S}_e + \vec{S}_p)^2 - \vec{S}_e^2 - \vec{S}_p^2)) \tag{0.4}$$

This interaction is diagonal in the basis where the total spin is diagonal. It is easily seen that the ground state has $S_T = 0$ and there are three degenerate excited states with $S_T = 1$. The energy difference between the ground and excited states is

$$\Delta E = \frac{4}{3}\left(\frac{e^2}{\hbar c}\right)^4 \frac{m_e}{m_p} g_p m_e c^2 \tag{0.5}$$

Replacing the known values we get:

$$\Delta E = 5.9 \times 10^{-6} eV \tag{0.6}$$

A photon from this transition has a wave length

$$\lambda = \frac{2\pi\hbar c}{\Delta E} = 0.21m = 21cm \tag{0.7}$$

which a famous wave-length in radio astronomy. It is a good exercise to repeat this calculation and check the figures.

The proposed **problem** is to compute the mean life of the $S_T = 1$ state. That is, you need to identify the interaction that produces the decay, use the Fermi Golden rule to compute the transition probability per unit time and from there the mean life τ.