String/gauge theory duality and QCD

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Summary

- Introduction
  - String theory
  - Gauge/string theory duality.
  - AdS/CFT correspondence.

- Mesons in AdS/CFT

- Chiral symmetry breaking

- Spin chains as a way to see strings in field theory

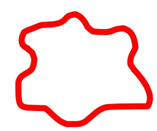
Introduction

String theory

●) Quantum field theory:

Relativistic theory of point particles.
Strong, weak and electromagnetic interactions are described in this way. In particular by gauge theories. Quantum gravity?

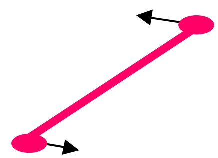
●) String theory:

Relativistic theory of extended objects: Strings 

Why?

Original motivation: Phenomenological model for hadrons (proton, neutron, pions, rho, etc.)

String picture → Fund. strings



(Susy, 10d, Q.G.)

mesons

π, ρ, ...

Quark model → QCD

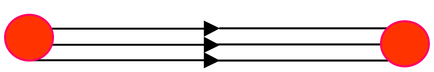
$q\,\bar{q}$

↓

Large N-limit

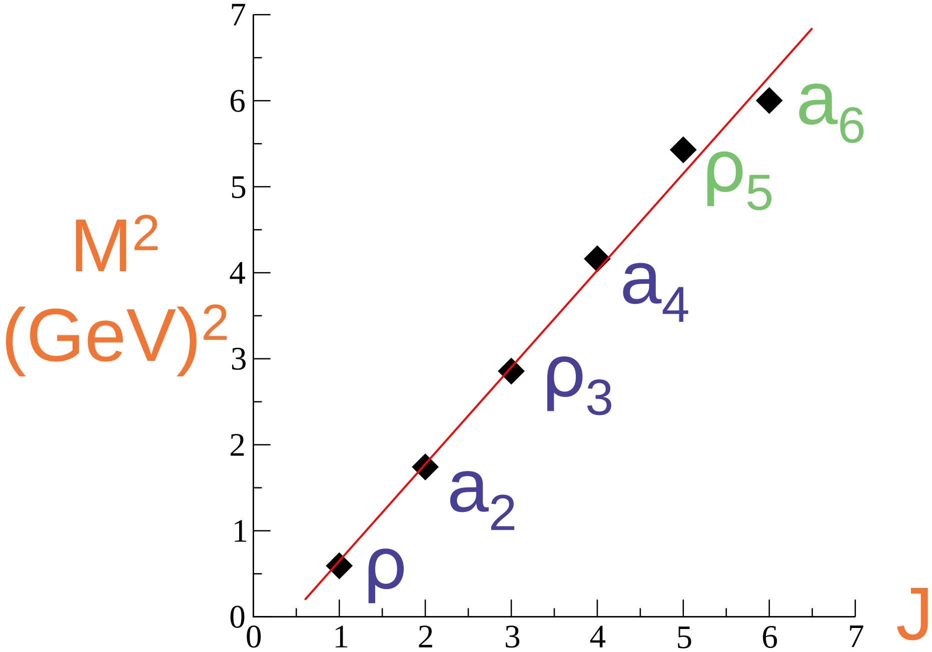
↓

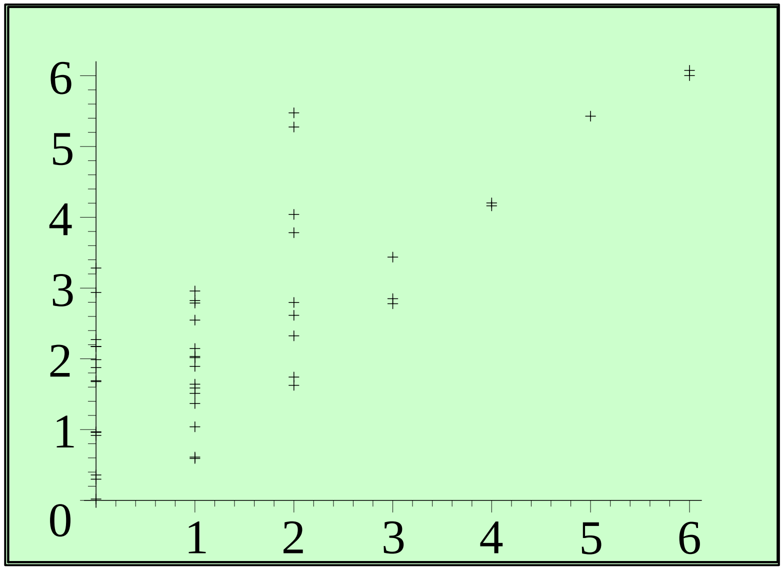
Effective strings

q  $\bar{q}$

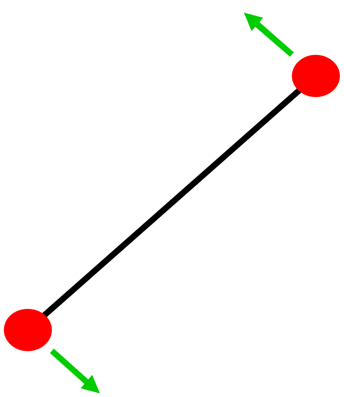
Strong coupling

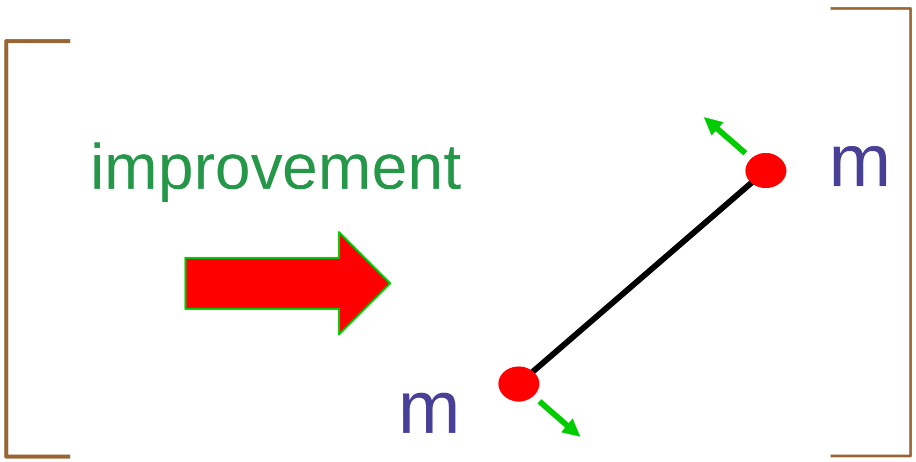
<u>Regge trajectories</u>





Simple model of rotating strings gives $E \approx \sqrt{J}$





Strings were thought as **<u>fundamental</u>**, however…

Strings as a fundamental theory

Only one fundamental object, strings (open and closed). Different modes of oscillation of the string give rise to the different observed particles.

Interactions are the splitting and rejoining of the strings.

Tachyons $\longrightarrow$ Taking care by supersymmetry

Quantum mechanically consistent only in 10 dim.

Unified models? Including gravity

Too many vacua.

What about hadrons?

Instead: bound states of quarks. mesons: $q\bar{q}$

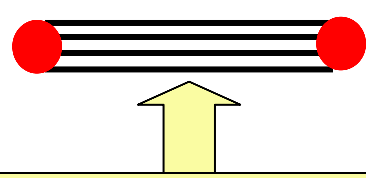
baryons: qqq

Interactions: SU(3); $q = \begin{bmatrix} - \\ - \\ - \end{bmatrix}$; $A_\mu = \begin{bmatrix} - & - & - \\ - & - & - \\ - & - & - \end{bmatrix}$

quarks gluons

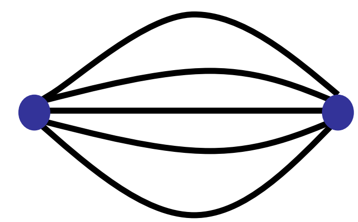
Coupling constant **small** at large energies (100 GeV) but **large** at small energies. No expansion parameter.

Confinement



(color) electric flux=string?

$V = k\,r$



$V = -\,k/r$

Idea ('t Hooft)

Take large-N limit, $q=\begin{bmatrix} - \\ - \\ - \\ - \\ - \\ - \\ - \\ - \\ - \end{bmatrix}$; $A_\mu=\begin{bmatrix} - & - & - & - \\ - & - & - & - \\ - & - & - & - \\ - & - & - & - \\ - & - & - & - \\ - & - & - & - \\ - & - & - & - \\ - & - & - & - \\ - & - & - & - \end{bmatrix}$

N N x N

$$N \to \infty, \quad \boxed{g_{YM}^2 N = \lambda}$$ fixed ('t Hooft coupling)

1/N: perturbative parameter.
Planar diagrams dominate (sphere)
Next: $1/N^2$ corrections (torus) + $1/N^4$ (2-handles) + …

Looks like a string theory
Can be a way to derive a string descriptions of mesons

AdS/CFT correspondence (Maldacena)

Gives a precise example of the relation between strings and a gauge theory. (Curiously non-confining).

Gauge theory	String theory
$\mathcal{N} = 4$ SYM SU(N) on R^4	IIB on $AdS_5 x S^5$
A_μ , Φ^i, Ψ^a	radius R
Operators w/ conf. dim. Δ	String states w/ $E = \frac{\Delta}{R}$

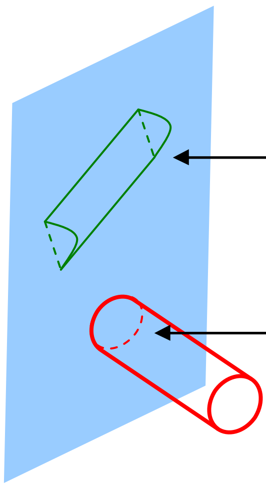
$$\boxed{g_s = g_{YM}^2; \qquad R / l_s = (g_{YM}^2 N)^{1/4}}$$

$N \to \infty, \lambda = g_{YM}^2 N$ fixed ➡

λ large → string th.
λ small → field th.

D-branes (Polchinski)

Region of space where strings world-sheets can end.



Open strings. Low energy: SU(N) gauge th.

Emission of graviton → D-branes have mass

If N, # of D-branes is large → mass large → deforms space

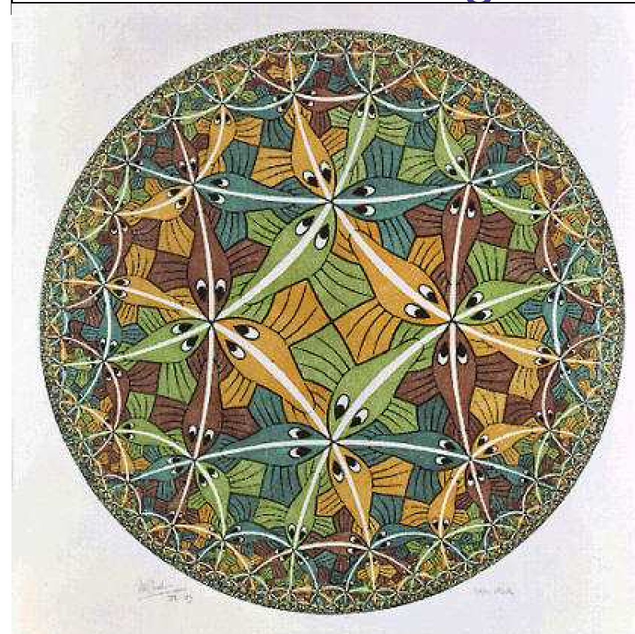
e.g. D3- brane:

$$ds^2 = \frac{1}{\sqrt{f}}\left(dX^+ dX^- + dX_{[2]}^2\right) + \sqrt{f}\, dY_{[6]}^2, \qquad f = \cancel{1} + 4\pi\alpha'^2 \frac{g_s N}{Y^4}$$

Suggests an interesting rep. of the large-N limit

AdS/CFT

$\mathcal{N} = 4$ SYM $\longleftrightarrow$ II B on $AdS_5 x S^5$



Escher

S^5: $X_1^2 + X_2^2 + \ldots X_6^2 = R^2$

AdS_5: $Y_1^2 + Y_2^2 + \ldots - Y_5^2 - Y_6^2 = -R^2$

How about hadrons and QCD?

deform

Strings?

AdS/CFT

Known examples

QCD

Other points in AdS/CFT

-) Allows gauge theory computations at strong coupling.

-) Describes confining gauge theories. Confinement transition in interpreted geometrically.

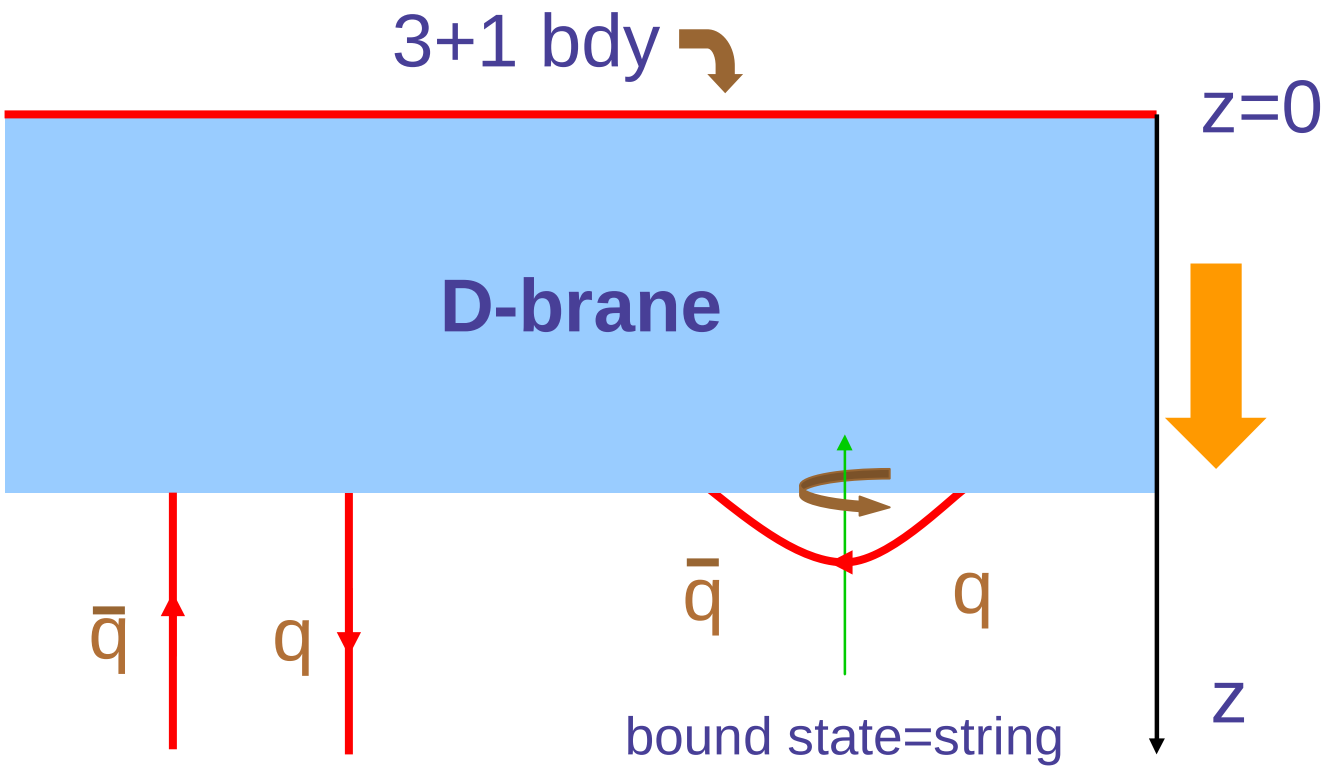
-) Finite temperature quark-gluon plasma is described by a black hole. (e.g. compute viscosity).

-) Quantum gravity in AdS is unitary.
  Usual QM rules apply to gravity!

Mesons (Non-confining case)

(w/ D. Mateos, R. Myers, D. Winters)

We need quarks (following Karch and Katz)



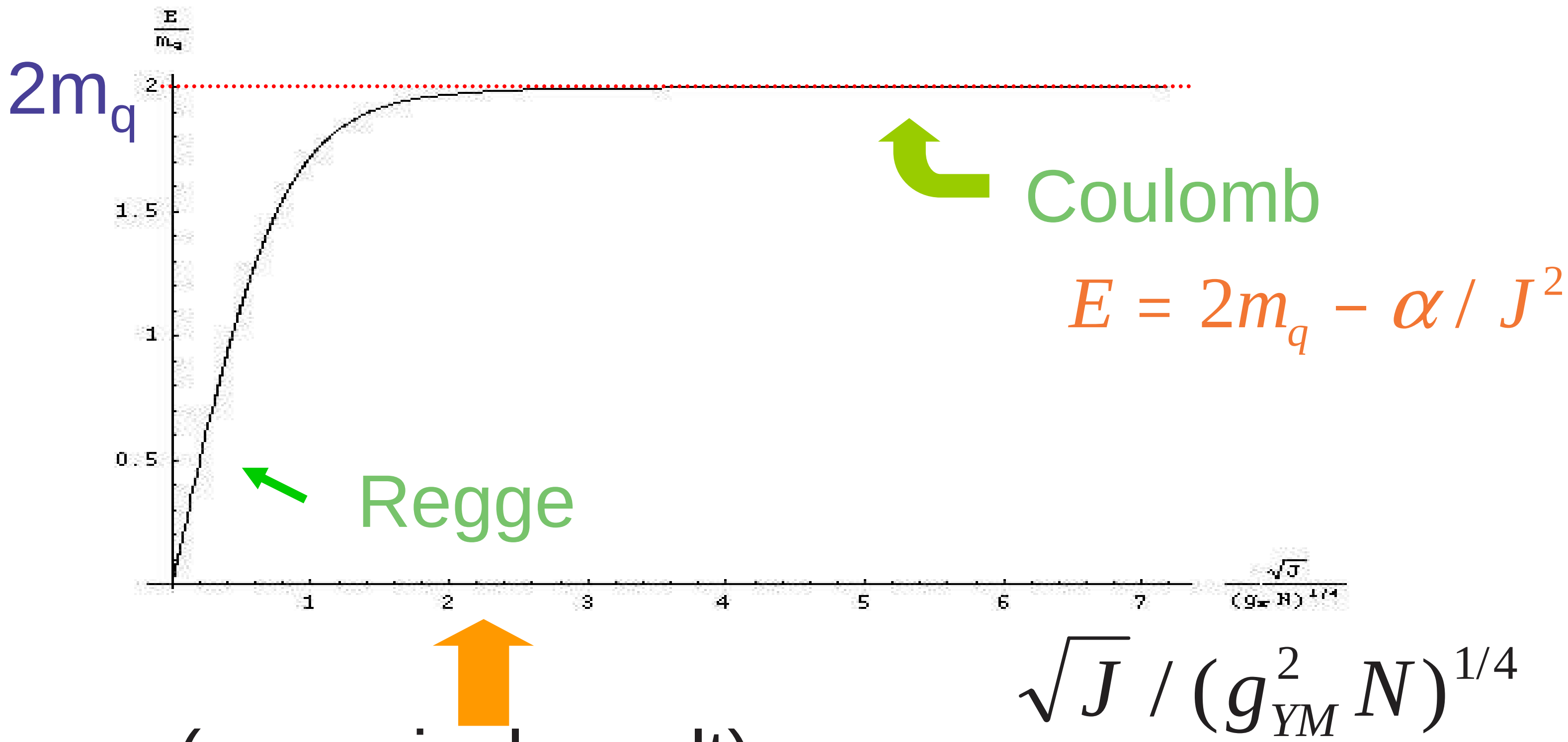
$$ds^2 =$$

$$= \frac{-dt^2 + d\vec{x}^2 + dz^2}{z^2}$$

So, in AdS/CFT, a meson is a string rotating in 5 dim.!

Meson spectrum

($\mathcal{N}$ = 4 is conformal → Coulomb force)



(numerical result)

The cases J=0, ½, 1 are special, very light , namely “tightly bound”. ($E_b \sim 2\, m_q$)

For J=0,1/2,1 we can compute the **exact** spectrum
(in 't Hooft limit and at strong coupling)

2 scalars	$(M/M_0)^2 = (n+m+1)\,(n+m+2)$	$, m \geq 0$
1 scalar	$(M/M_0)^2 = (n+m+1)\,(n+m+2)$	$, m \geq 1$
1 scalar	$(M/M_0)^2 = (n+m+2)\,(n+m+3)$	$, m \geq 1$
1 scalar	$(M/M_0)^2 = (n+m)\,(n+m+1)$	$, m \geq 1$
1 vector	$(M/M_0)^2 = (n+m+1)\,(n+m+2)$	$, m \geq 0$
1 fermion	$(M/M_0)^2 = (n+m+1)\,(n+m+2)$	$, m \geq 0$
1 fermion	$(M/M_0)^2 = (n+m+2)\,(n+m+3)$	$, m \geq 0$

$n \geq 0$; there is a mass gap of order M_0 for $m_q \neq 0$

$$M_0 = \frac{L}{R} = \frac{m_q}{\sqrt{g_{YM}^2 N}} << m_q \qquad for \quad \sqrt{g_{YM}^2 N} >> 1$$

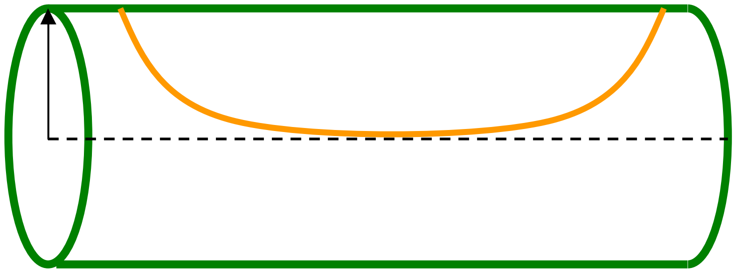
Confining case (w/ D. Mateos, R. Myers, D. Winters)

Add quarks to Witten's confining bkg.

- Spectrum is numerical
- We see $U(1)_A$ chiral symmetry geometrically
- For $m_q=0$ there is a Goldstone boson Φ. ($M_\Phi=0$)
- For $m_q \neq 0$ $$M_\phi^2 = -\frac{m_q}{f_\phi^2}\langle\bar{\psi}\psi\rangle$$ GMOR- relation
- Rot. String (w/ Vaman, Pando-Zayas, Sonnenschein)

reproduces "improved model":

Witten's confining background



Extra dimension τ.
Breaks susy. Two bkgs.
(conf. & non conf.)

Background metric (not AdS any more):

$$ds^2 = \left(\frac{U}{R}\right)^{3/2} \left(\eta_{\mu\nu}\, dx^\mu dx^\nu + f(U) d\tau^2\right) + \left(\frac{R}{U}\right)^{3/2} \frac{dU^2}{f(U)} + R^{3/2} U^{1/2}\, d\Omega_4^2 \,,$$

$$e^\phi = g_s \left(\frac{U}{R}\right)^{3/4} , \qquad F_4 = \frac{N_c}{V_4} \varepsilon_4 \,, \qquad f(U) = 1 - \frac{U_{KK}^3}{U^3} \,.$$

τ U, S^4

D4 branes: 0123456789 + D6 branes (quarks)

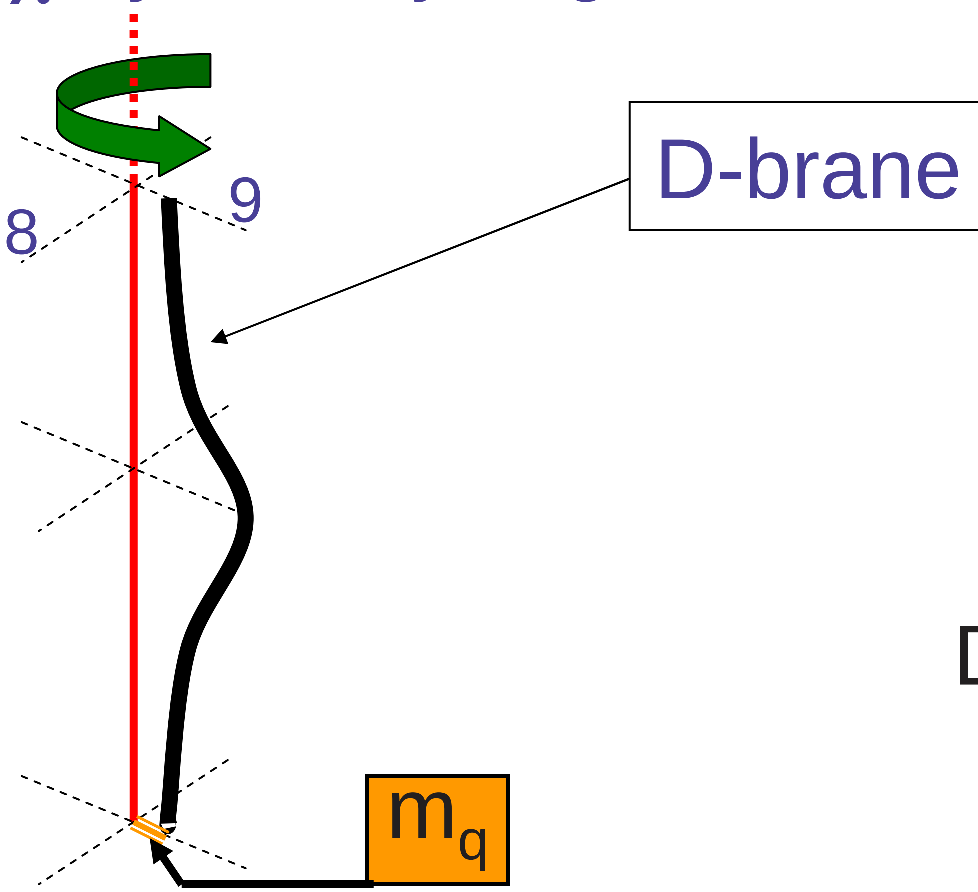
Chiral symmetry breaking $U(1)_A$

$$\psi_L \to e^{i\alpha}\,\psi_L\,,\ \psi_R \to e^{-i\alpha}\,\psi_R$$

$m_q = 0 \rightarrow$ spontaneous

$m_q \neq 0 \rightarrow$ explicit

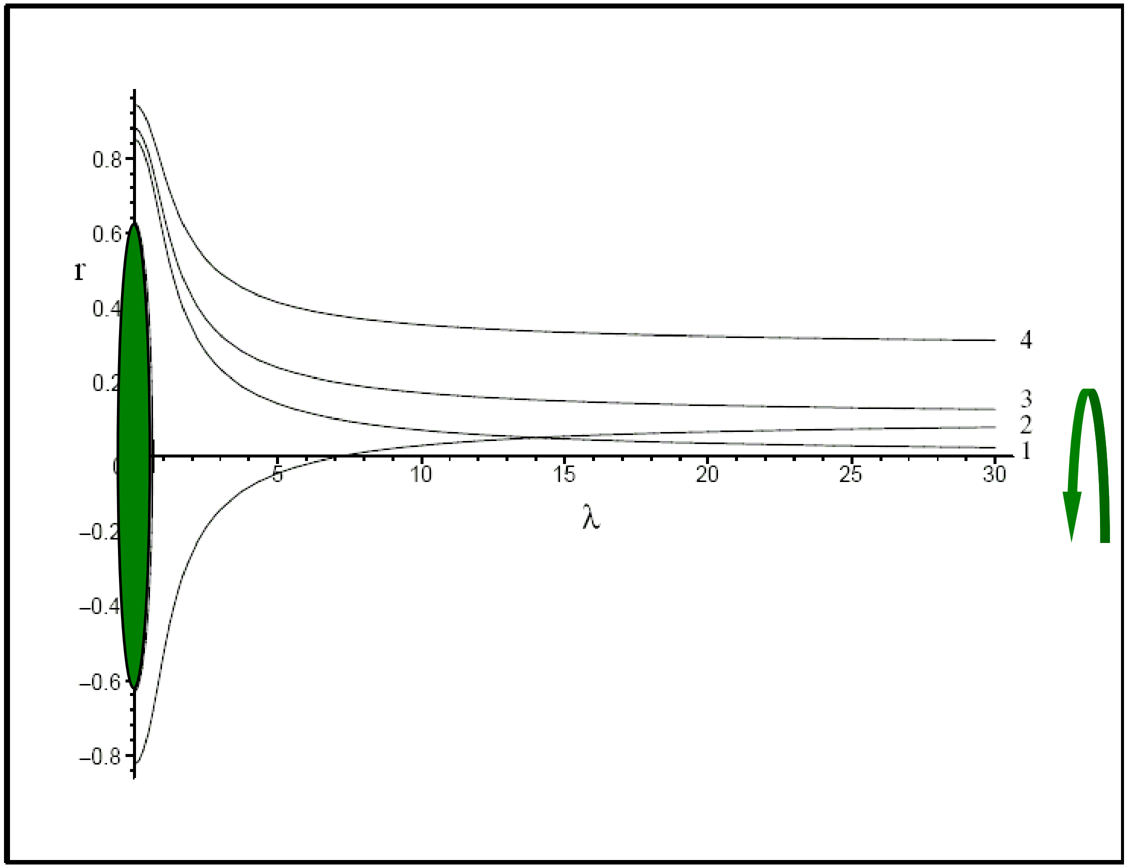
Here χ-symmetry is geometric, a rotation:



0123456789

D6 ------- | -----

Brane embeddings



Quark mass

When m_q=0 there is a massless mode, D-brane rotation

$$r(\lambda) \simeq r_\infty + \frac{c}{\lambda}$$

$$m_{\mathrm{q}} = \frac{L}{2\pi\ell_s^2} = \frac{U_{\mathrm{KK}}\, r_\infty}{2\pi\ell_s^2}$$

$$\frac{1}{N_{\mathrm{c}}}\langle\bar{\psi}\psi\rangle \simeq g_{\mathrm{YM}}^2 N_{\mathrm{c}} M_{\mathrm{KK}}^3 c$$

Quark condensate (as a function of quark mass)

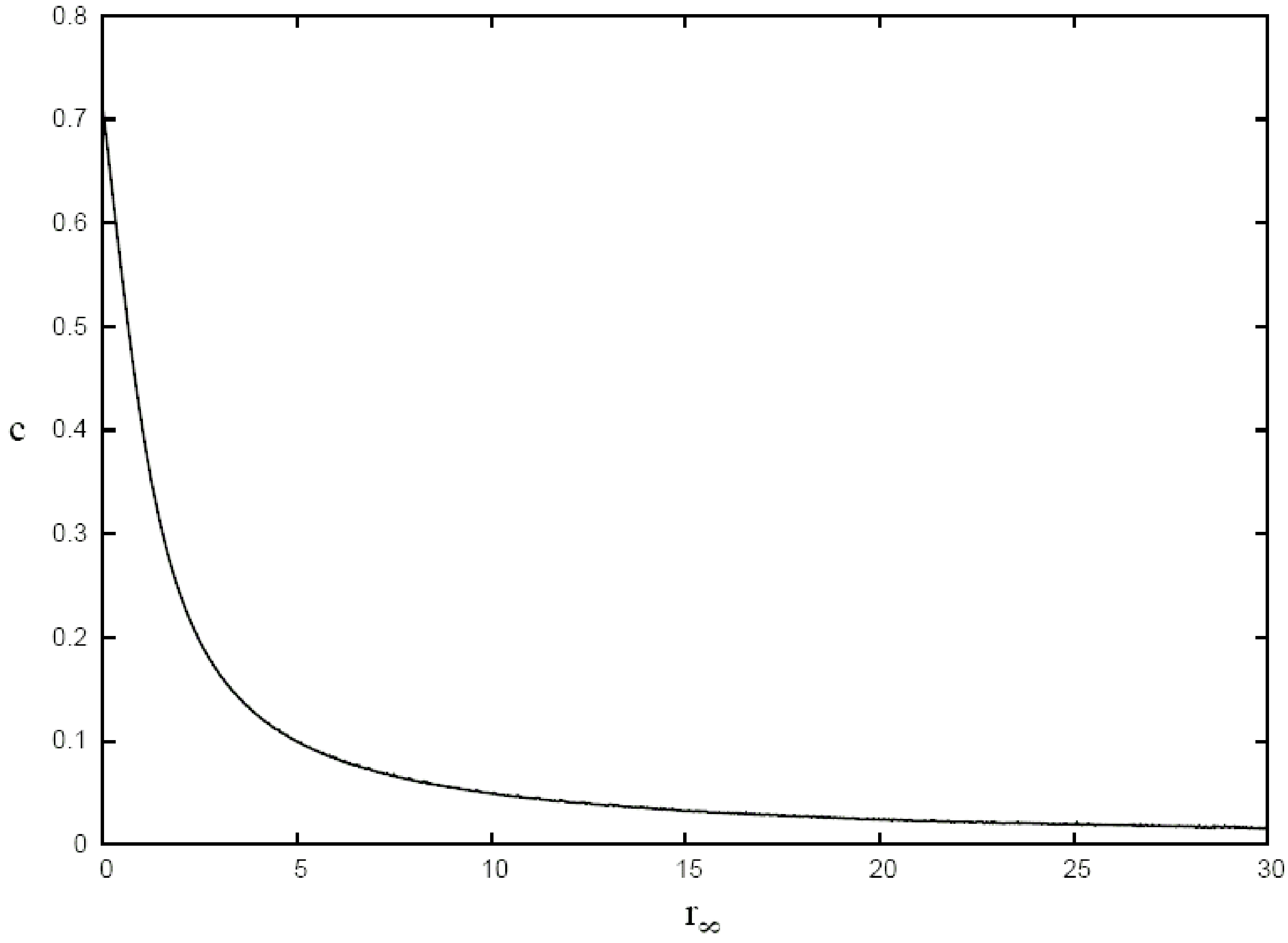


Figure 2: The quark condensate $\langle\bar{\psi}\psi\rangle \propto c$ as a function of the quark mass $m_q \propto r_\infty$.

Creating a condensate costs energy when the quark mass is non-zero

Gell-Mann Oakes Renner relation

When m_q is non-zero the pion acquires a mass given by:

$$M_\pi^2 = -\frac{m_\mathrm{q} \langle \bar{\psi}\psi \rangle}{f_\pi^2}$$

Here we can compute all quantities and it is satisfied.

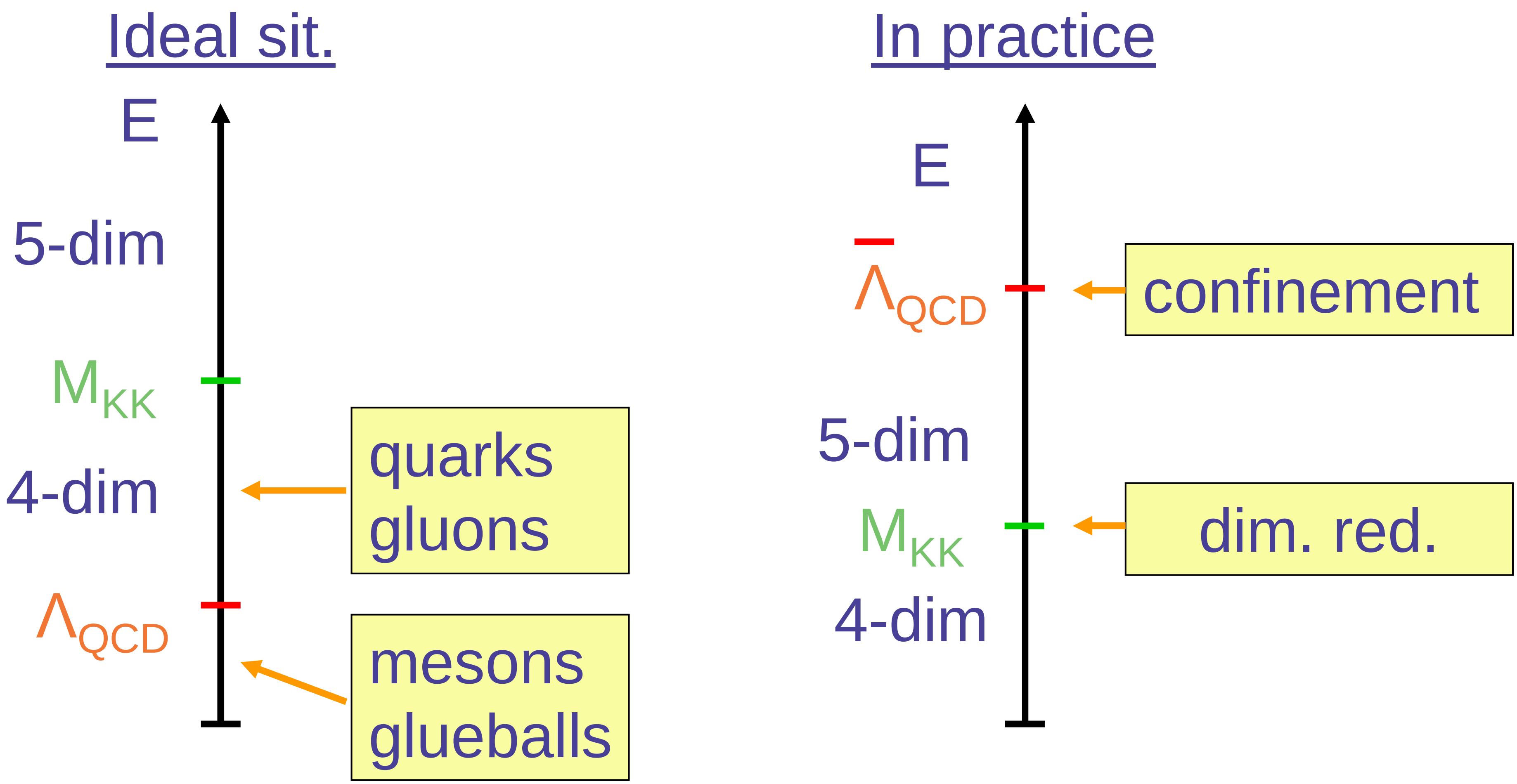
M_π is computed using pert. theory.

f_π is the normalization of ϕ in the eff. action.

$\langle \bar{\psi}\psi \rangle$ was already computed.

We can compute meson spectrum at strong coupling. In the confining case results are similar to QCD, including qualitative features. (Also Sakai-Sugimoto)

How close are we to QCD?



Can we derive the string picture from the field theory?

Study known case: $\mathcal{N} = 4$ SYM

Take two scalars $X = \Phi_1 + i\,\Phi_2\,;\; Y = \Phi_3 + i\,\Phi_4$

$O = \mathbf{Tr}(XX\ldots Y..Y\ldots X)$, J_1 X's , J_2 Y's, J_1+J_2 large

Compute 1-loop conformal dimension of O , or equiv. compute energy of a bound state of J_1 particles of type X and J_2 of type Y (but on a three sphere)

$$R^4 \longleftrightarrow S^3 x R$$

$$\Delta \longleftrightarrow E$$

Large number of ops. (or states). All permutations of Xs and Ys mix so we have to diag. a huge matrix.

Nice idea (Minahan-Zarembo). Relate to a phys. system

Tr(X X…Y X X Y)	$\longleftrightarrow$	$\vert \uparrow\uparrow \ldots \downarrow \uparrow\uparrow\downarrow \rangle$
operator	$\longleftrightarrow$	conf. of spin chain
mixing matrix	$\longleftrightarrow$	op. on spin chain

$$H = \frac{\lambda}{4\pi^2} \sum_{j=1}^{J} \left(\frac{1}{4} - \vec{S}_j \cdot \vec{S}_{j+1} \right)$$

Ferromagnetic Heisenberg model !

Ground state (s)

$|\uparrow\uparrow\ldots\uparrow\uparrow\uparrow\uparrow\rangle \longleftrightarrow \mathbf{Tr}(\,X\,X\ldots X\,X\,X\,X\,)$

$|\downarrow\downarrow\ldots\downarrow\downarrow\downarrow\downarrow\rangle \longleftrightarrow \mathbf{Tr}(\,Y\,Y\ldots Y\,Y\,Y\,Y\,)$

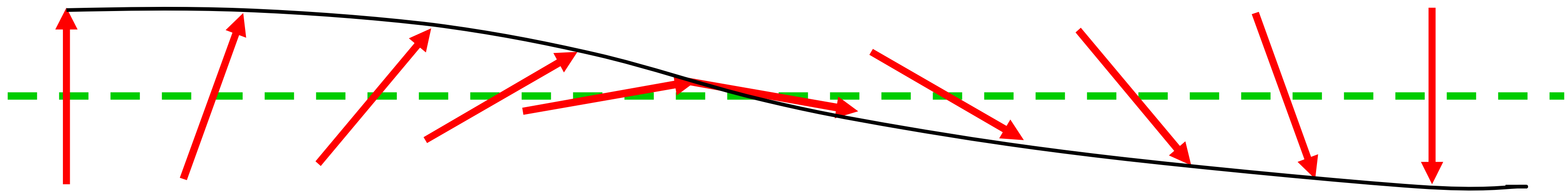
First excited states

$$|k\rangle = \sum e^{ikl}|\uparrow\uparrow\ldots\underset{l}{\downarrow}\ldots\uparrow\uparrow\rangle, \quad k = \frac{2\pi n}{J};\,(J = J_1 + J_2)$$

$$\varepsilon(k) = \frac{\lambda}{J^2}\left(-1+\cos k\right) \xrightarrow[k\to 0]{} \frac{\lambda n^2}{2J^2} \qquad \text{(BMN)}$$

More generic (low energy) states: Spin waves

Other states, e.g. with $J_1=J_2$



Spin waves of long wave-length have low energy and are described by an effective action in terms of two angles θ, φ: direction in which the spin points.

$$S_{eff.} = J\left\{ -\frac{1}{2}\int d\sigma d\tau \cos\theta \partial_\tau \phi - \right.$$

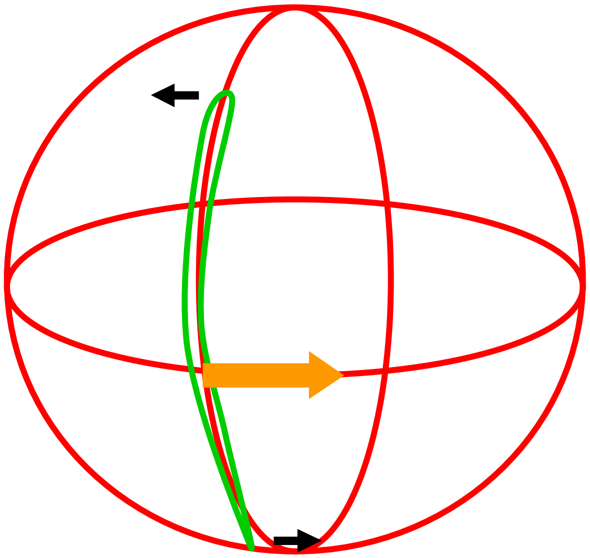
$$\left. -\frac{\lambda}{32\pi J^2}\int d\sigma d\tau \left[(\partial_\sigma \theta)^2 + \sin^2\theta (\partial_\sigma \phi)^2 \right] \right\}$$

Taking J large with λ/J^2 fixed: classical solutions

According to AdS/CFT there is a string description

particle: X(t)   string: X(σ,t)

We need S^3: $\underbrace{X_1^2+X_2^2}_{J_1}+\underbrace{X_3^2+X_4^2}_{J_2} = R^2$



CM: J_1
Rot: J_2

Action: S[θ(σ,t), φ(σ,t)], which,

for large J is: (agrees w/ f.t.)

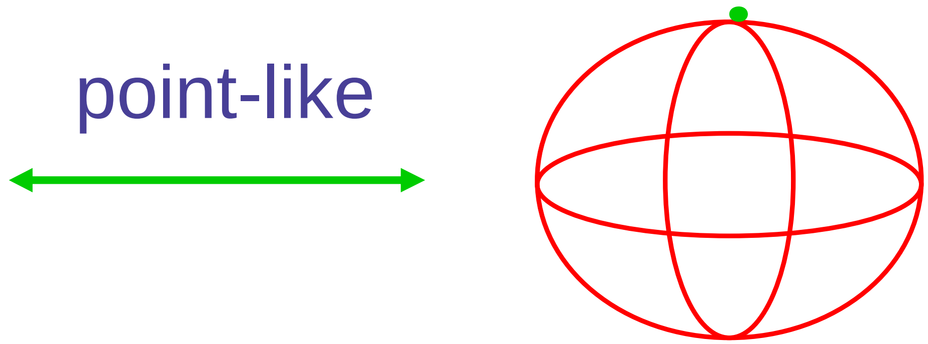
$$S_{eff.} = J\left\{-\frac{1}{2}\int d\sigma\, d\tau \left[\cos\theta\, \partial_\tau \phi - \frac{\lambda}{32\pi J^2}\left[(\partial_\sigma \theta)^2 + \sin^2\theta (\partial_\sigma \phi)^2\right]\right]\right\}$$

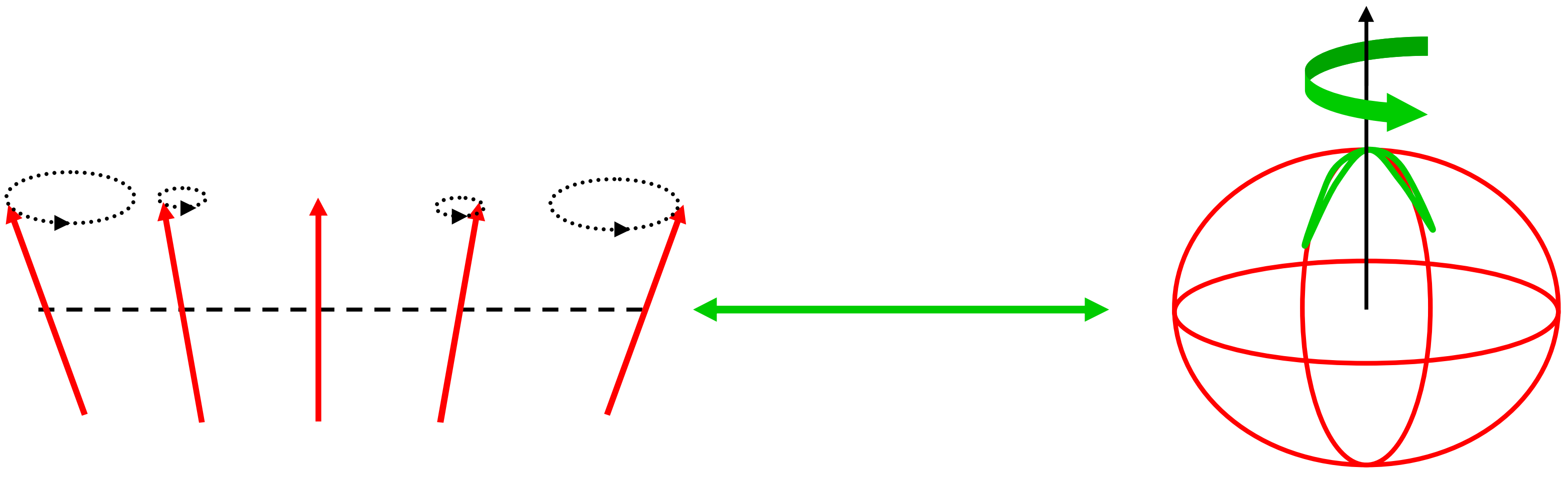
Suggests that (θ, φ) = (θ, φ) namely that

$\langle \vec{S} \rangle$ is the position of the string

<u>Examples</u>

$|\uparrow\uparrow \ldots \uparrow\uparrow\uparrow\uparrow\rangle$





Strings as bound states

Fields create particles:

$$X \rightarrow |\mathbf{x}\rangle \;, \qquad\qquad Y \rightarrow |\mathbf{y}\rangle$$

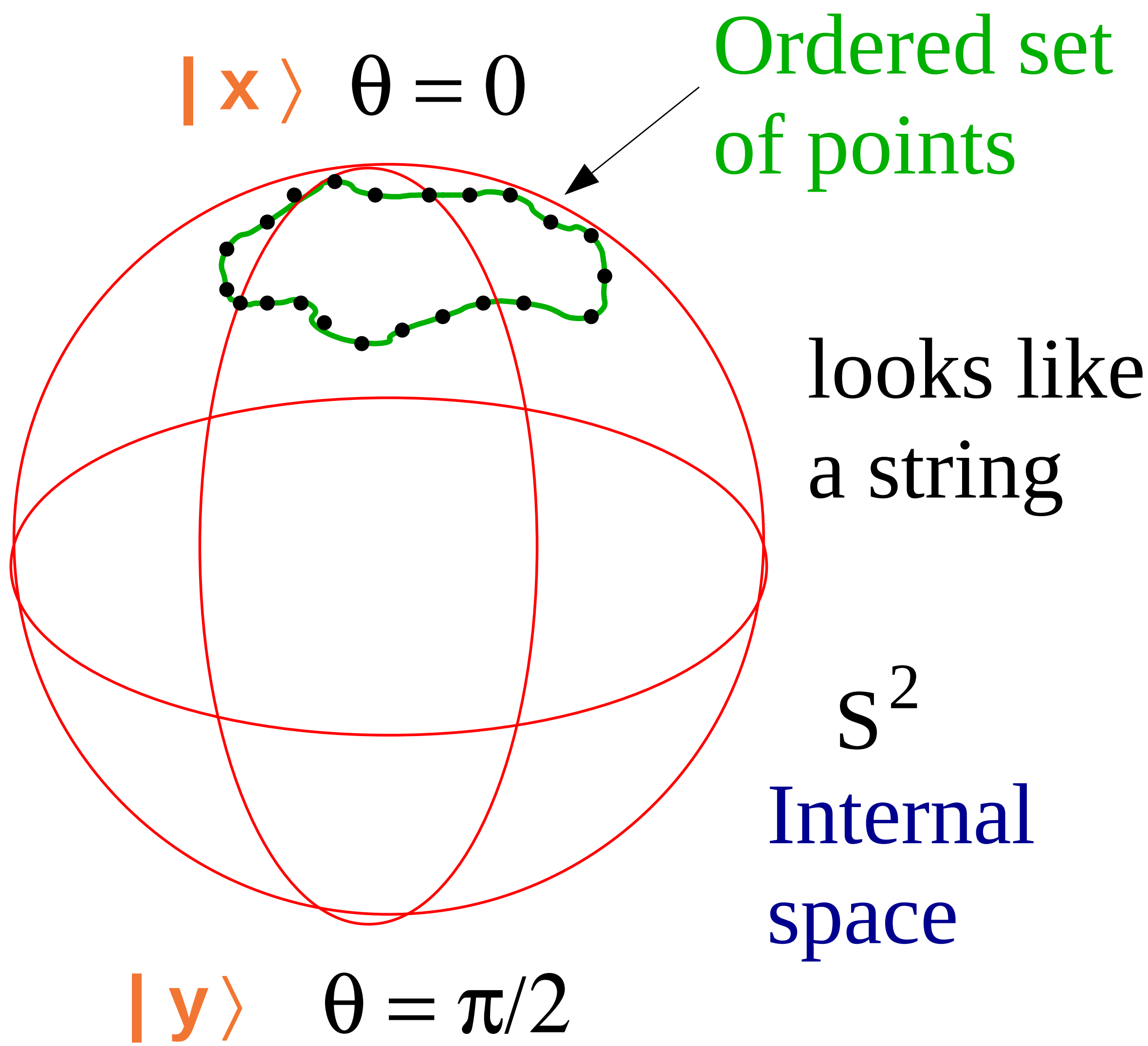
Q.M. : $|\psi\rangle = \cos(\theta/2)\exp(i\,\phi/2)\;|\mathbf{x}\rangle$

$$+ \sin(\theta/2)\exp(-i\,\phi/2)\;|\mathbf{y}\rangle$$

We consider a state with a large number of particles

$i=1\ldots J$ each in a state $v_i = |\psi(\theta_i, \phi_i)\rangle$. (Coherent state)

Can be thought as created by $\mathcal{O} = \mathrm{Tr}\,(v_1\, v_2\, v_3 \ldots v_n)$



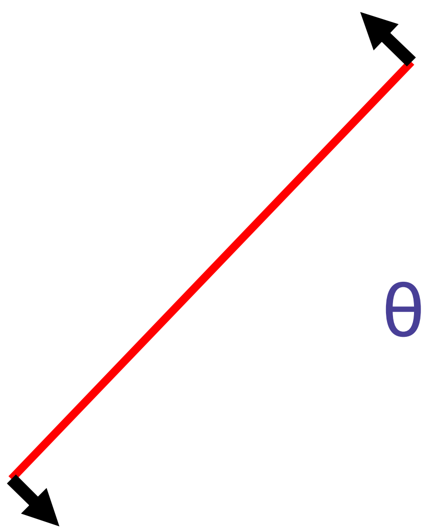
Strings are useful to describe states of a large number of particles (in the large–N limit)

Rotation in AdS$_5$? (Gubser, Klebanov, Polyakov)

$$Y_1^2 + Y_2^2 + Y_3^2 + Y_4^2 - Y_5^2 - Y_6^2 = -R^2$$

$\sinh^2\rho;\ \Omega_{[3]}$ $\qquad$ $\cosh^2\rho;\ t$

$$\boxed{ds^2 = -\cosh^2\rho\, dt^2 + d\rho^2 + \sinh^2\rho\, d\Omega_{[3]}^2}$$



$\theta = \omega\, t$

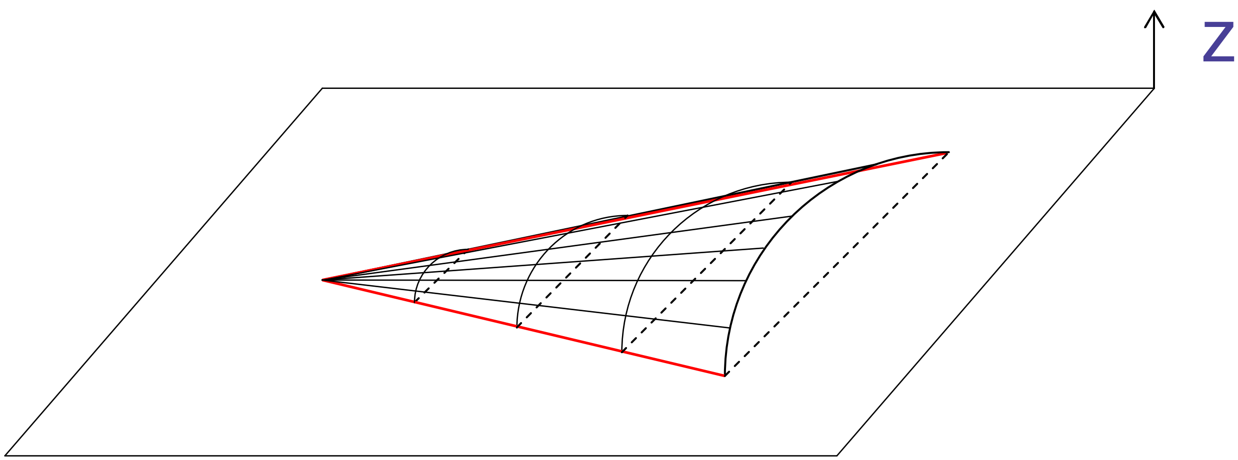
$$E \cong S + \frac{\sqrt{\lambda}}{2\pi}\ln S, \quad (S \to \infty)$$

$$O = Tr\left(\Phi\, \nabla_+^S \Phi\right), \quad x_+ = z + t$$

Verification using Wilson loops (MK, Makeenko)

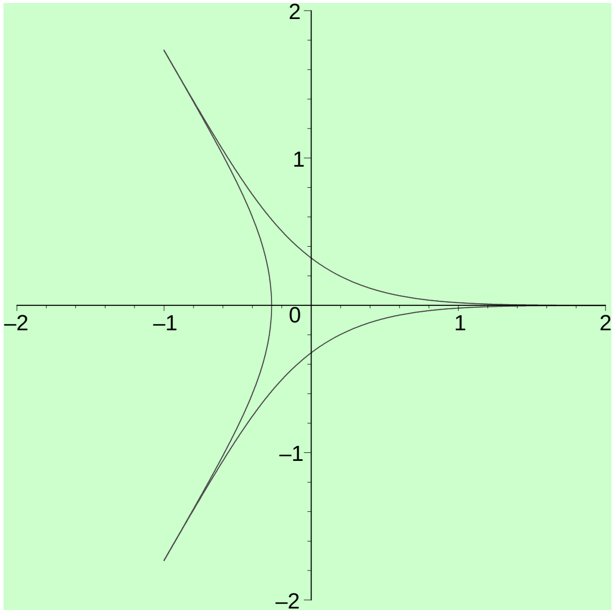
The anomalous dimensions of twist two operators can also be computed by using the **cusp anomaly** of light-like Wilson loops (Korchemsky and Marchesini).

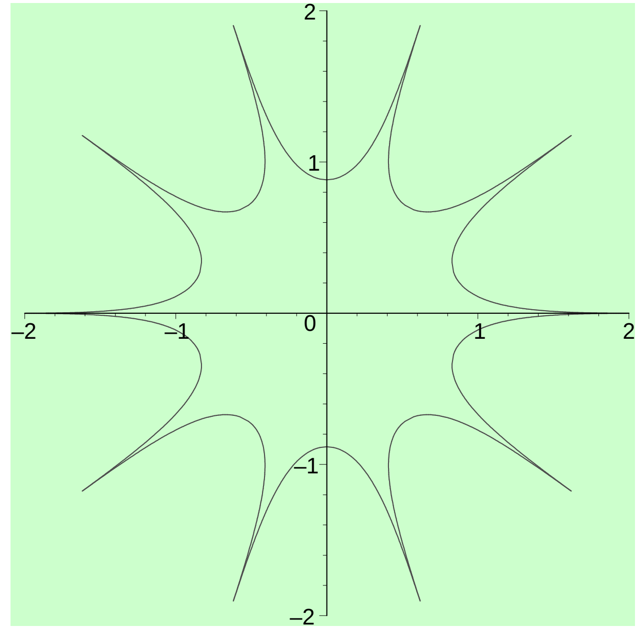
In **AdS/CFT** Wilson loops can be computed using surfaces of minimal area in AdS_5 (Maldacena, Rey, Yee)



The result **agrees** with the rotating string calculation.

Generalization to higher twist operators (MK)





$$E \cong S + \left(\frac{n}{2}\right)\frac{\sqrt{\lambda}}{2\pi}\ln S\,, \quad (S \to \infty)$$

$$O = Tr\left(\nabla_{+}^{S/n}\Phi\ \nabla_{+}^{S/n}\Phi\ \nabla_{+}^{S/n}\Phi \ldots \nabla_{+}^{S/n}\Phi\right)$$

Also Belitsky, Gorsky, Korchemsky

Strings rotating on AdS_5, in the field theory side are described by operators with large spin.

Operators with large spin in the SL(2) sector

Spin chain representation

$$\mathrm{Tr}\left[\nabla_+^{s_1} X\, \nabla_+^{s_2} X\, \nabla_+^{s_3} X \ldots \nabla_+^{s_L} X\right] \quad \rightarrow \quad |s_1 s_2 s_3 \ldots s_L\rangle$$

s_i non-negative integers.

Spin	$S=s_1+\ldots+s_L$
Conformal dimension	E=L+S+anomalous dim.

Belitsky, Korchemsky, Pasechnik described in detail the L=3 case using Bethe Ansatz.

It can be generalized to all loops (Beisert, Eden, Staudacher $\rightarrow$ $E = S + (n/2)\, f(\lambda) \ln S$

Large spin means large quantum numbers so one can use a semiclassical approach (coherent states).

Conclusions

AdS/CFT provides a unique possibility of analytically understanding the low energy limit of non-abelian gauge theories (confinement).

Two results:

- Computed the masses of quark / anti-quark bound states at **strong** coupling.

- Showed a way in which strings directly emerge from the gauge theory.