661, Spring 2016, Homework VI, (4 problems)

Problem 1

Consider a potential $V(r)$ of finite range R ($V(r > R) = 0$). Assuming that the Born approximation is valid for such a potential:

- Give a formula for the scattering length a_s in the low energy limit.
- Using the previous result and the optical theorem calculate the imaginary part of the forward scattering amplitude at second order in the potential.

Problem 2

Consider a potential

$$V(r) = \begin{cases} -V_0 & \text{if} \quad r < a \\ 0 & \text{if} \quad r > a \end{cases} \tag{0.1}$$

with $V_0 > 0$ (attractive potential).

- Solve exactly the Schroedinger equation for the s-wave ($\ell = 0$) and find the phase-shift $\delta_0(E)$.
- There is theorem that states that

  $$\pi N_0 = \delta_0(E = 0) - \delta_0(E = \infty) \tag{0.2}$$

  where N_0 is the number of bound states in the potential with zero angular momentum. Check the theorem by giving different values to the parameters of the problem and verifying the relation between bound states and phase-shift. Can you show that it is valid for any value of the parameters?

Problem 3

Assume that for a certain central potential $f(\theta)$ is independent of θ.

- Show that the optical theorem puts an upper bound on the total cross section, and that, in particular it should vanish for large energies.
- Assume now that $f(\theta)$ is approximately constant inside a solid angle $\Delta\Omega$ around $\theta = 0$ (forward direction). Show that, for a given total cross section $\sigma(k)$ the optical theorem puts an upper bound on $\Delta\Omega$.
- Using the previous result show that if $\sigma(k)$ becomes constant for large energies (independent of k) then the scattering probability has a large peak in the forward direction.

Problem 4

Consider scattering by a repulsive delta-function potential

$$V(r) = \frac{\hbar^2}{2m}\,\gamma\,\delta(r-R), \qquad \gamma > 0 \tag{0.3}$$

- Find the equation that determines the s-wave phase shift $\delta_0(E)$
- Consider the case where γ is large compared to $1/R$ and k. Give numerical values to the parameters and plot $\delta_0(E)$ and $\sigma_0(E)$ numerically. Here $\sigma_0(E)$ is the contribution of $\ell = 0$ to the total cross section.
- Vary the parameters in the previous point so that resonances are clearly seen when plotting $\sigma_0(E)$. At what energies should the resonances be?