Shock waves in strongly coupled plasmas

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Based on: arXiv:1004.3803

(S. Khlebnikov, G. Michalogiorgakis, M.K.)

Quantum Gravity in the Southern Cone **V**, *Buenos Aires,2010*

Summary

- **Introduction**

  Shock waves (fluids, plasmas) → black holes

  String / gauge theory duality (**AdS/CFT**)

  Strongly coupled plasmas in AdS/CFT

- **Shock waves in AdS/CFT**

  Dual description

- **More recent work**

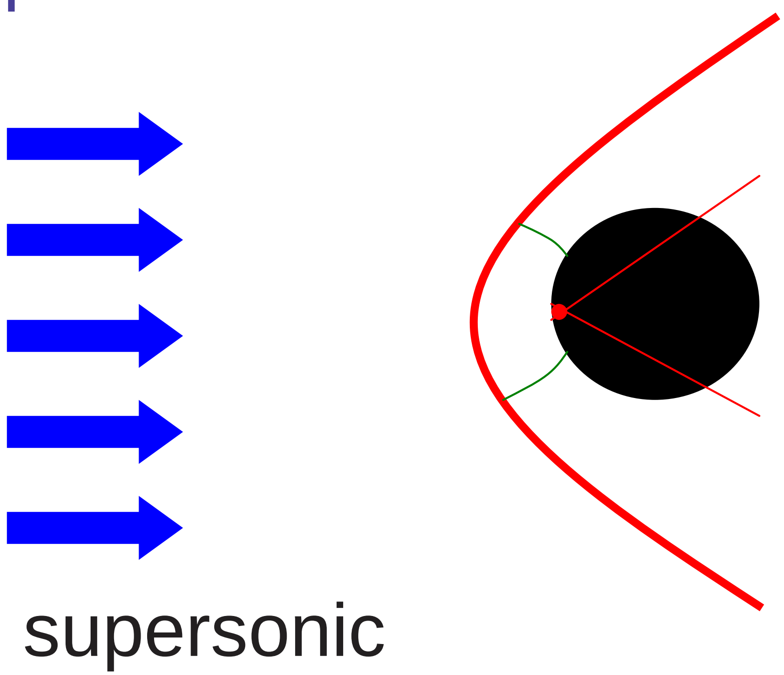
Other dimensionality

Gravity discussion (surface gravity)

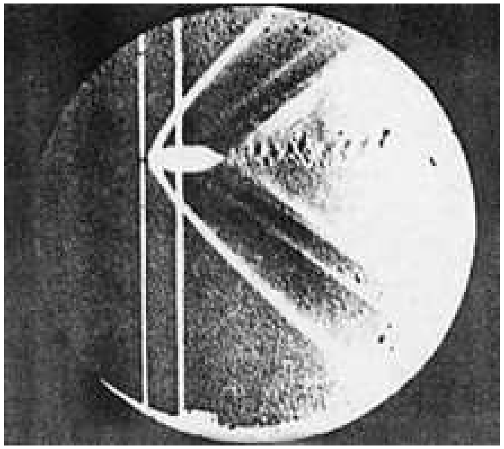
- **Conclusions**

Shock waves in fluids (Landau-Lifshitz)

When an object moves supersonically in a fluid generically creates shock waves. These shocks are perturbations that propagate supersonically and (usually) seen as discontinuities in the hydrodynamic quantities.



Examples



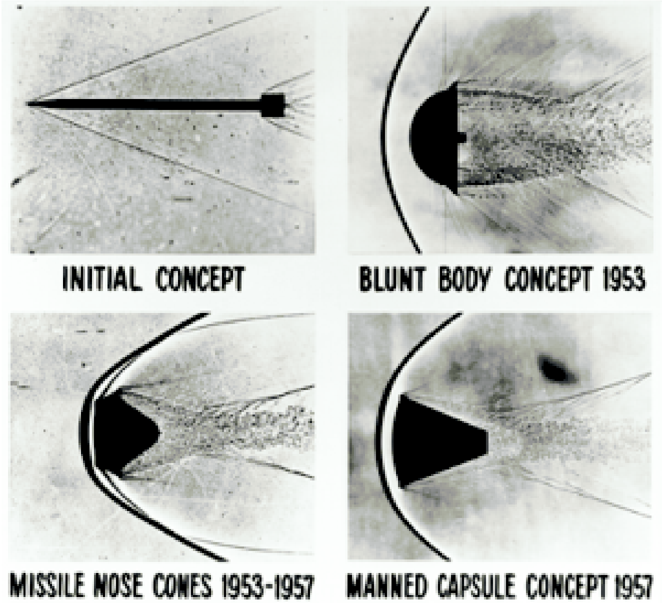
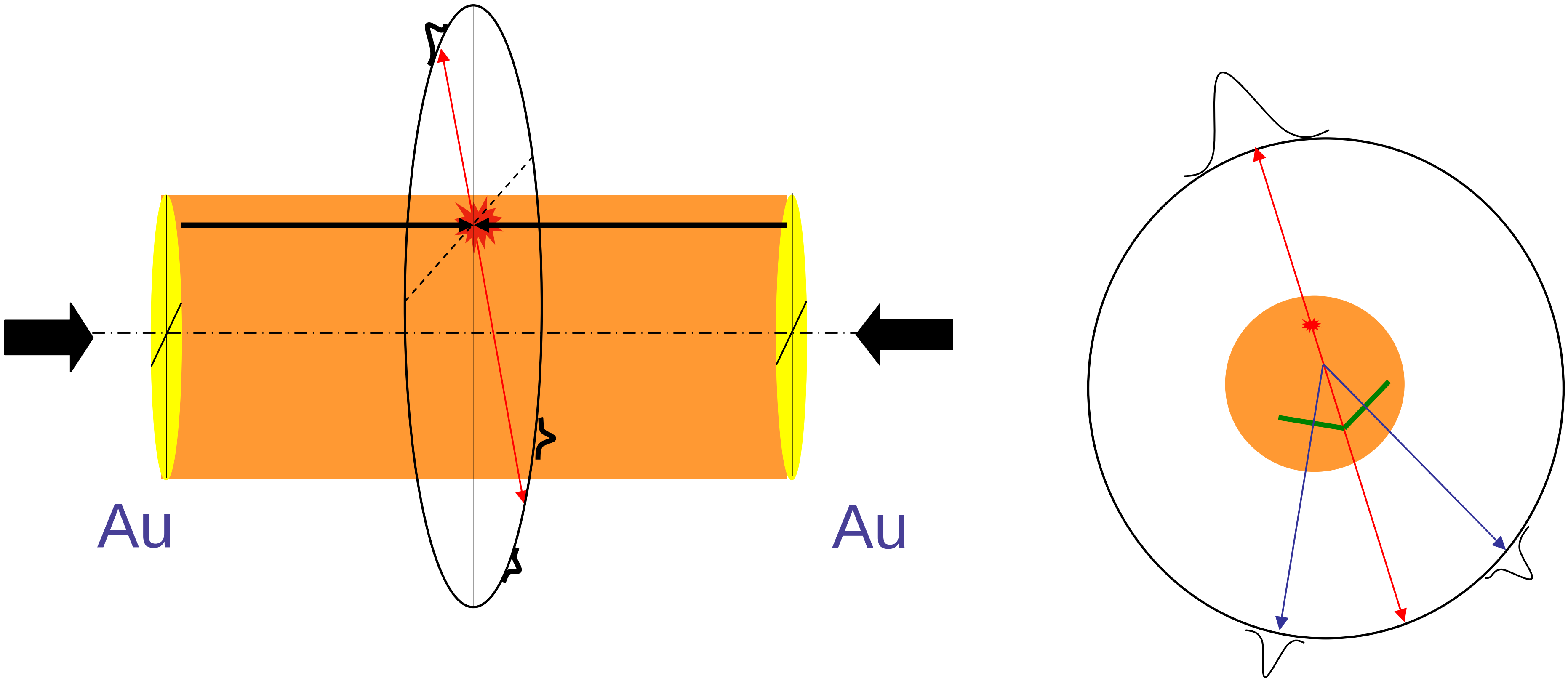


Photo by John Gay

Photo by spacex

Shock waves in plasmas (RHIC Mach cone)

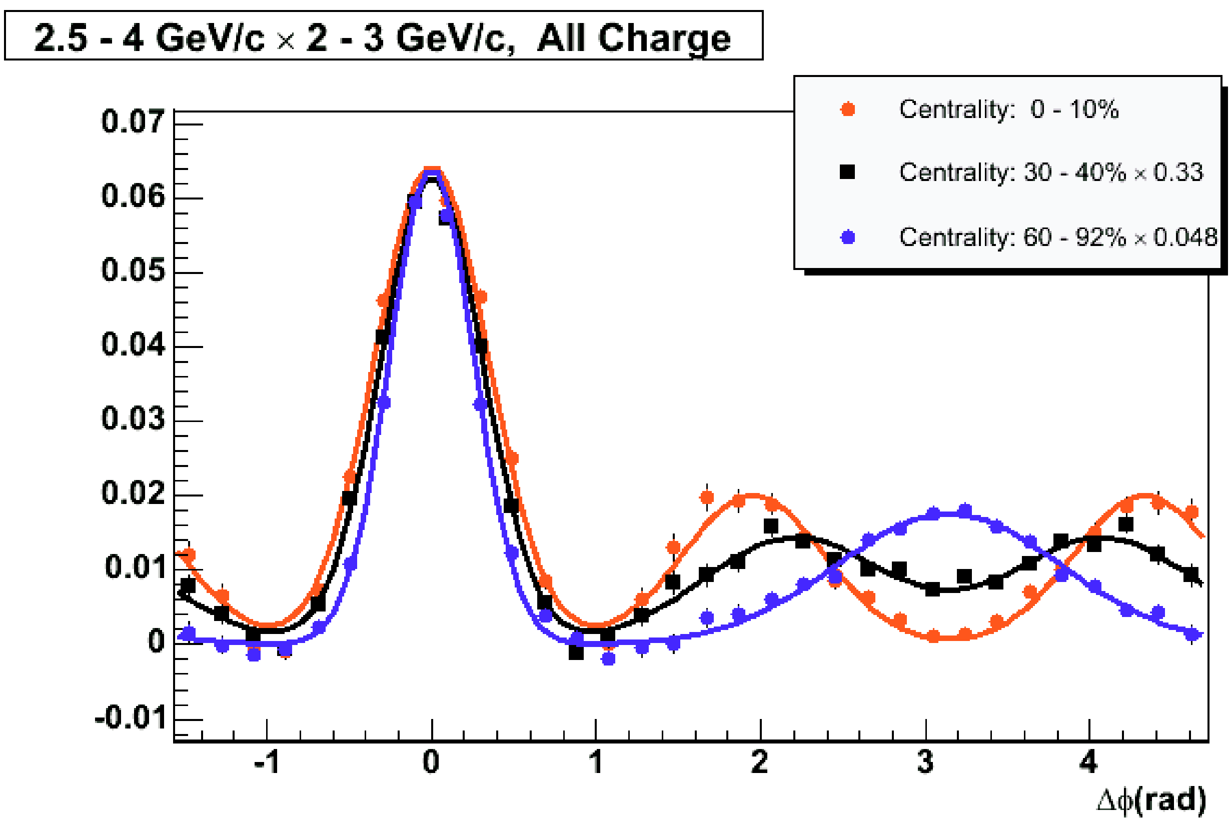
There is recent evidence for the existence of a Mach cone produced by a quark propagating inside the QGP.



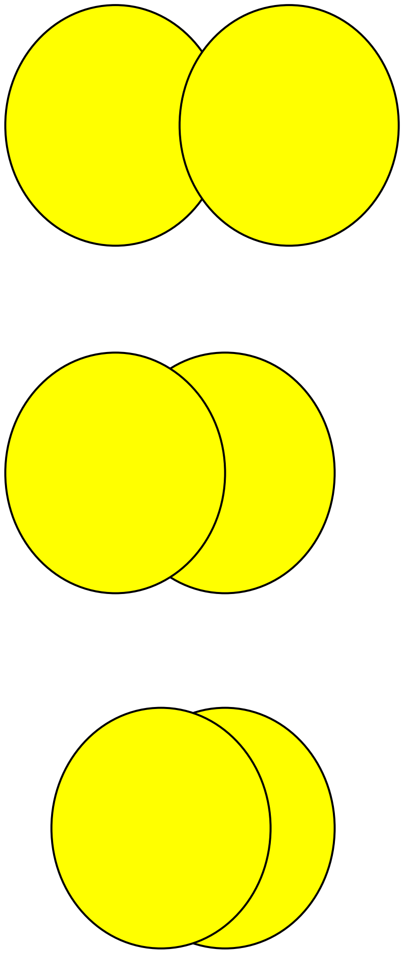
nucl-ex/0404010, Fuqiang Wang (for the STAR coll.)
nucl-ex/0510019, Jiangjong Jia (for the PHENIX coll.) (Figure)

hep-ph/0411315, Casadelrrey-Solana, Shuryak, Teaney.

Also STAR: Three jet correlations.



Talk by H. Buesching, QM2005

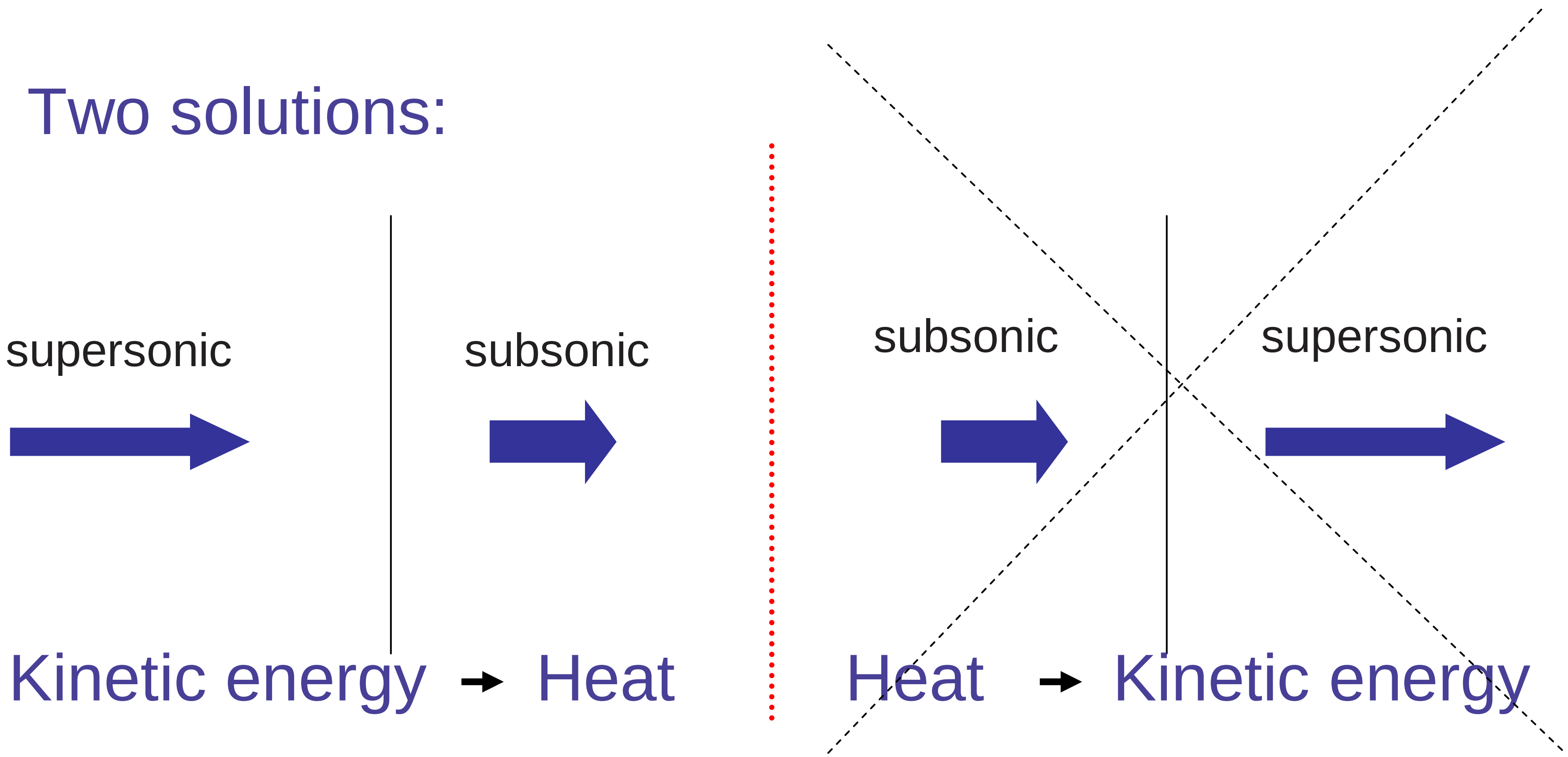


In AdS/CFT: Gubser, Pufu, Yarom, Michalogiorgakis, Friess; Chesler, Yaffe

Description of shock waves

Impose continuity in the energy and momentum across the shock (also charge, particle number, etc.).

Two solutions:



They create entropy even in ideal hydrodynamics.

The discontinuity in the shock wave means that the hydrodynamic description breaks down at the shock. The fields change across distances smaller than the hydrodynamic scale (~ mean free path). A microscopic description can resolve the singularity (e.g. Boltzmann equation).

The AdS/CFT correspondence relates the dynamics of certain fluids with the dynamics of black holes. Hydrodynamics breaks down at a scale $1/T$ but gravity does not. For that reason, gravity should resolve the discontinuity and shock waves should propagate on black holes.

AdS/CFT correspondence (Maldacena)

Gives a precise example of the relation between strings and gauge theory.

Gauge theory	String theory
$\mathcal{N} = 4$ SYM SU(N) on R^4	IIB on $AdS_5 x S^5$
A_μ , Φ^i, Ψ^a	radius R
Operators w/ conf. dim. Δ	String states w/ $E = \frac{\Delta}{R}$

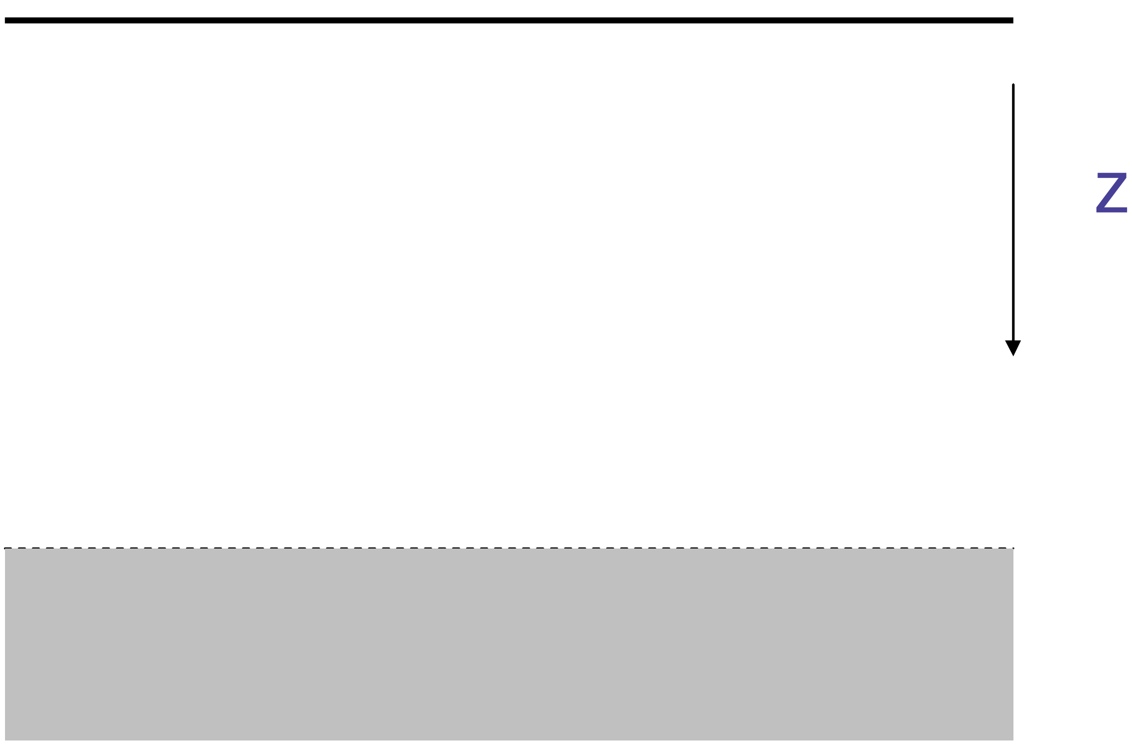
$$g_s = g_{YM}^2 ; \qquad R / l_s = (g_{YM}^2 N)^{1/4}$$

$N \to \infty , \lambda = g_{YM}^2 N$ fixed ➡

λ large → string th.
λ small → field th.

AdS/CFT correspondence and plasma physics

$\mathcal{N}$ =4 SYM at finite temperature is a conformal plasma which is dual to a black hole in AdS space.



Hydrodynamics of the $\mathcal{N}$ =4 conformal plasma

Low energy excitations of a plasma are given by temperature variations and displacements characterized by T and u_μ (4 variables). $T_{\mu\nu}$ has nine indep.comp. We define T and u_μ through:

$$T^{\mu\nu}u_\nu = -3(\pi T)^4 u^\mu$$

There are 3 more eigenvectors u_1, u_2, u_3 (3 variables) and 3 eigenvalues λ_1, λ_2, λ_3 (2 variables). (4+3+2=9)

Hydrodynamics determines these 5 variables in terms of u_μ and T. Then we can use 4 equations:

$$\partial_\mu T^{\mu\nu} = 0$$

Hydrodynamics from gravity (Battacharyya, Hubeny, Minwalla, Rangamani)

For any conserved $T_{\mu\nu}$ we can find a dual metric $g_{\mu\nu}$ (asymptotically AdS + $\delta g_{\mu\nu} \rightarrow T_{\mu\nu}$). However those metrics are generically singular. For long wavelengths (along the boundary direction) we can systematically find the $T_{\mu\nu}$ that gives rise to a non-singular metric.

$$T^{\mu\nu} = (\pi T)^4 \left(\eta^{\mu\nu} + 4u^\mu u^\nu\right) - 2(\pi T)^3 \sigma^{\mu\nu} + (\pi T)^2 \left((\ln 2) T_{2a}^{\mu\nu} + 2T_{2b}^{\mu\nu} + (2 - \ln 2)\left(\frac{1}{3} T_{2c}^{\mu\nu} + T_{2d}^{\mu\nu} + T_{2e}^{\mu\nu} \right) \right)$$

Policastro, Son, Starinets

$$\sigma^{\mu\nu} = P^{\mu\alpha} P^{\nu\beta} \partial_{(\alpha} u_{\beta)} - \frac{1}{3} P^{\mu\nu} \partial_\alpha u^\alpha$$

$$P^{\mu\nu} = \eta^{\mu\nu} + u^\mu u^\nu$$

$$T_{2a}^{\mu\nu} = \epsilon^{\alpha\beta\gamma(\mu}\sigma_{\gamma}^{\nu)}u_\alpha l_\beta \, , T_{2c}^{\mu\nu} = \partial_\alpha u^\alpha \sigma^{\mu\nu} \, ,$$

$$T_{2b}^{\mu\nu} = \sigma^{\mu\alpha}\sigma_{\alpha}^{\nu} - \frac{1}{3}P^{\mu\nu}\sigma^{\alpha\beta}\sigma_{\alpha\beta} \, ,$$

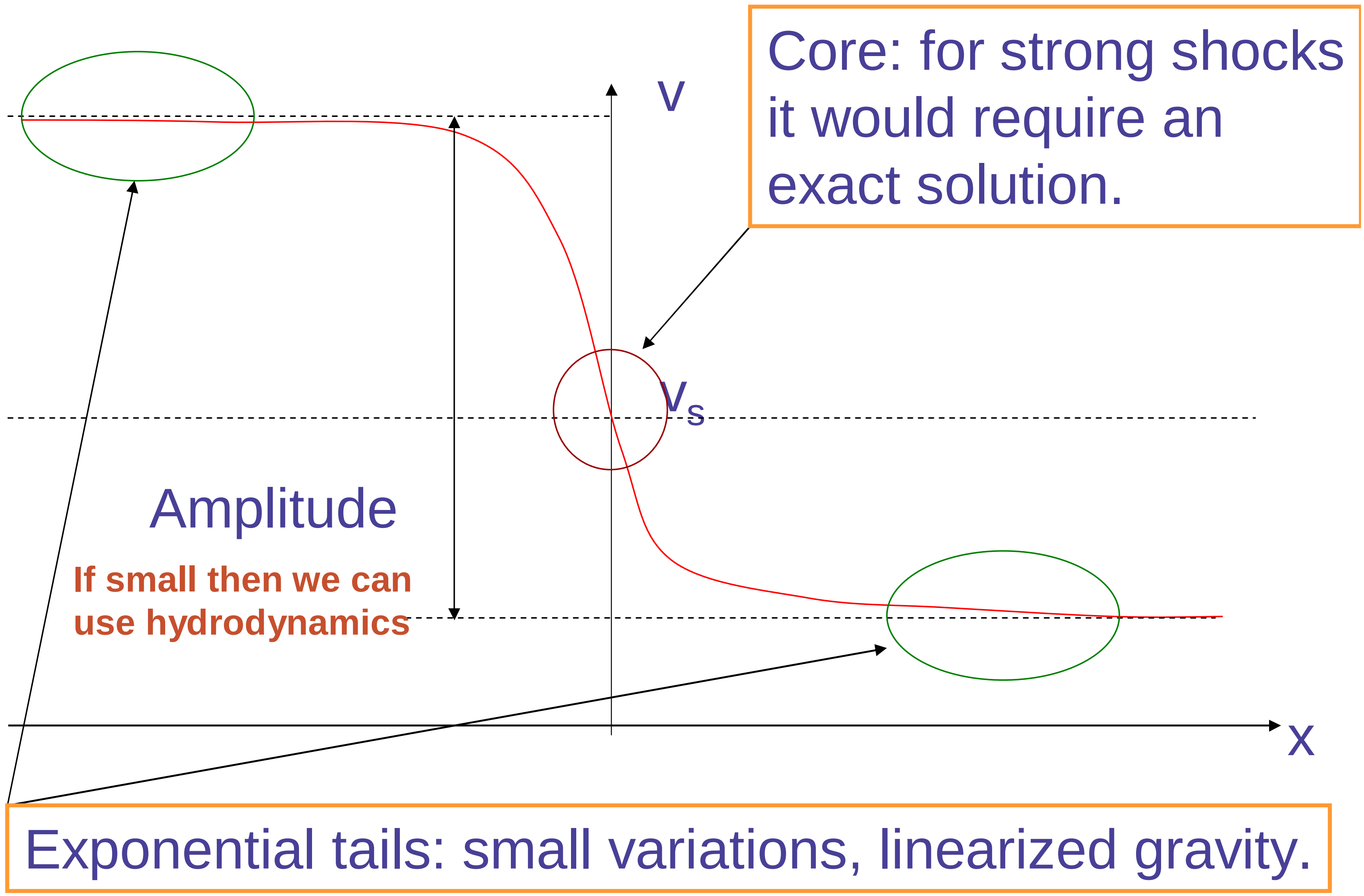
$$T_{2d}^{\mu\nu} = \mathcal{D}^\mu \mathcal{D} u^\nu - \frac{1}{3}P^{\mu\nu}\mathcal{D}u^\alpha \mathcal{D}u_\alpha \, ,$$

$$T_{2e}^{\mu\nu} = P^{\mu\alpha}P^{\nu\beta}\mathcal{D}\left(\partial_{(\alpha}u_{\beta)}\right) - \frac{1}{3}P^{\mu\nu}P^{\alpha\beta}\mathcal{D}\left(\partial_\alpha u_\beta\right)$$

$$\mathcal{D} = u^\alpha \partial_\alpha$$

$$l_\mu = \epsilon_{\alpha\beta\gamma\mu}u^\alpha \partial^\beta u^\gamma$$

Gravitational resolution of the shock waves



Weak Shock waves in (gravitational) hydrodynamics

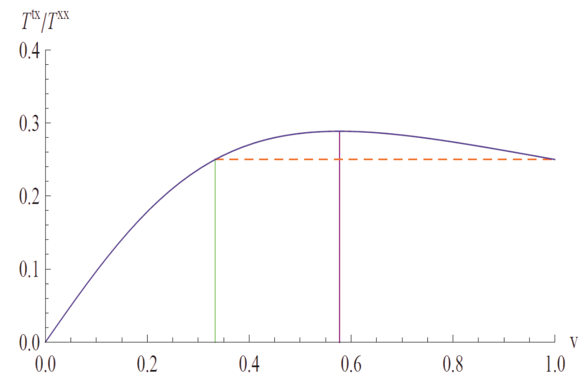
Ideal hydro $T^{\mu\nu} = (\pi T)^4 \left(\eta^{\mu\nu} + 4u^\mu u^\nu\right)$

$$v_s = \frac{1}{\sqrt{3}}$$

Matching

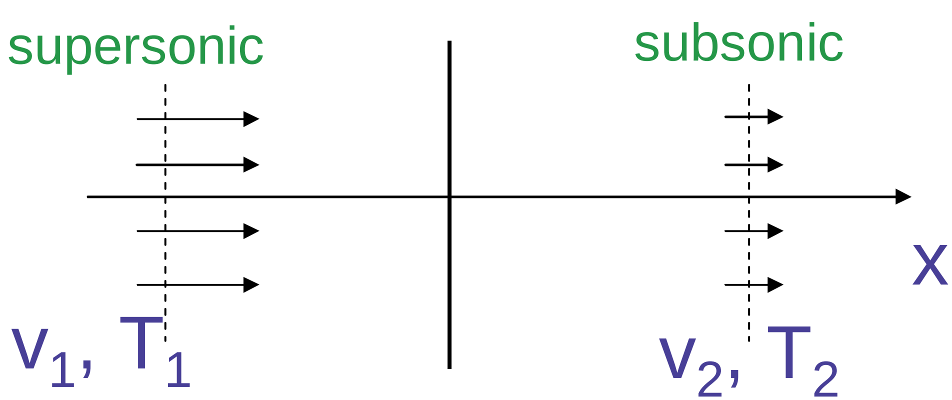
$$T^{tx} = 4pu^t u^x = 4p\frac{v}{1-v^2}\,,$$

$$T^{xx} = p\left(1+4u_x^2\right) = p\frac{1+3v^2}{1-v^2}\,.$$



gives

$$v_2 = \frac{1}{3v_1}\,, \quad p_2 = p_1\frac{9v_1^2-1}{3(1-v_1^2)}\,, \quad T_2 = T_1\left(\frac{9v_1^2-1}{3(1-v_1^2)}\right)^{1/4}$$

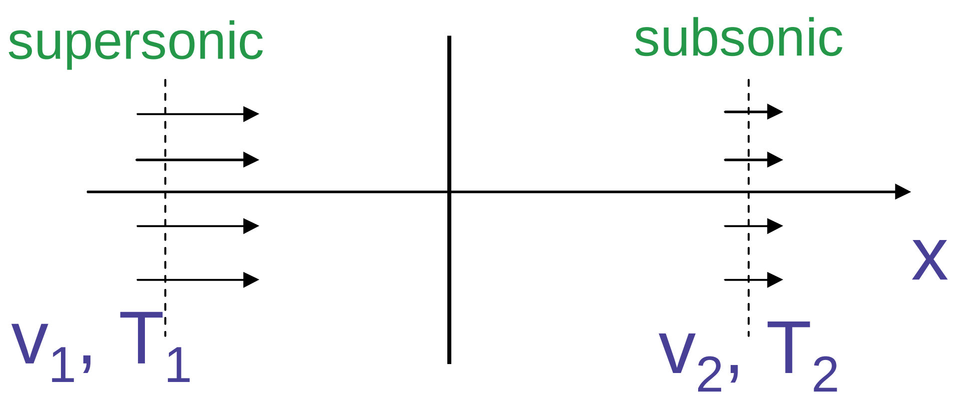


Entropy generation:

Total entropy in a volume is constant for a stationary solution but more entropy comes out than in. Therefore entropy is generated.

$$s_\mu = 4\pi^4 T^3 u_\mu$$

$$\Delta s^x = s^x_{subsonic} - s^x_{supersonic} = \frac{4\pi^2 T_1^3}{\sqrt{1-v_1^2}}\left(3^{-3/4}\left(\frac{9v_1^2-1}{1-v_1^2}\right)^{1/4} - v_1\right)$$



Higher order hydrodynamics: (ν breaks time rev.)

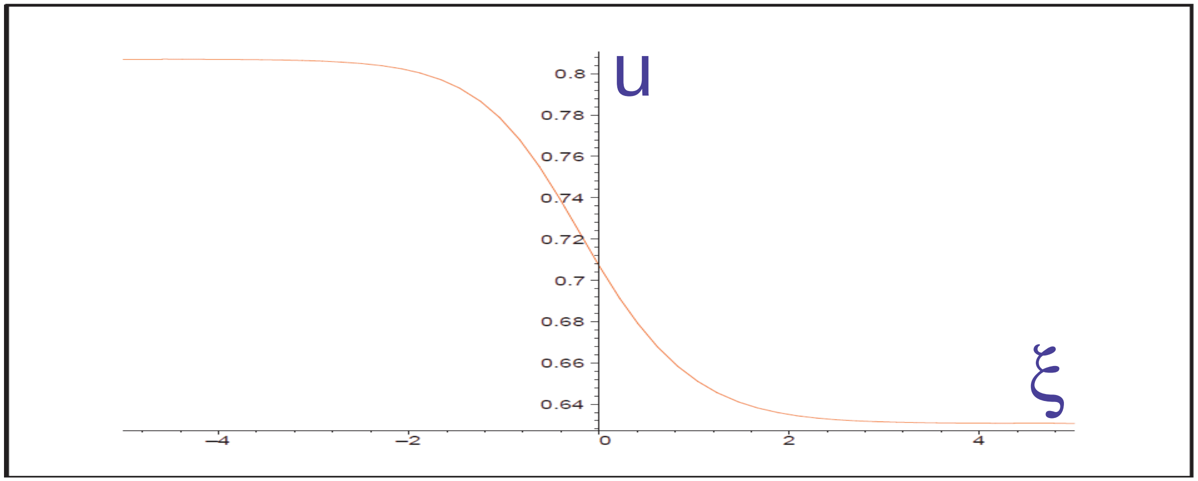
We perform a systematic expansion in the amplitude of the shock: $u = u_{(0)} + u_{(1)} + u_{(2)}$

$$u_{(0)} = \frac{1}{\sqrt{2}}\,,$$

$$u_{(1)} = -u_\infty \tanh\xi\,,$$

$$u_{(2)} = \frac{u_\infty^2}{6}\left[4\sqrt{2}(1-\ln 2)\frac{\ln\cosh\xi}{\cosh^2\xi} + 5\sqrt{2}\left(\tanh^2\xi + \tanh\xi + \frac{\xi}{\cosh^2\xi}\right)\right]$$

$$T = T_{(0)} - \frac{\sqrt{2}}{3}T_{(0)}u_{(1)} - \frac{\sqrt{2}}{3}T_{(0)}u_{(2)} + \frac{1}{3\pi}u'_{(1)}$$



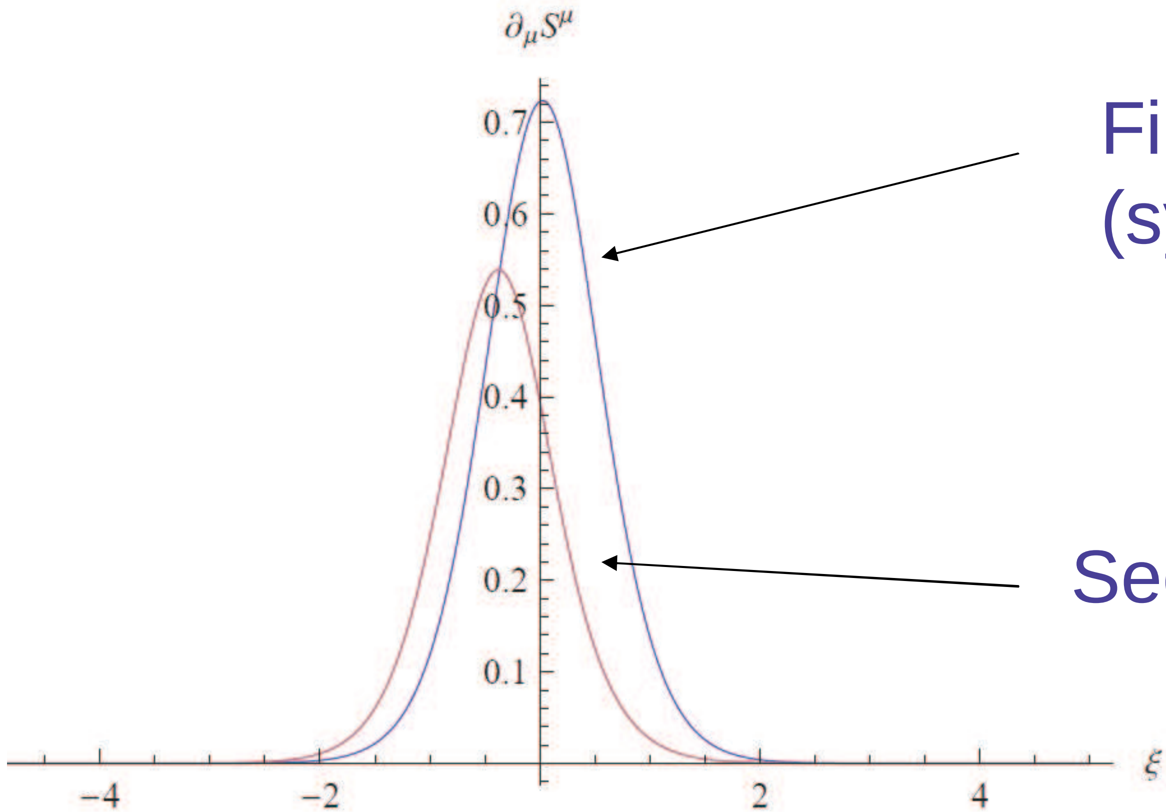
$$u = \frac{v}{\sqrt{1-v^2}}$$

$$\xi = \frac{4\pi T_{(0)}u_\infty}{3}x$$

Entropy generation:

Current: $s^\mu = 4\pi\eta u^\mu - \frac{\tau_\pi \eta}{4T}\sigma^{\kappa\nu}\sigma_{\kappa\nu}u^\mu$ (Loganayagam)

$$\partial_\mu s^\mu = \frac{\eta}{2T}\sigma^{\mu\nu}\sigma_{\mu\nu} \; .$$



First order hydro (symmetric)

Second order hydro

Gravity dual: (using BHMR construction)

$$ds^2 = -2u_\mu dx^\mu dr + \frac{1}{r^2}((\pi T)^4 + k)u_\mu u_\nu dx^\mu dx^\nu + r^2\eta_{\mu\nu}dx^\mu dx^\nu + \frac{1}{r^2}j\tilde{u}_\mu u_\nu dx^\mu dx^\nu$$
$$+\alpha r^2(\tilde{u}_\mu \tilde{u}_\nu dx^\mu dx^\nu - \frac{1}{2}(dy^2 + dz^2)) ,$$

$$\tilde{u}^\mu = (u(x), u^0(x), 0, 0)$$

$$k = \frac{2}{3}r^3(u'_{(1)} + u'_{(2)}) - \frac{\sqrt{2}}{3}r^2 u''_{(1)} ,$$

$$j = -\frac{2}{\sqrt{3}}r^3(u'_{(1)} + u'_{(2)}) + \frac{4\sqrt{2}}{3\sqrt{3}}u_{(1)}u'_{(1)}(\pi T_{(0)})^3 F_2\left(\frac{r}{\pi T_{(0)}}\right) ,$$

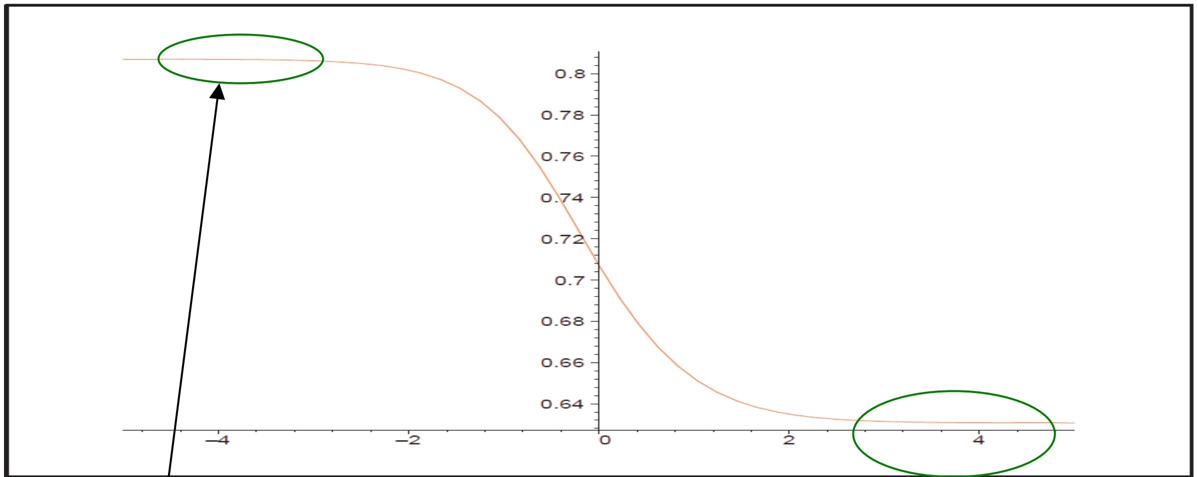
$$\alpha = \frac{u'_{(1)} + u'_{(2)}}{3\pi T_{(0)}}F_1\left(\frac{r}{\pi T_{(0)}}\right) - \frac{4\sqrt{2}}{9\pi T_{(0)}}u_{(1)}u'_{(1)}F_3\left(\frac{r}{\pi T_{(0)}}\right) ,$$

$$F_1(y) = \ln\left(\frac{(1+y^2)(1+y)^2}{y^4}\right) - 2\arctan y + \pi$$

$$F_2(y) = \frac{1}{2}(y^4 - 1)\left(2\arctan y + \ln\left(\frac{1+y^2}{(1+y)^2}\right)\right) - \frac{1}{2}\pi y^4 + y^3 + y^2 - \frac{25}{12} ,$$

$$F_3'(y) = \frac{1}{y^5 - y}\left\{2(1-y^3)\left(\arctan y - \frac{\pi}{2} - \ln(1+y)\right) + (1+y^3)\ln(1+y^2)\right.$$
$$\left. -4y^3\ln y + 7 - 2\ln 2 - \frac{2(1+y)}{1+y^2} - \frac{2}{1+y} - 4y^2\right\} .$$

Strong shock waves (linearized gravity)



$v=v_1+\delta v_1$

$v=v_2+\delta v_2$

Even for strong shocks $\delta v_1/v_1 << 1$ ($\delta v_2/v_2 << 1$)
We can use linearized gravity.

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}, \quad h_{\mu\nu}(t,x,r) = r^2 H_{\mu\nu}(r) e^{-i\omega t + iqx}$$

Boosted black hole

$\omega=0$, q imag.

Boundary conditions:

Infalling boundary conditions at the horizon are analytically continued to give the correct b.c. breaking the time reversal symmetry leading to the correct shock wave. Equivalently we ask regularity in Eddington-Filkenstein coordinates.

$$ds_0^2 = r^2 \eta_{\mu\nu} dx^\mu dx^\nu + \frac{r_0^4}{r^2}(dt \cosh\beta - dx \sinh\beta)^2 + \frac{dr^2}{r^2(1 - r_0^4/r^4)}$$

$$ds_1^2 = r^2 \left[H_{00} dt^2 + H_{11} dx^2 + 2H_{01} dt dx + H(dy^2 + dz^2)\right] e^{iqx}$$

$$Z(r) = H_{00}(r) + \left(1 + \frac{r_0^2}{r^4}\gamma^2\right) H(r)$$

$$Z'' + P(u)Z' + Q(u)Z = 0$$

$$P(u) = \frac{3 + 3u^2 - 5\gamma^2 u^2 + 3\gamma^2 u^4}{u f(u)(\gamma^2 u^2 - 3)},$$

$$Q(u) = -\frac{4\gamma^2 u^2}{f(u)(\gamma^2 u^2 - 3)} + q^2 \frac{\gamma^2 u^2 - 1}{4u f^2(u)}$$

$$u = r_0^2/r^2 \qquad \gamma = \cosh\beta$$

$$f(u) = 1 - u^2$$

Results:

$v = 0$ (fluid at rest) $iq = 2.3361$

$v = 1/\sqrt{3}$ (the speed of sound) $q = 0,\ Z(u) = u^2$

$v = \sqrt{2/3}$ (the singular point) $iq = \sqrt{2}$

$v \to 1$ (ultrarelativistic limit) $iq_0(v) = 1.895\sqrt{\gamma}$

$$x = \frac{u^2}{u_1^2} = \frac{1}{3}u^2 \cosh^2 \beta$$

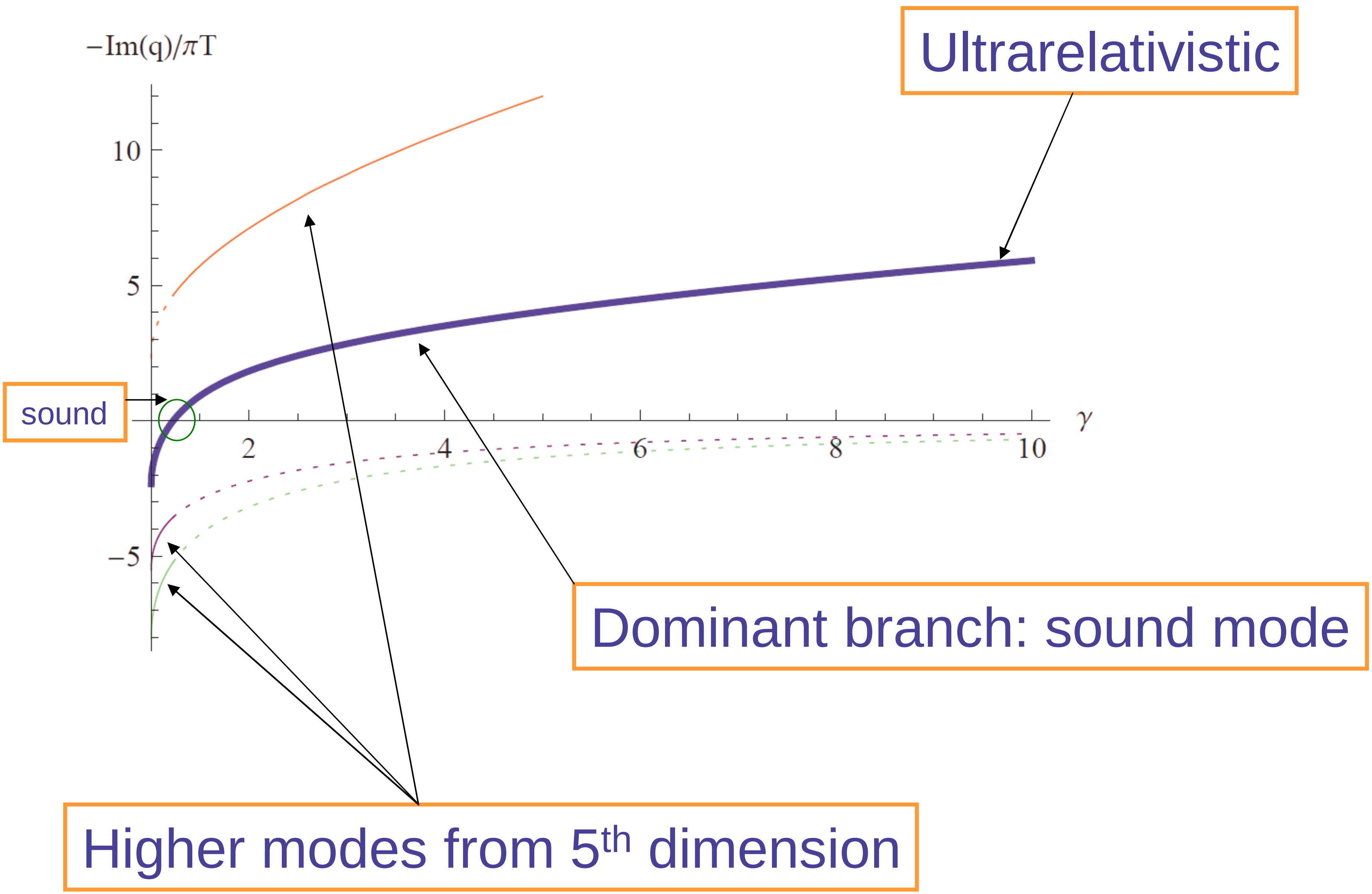
$$\gamma = \frac{1}{\sqrt{1-v^2}}$$

Scaling

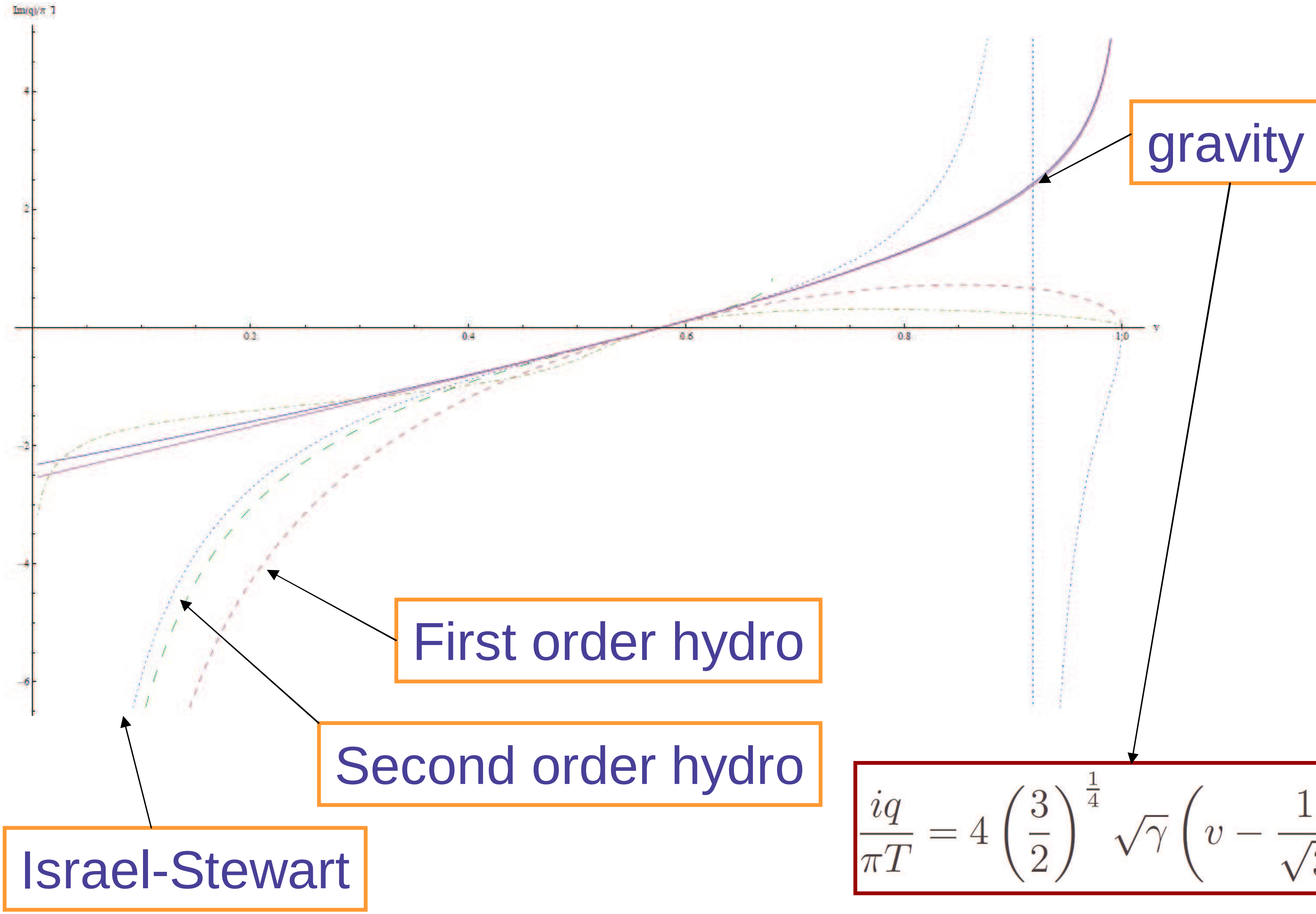
$$Z'' + \frac{2}{1-x}Z' + \frac{p^2}{16}\left(3 - \frac{1}{x}\right)\frac{1}{\sqrt{x}}Z = 0$$

$$p^2 \equiv q^2 u_1 = \frac{q^2\sqrt{3}}{\cosh \beta}$$

Numerically:



Comparison with hydrodynamics:



$$\frac{iq}{\pi T} = 4\left(\frac{3}{2}\right)^{\frac{1}{4}} \sqrt{\gamma}\left(v - \frac{1}{\sqrt{3}}\right)$$

More recent work, preliminary results:

The scaling of q for $\gamma \to \infty$ probes the ultraviolet of the theory and should be interesting to study in other cases. Even in the conformal case the exponent depends on the dimension as $q \sim \gamma^{2/d}$

For a stationary solution the surface gravity should be constant? This would mean the temperature is uniform. However the very definition of surface gravity requires the existence of a Killing vector becoming light-like at the horizon. We do not seem to have any.

Oscillations of shock waves generate sound. We have the same here. Analogous to super-radiance?

Conclusions

We performed a systematic study of how gravity resolves shocks in AdS/CFT for the $\mathcal{N}=4$ conformal plasma.

For weak shocks we solve for the hydrodynamic shock and reconstructed the metric.

For strong shocks we computed the exponential tails and found an interesting scaling for large γ factor:
$i\,q \sim \gamma^{1/2}$

Shock waves are important probes of the microscopic description of the theory and should be carefully studied.