Summing planar diagrams

in light-cone gauge

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Summary

- **Introduction**

  Motivation: large-N, D-branes, AdS/CFT, results

  D-brane interactions: lowest order, light-cone gauge

- **D-brane interactions in planar approximation**

  Dual closed string Hamiltonian: $H = H_0 - \lambda P$

  P: hole insertion

- **Supergravity result:** $H = H_0 - \lambda P'$

- **Calculation of** P **in the bosonic string**
(Neumann coeff. , scattering from D-branes)

- **Comparison of** P **and** P'

- **Calculation of** P **in the D3-brane open superstring**
(Scattering of generic closed strings states
from D-branes)

- **Comments on possible field theory applications**

- **Conclusions**

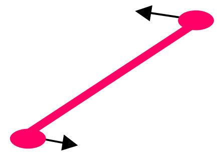
Large N limit (‘t Hooft)

mesons

π, ρ, ...

String picture → Fund. strings

(Susy, 10d, Q.G.)

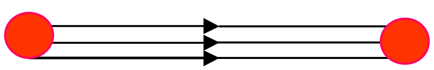


Quark model → QCD [SU(3)]

$q\,\bar{q}$

Large N-limit [SU(N)]

Effective strings

q  $\bar{q}$

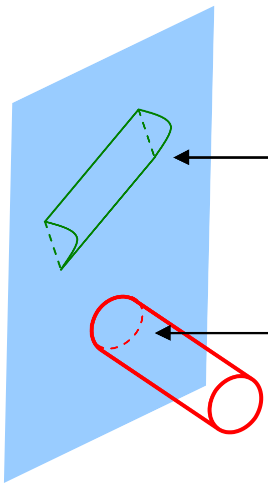
Strong coupling

Lowest order: sum of planar diagrams (infinite number)

Suggested using light-cone gauge / frame.

D-branes (Polchinski)

Region of space where strings world-sheets can end.



Open strings. Low energy: SU(N) gauge th.

Emission of graviton → D-branes have mass

If N, # of D-branes is large → mass large → deforms space

e.g. D3- brane:

$$ds^2 = \frac{1}{\sqrt{f}}\left(dX^+ dX^- + dX_{[2]}^2\right) + \sqrt{f}\, dY_{[6]}^2, \qquad f = 1 + 4\pi\alpha'^2 \frac{g_s N}{Y^4}$$

Suggests an interesting rep. of the large-N limit

AdS/CFT (Maldacena)

We can extract the gauge theory, namely the low energy limit and obtain a duality (no direct derivation)

Open strings | Sugra background

$\mathcal{N} = 4$ SYM ⟷ II B on $AdS_5 x S^5$

$$g_s = g_{YM}^2 ;$$
$$R / l_s = (g_{YM}^2 N)^{1/4}$$

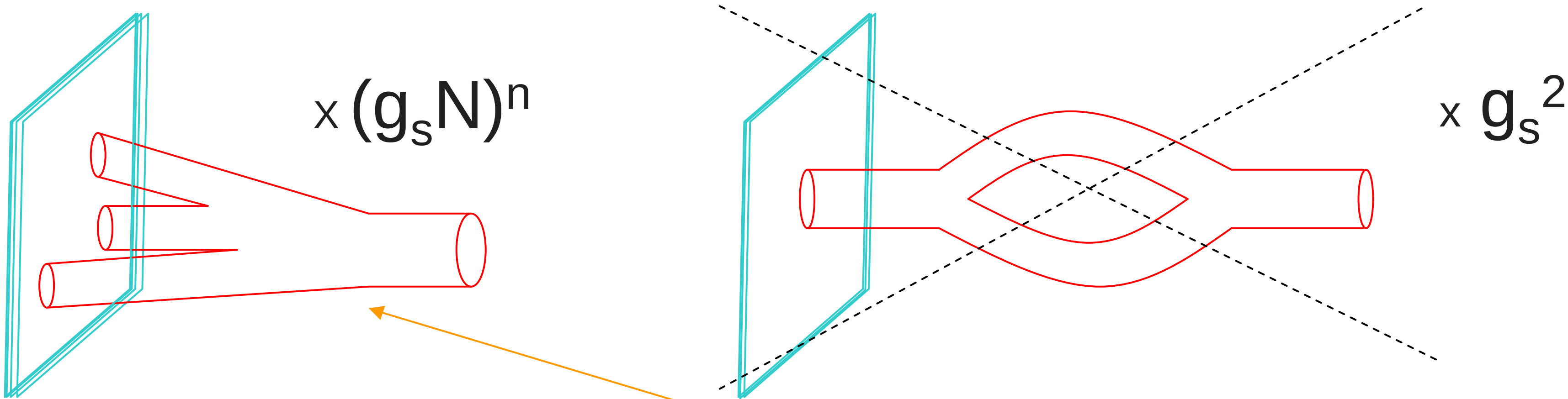
S^5: $X_1^2 + X_2^2 + \ldots X_6^2 = R^2$

AdS_5: $Y_1^2 + Y_2^2 + \ldots - Y_5^2 - Y_6^2 = -R^2$

$N \to \infty, \lambda = g_{YM}^2 N$ fixed ➡

λ large → string th.
λ small → field th.

Planar approximation ($g_s \to 0$, $N \to \infty$, $g_s N$ fixed)



We expect that summing these diagrams gives the propagation of closed strings in the supergravity background. In the bosonic case we get: $H = H_0 - \lambda P$

$$\hat{P} \simeq -\int_{-\pi}^{\pi} d\sigma \frac{16\pi^3 c_3}{Y^4}\left((4\pi\Pi_X)^2 + Y'Y' - 4 + 4\frac{YY''}{Y^2}\right), \qquad (q^2 \to 0 \text{ pole part})$$

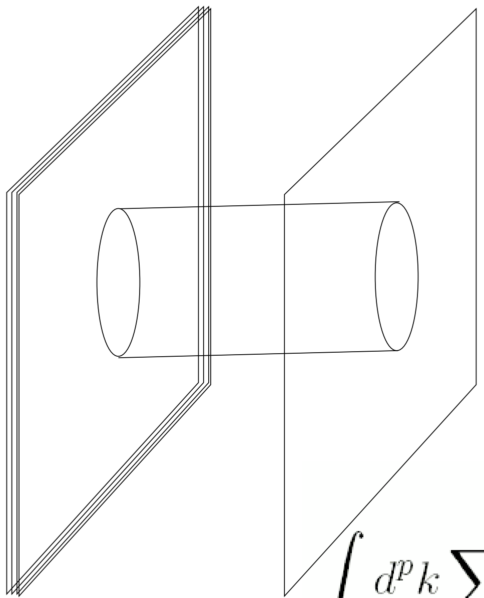
(open strings)

$$\hat{P} = -\alpha' \int d\sigma \frac{1}{Y^4}\left[(2\pi\alpha'\Pi_X)^2 + Y'^2\right]$$

(supergravity)

In the supersymmetric case they agree

D-brane interactions



N

Open string: zero point energy

$$\omega_k = \sqrt{\vec{k}^2 + m^2}, \quad m^2 = \sum_{n\geq 1, i} N_n^i n - a + L^2$$

$$\int d^p k \sum_{m^2} \omega_k = \int d^p k \sum_{m^2} \int_0^\infty \frac{d\ell}{\ell^{\frac{3}{2}}} e^{-\ell(\vec{k}^2+m^2)} = \begin{cases} \int d^{p+1}k \sum_{m^2} \int_0^\infty \frac{d\ell}{\ell} e^{-\ell(k^2+m^2)} \\ \int d^{p-1}k_\perp \sum_{m^2} \int_0^\infty \frac{d\ell}{\ell^2} e^{-\ell(k_\perp^2+m^2)} \end{cases}$$

[schematic: divergences have to be regulated]

$$\int d^{p-1}k_\perp \sum_{m^2} \int_0^\infty dp^+ e^{-\frac{1}{p^+}(k_\perp^2+m^2)} = \mathrm{Tr}\, e^{-\tau H_{l.c.}}, \quad H_{l.c.} = P^-, \quad \tau = 1, \quad (p^+ = \frac{1}{\ell})$$

$$\mathrm{Tr}\, e^{-\tau H_{l.c.}} = \int \mathcal{D}X_\perp e^{-\int d\sigma d\tau \left[(\partial_\tau X_\perp)^2 + (\partial_\sigma X_\perp)^2\right]}$$

σ: 0 $\longrightarrow$ p^+

$$\int \mathcal{D}X e^{-i\int d\sigma d\tau (X^- \Box X^+ + X_\perp \Box X_\perp)}$$

$$\Box X^+ = 0 \Rightarrow X^+ = x^+(\tau+\sigma) + x^+(\tau-\sigma)$$
$$= \frac{1}{2}(\tilde{\tau}+\tilde{\sigma}) + \frac{1}{2}(\tilde{\tau}-\tilde{\sigma}) = \tilde{\tau}$$

$$P^+ = \int d\sigma (\partial_\tau X^+) = \int_0^{p^+} d\sigma = p^+$$

Length = p^+

- **Open string**

$$H_{\text{open}} = \frac{1}{4p^+}\left[p_\perp^2 + \sum_{n\geq 1,i} N_n^i n - \frac{d-2}{24\alpha'} + \frac{Y^2}{4\pi^2\alpha'^2}\right]$$

$$Z = \int d^{p-1}p_\perp \sum_{N_n^i} e^{-\tau\, H_{\text{open}}}, \quad \tau = 2\pi\sqrt{\alpha'}$$

Agree

- **Closed string**

$$H_{\text{closed}} = \frac{1}{\sqrt{\alpha'}}\left[\frac{1}{2}\alpha' p^2 + \sum_{n\geq 1,i}\left(N_n^{Ii} n + N_n^{IIi} n\right) - \frac{d-2}{12}\right]$$

$$Z = \langle B_f|\, e^{-H\tau}\, |B_i\rangle, \quad \tau = 4\alpha' p^+$$

Boundary states

$$X = x_0 + \sum_{n \neq 1} \left[\cos(n\sigma) x_n^I + \sin(n\sigma) x_n^{II} \right],$$

$$P = \frac{1}{2\pi} p_0 + \frac{1}{\pi} \sum_{n \geq 1} \left[\cos(n\sigma) p_n^I + \sin(n\sigma) p_n^{II} \right]$$

$$X \left| B \right\rangle_D = 0, \qquad P \left| B \right\rangle_N = 0$$

Conditions for boundary state

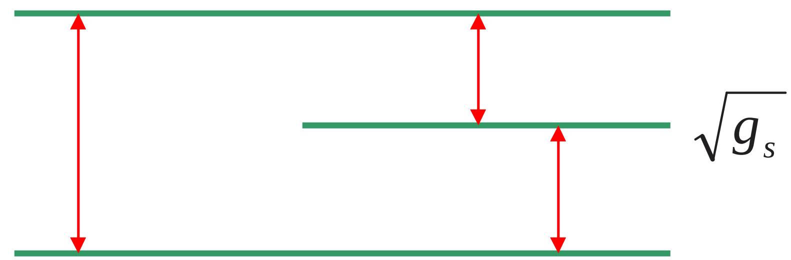
Solutions

$$\left| B \right\rangle_D = \delta(x_0 - Y) e^{-\frac{1}{2} \sum_{n \geq 1} \left(a^\dagger_{In} a^\dagger_{In} + a^\dagger_{IIn} a^\dagger_{IIn} \right)} \left| 0 \right\rangle$$

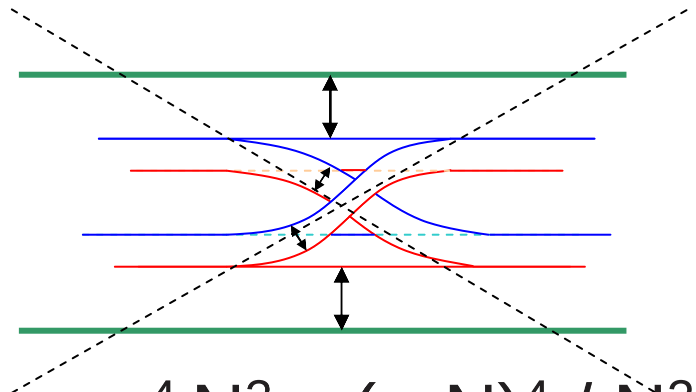
$$\left| B \right\rangle_N = \delta(p_0) e^{\frac{1}{2} \sum_{n \geq 1} \left(a^\dagger_{In} a^\dagger_{In} + a^\dagger_{IIn} a^\dagger_{IIn} \right)} \left| 0 \right\rangle$$

<u>Higher orders</u> (Include open string interactions)

Open strings can split and join.



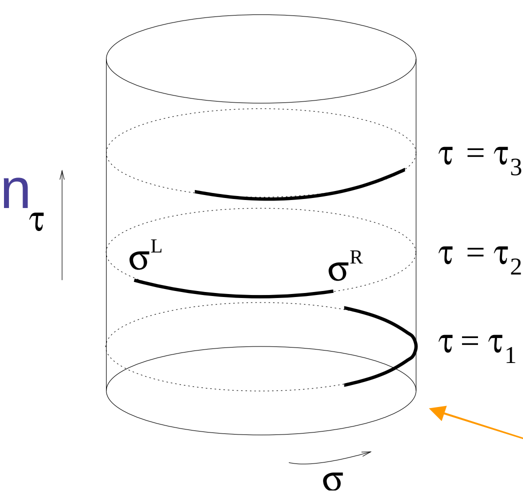
p^+, the length is conserved,



$(g_sN)^4$

$g_s^4 N^2 = (g_sN)^4 / N^2$

$$\sum_n (g_sN)^n$$



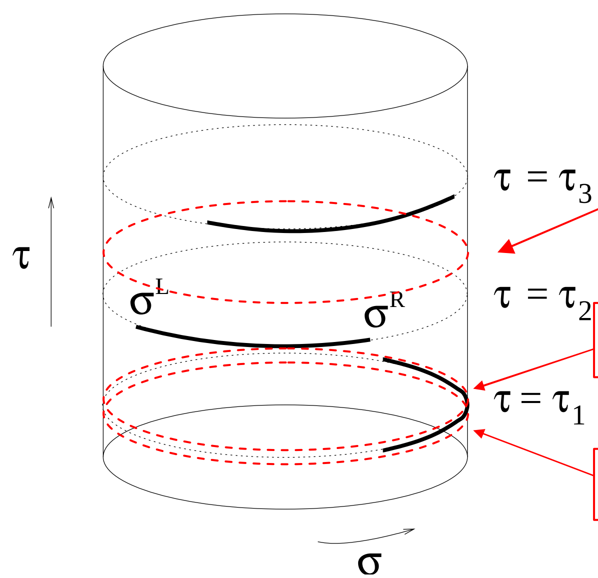
= propagation of a single closed string

n slits

This gives:

$$Z_0 = \sum_{n=0}^{\infty} (g_s N)^n \int \prod_{i=1}^{n} d\sigma_i^L d\sigma_i^R d\tau_i \quad \int \mathcal{D}X_\perp e^{-\int [\dot{X}_\perp^2 + X_\perp'^2]}$$

$$= \sum_{n=0}^{\infty} (g_s N)^n \int \prod_{i=1}^{n} d\sigma_i^L d\sigma_i^R d\tau_i \quad \mu(\sigma_i^L, \sigma_i^R, \tau_i)$$



$$\sum \; |\psi\rangle\langle\psi|$$
$$|\psi\rangle$$

$$\tau = \tau_1 + \epsilon$$

$$\tau = \tau_1 - \epsilon$$

We define the operator $P(\sigma_L,\sigma_R)$ that maps the string from τ_1-ε to τ_1+ε

To sum we use the closed string point of view

$$Z_0 = \sum_{n=0}^{\infty}(g_s N)^n \int \prod_{i=1}^{n} d\sigma_i^L d\sigma_i^R d\tau_i \;\; \langle B_f|\, e^{-H_0(\tau-\tau_n)} \ldots P(\sigma_2^L,\sigma_2^R) e^{-H_0(\tau_2-\tau_1)} P(\sigma_1^L,\sigma_1^R) e^{-H_0\tau_1}\, |B_i\rangle$$

$$= \sum_{n=0}^{\infty}(g_s N)^n \int \prod_{i=1}^{n} d\tau_i \;\; \langle B_f|\, e^{-H_0(\tau-\tau_n)} \ldots P\, e^{-H_0(\tau_2-\tau_1)}\, P\, e^{-H_0\tau_1}\, |B_i\rangle$$

Where $P = \int d\sigma^L d\sigma^R\, P(\sigma^L, \sigma^R)$. Define $P(\tau) = e^{H_0\tau}\, P\, e^{-H_0\tau}$

$$Z_0 = \sum_{n=0}^{\infty}(g_s N)^n \int \prod_{i=1}^{n} d\tau_i \;\; \langle B_f|\, e^{-H_0\tau} P(\tau_n) \ldots P(\tau_1)|B_i\rangle$$

$$= {}_I\langle B_f|\, \hat{T} e^{g_s N \int_0^\tau P(\tau)\, d\tau} |B_i\rangle_I = \langle B_f| e^{-(H_0 - \lambda P)\tau} |B_i\rangle$$

$\lambda = g_s N$

We can define:

$$H_{closed} = H_0 - \lambda P$$

P essentially inserts a hole

Possible problems

We need $\mu(\sigma_i^L, \sigma_i^R, \tau_i) = \int \mathcal{D}X_\perp e^{-\int\left[\dot{X}_\perp^2 + X_\perp'^2\right]}$

This may need corrections if the path integral is not well defined. For example if two slits collide there can be divergences that need to be subtracted. This can modify P and include higher order corrections in λ .

In fact, at first sight this seems even necessary since the propagation of closed strings in the supergravity bakg. Depends on the metric that has non-trivial functions of λ. We analyze this problem now.

Even if there are extra corrections, P as defined contains important information as we will see.

Closed strings in the D3-brane background

$$ds^2 = \frac{1}{\sqrt{f}}\left(dX^+dX^- + dX^2\right) + \sqrt{f}dY^2, \qquad f = 1 + 4\pi\alpha'^2\frac{g_sN}{Y^4}$$

$$S = \frac{1}{4\pi\alpha'}\int d\sigma d\tau \sqrt{h}h^{ab}G_{\mu\nu}\partial_a X^\mu \partial_b X^\nu$$

Take: $h_{01} = 0,\ X^+ = \tau$

$$S = \frac{1}{4\pi\alpha'}\int d\sigma d\tau \left[E(\dot{X}^- + \dot{X}^2) - \frac{1}{f}EX'^2 + Ef\dot{Y}^2 - \frac{1}{E}Y'^2\right]$$

$$E = \sqrt{-\frac{h_{11}}{h_{00}f}}$$

Since $\dot{E} = 0$ we set E=1 and get:

$$S = \frac{1}{4\pi\alpha'}\int d\sigma d\tau \left[-X'^2 + \frac{1}{f}\dot{X}^2 - fY'^2 + \dot{Y}^2\right]$$

and

Not good

$$H = \frac{1}{4\pi\alpha'}\int d\sigma \left\{(2\pi\alpha'\Pi_Y)^2 + X'^2 + f\left[(2\pi\alpha'\Pi_X)^2 + Y'^2\right]\right\}$$

Which is indeed of the form $H_{closed} = H_0 - \lambda P$

with

$$H_0 = \frac{1}{4\pi\alpha'}\int d\sigma \left\{(2\pi\alpha'\Pi_Y)^2 + X'^2 + \left[(2\pi\alpha'\Pi_X)^2 + Y'^2\right]\right\}$$

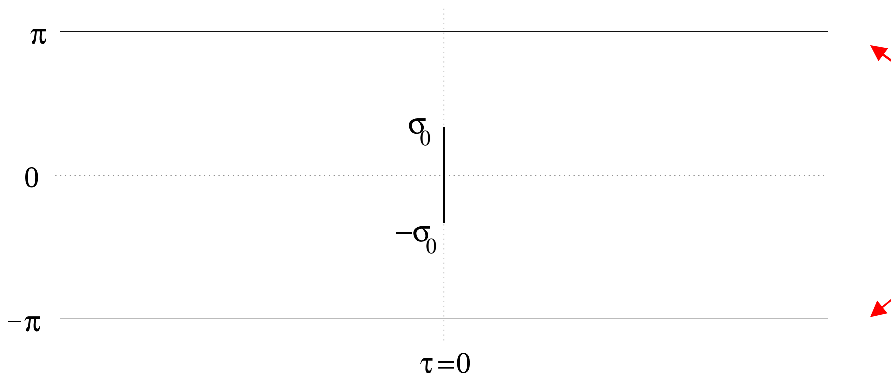
$$\hat{P} = -\alpha' \int d\sigma \frac{1}{Y^4}\left[(2\pi\alpha'\Pi_X)^2 + Y'^2\right]$$

The near horizon (field th.) limit is:

$$H = \frac{1}{4\pi\alpha'}\int d\sigma \left\{(2\pi\alpha'\Pi_Y)^2 + X'^2 + \frac{4\pi\alpha'^2 g_s N}{Y^4}\left[(2\pi\alpha'\Pi_X)^2 + Y'^2\right]\right\}$$

which describes closed strings in **AdS_5xS^5**

What is P in the open string side ?



Identified (cylinder)

Scattering of a closed string by a D-brane

Vertex representation of P: $|V\rangle = \sum_{|1\rangle|2\rangle} \langle 2|\hat{P}|1\rangle \; |1\rangle \otimes |2\rangle,$

Neumann
Dirichlet:

$$(X_1(\sigma,\tau=0) - X_2(\sigma,\tau=0))\,|V\rangle = 0, \quad \sigma_0 \le |\sigma| \le \pi,$$

$$(P_1(\sigma,\tau=0) + P_2(\sigma,\tau=0))\,|V\rangle = 0, \quad \sigma_0 \le |\sigma| \le \pi,$$

$$P_1(\sigma,\tau=0)|V\rangle = 0, \quad |\sigma| \le \sigma_0,$$

$$P_2(\sigma,\tau=0)|V\rangle = 0, \quad |\sigma| \le \sigma_0,$$

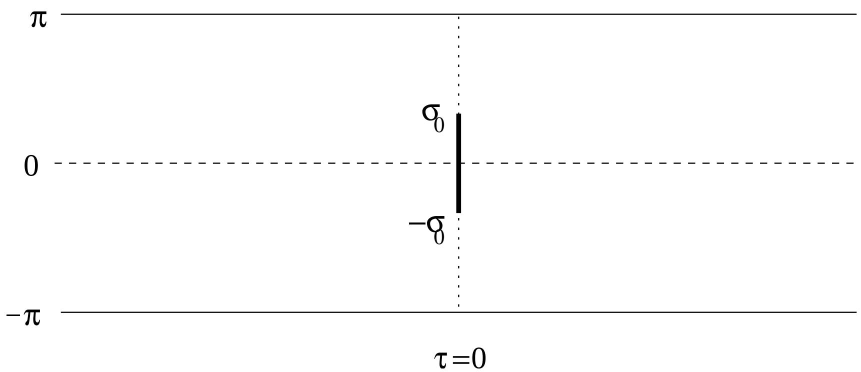
Solution:

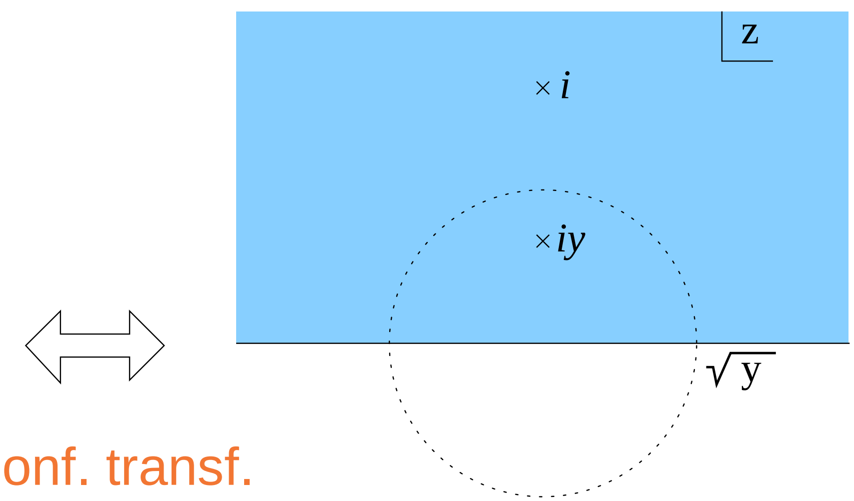
$$|V\rangle = e^{\sum_{rs,imn} N^{rs}_{i,nm} a^{\dagger}_{irn} a^{\dagger}_{ism}} \prod_{i/\varepsilon_i=+1} \delta(p^i_1 + p^i_2)|0\rangle,$$

(Neumann)

Compute coefficients.

$$\rho = \tau + i\sigma \quad , \quad u = e^{\rho}$$





$$G(z, z') = \ln|z - z'| + \varepsilon_i \ln|z - \bar{z}'|$$

r,s=1,2

$$\mathrm{N}^{rs}(u, u') = G(z_r(u), z_s(u')) - \delta^{rs} \ln|u - u'| = \sum_{m,n=\infty}^{\infty} \mathrm{N}^{rs}_{mn}(\varepsilon_i, \sigma_0) e^{in\sigma} e^{im\sigma'}$$

Result:

$$\mathrm{N}^{rs}_{mn}(\varepsilon_i,\sigma_0) = -\frac{i}{8}\frac{(1+\varepsilon_i)}{m+n}\left(a^r_m\delta_{n0}+a^s_n\delta_{m0}\right)+\frac{1}{(m+n)\sin\sigma_0}\mathrm{Im}\left(f^r_m f^s_n\right),$$

with

$$a^{1,2}_m = 2i \pm \frac{i}{\sin\frac{\sigma_0}{2}}\sum_{l=1}^{m}\frac{1}{l}\left(\mathrm{P}_{l-2}(\cos\sigma_0)-\cos\sigma_0\mathrm{P}_{l-1}(\cos\sigma_0)\right), \qquad m>0,$$

$$f^1_{m>0} = -\bar{f}_m, \qquad f^1_{m<0} = -\varepsilon_i f_{-m}, \qquad f^2_{m\neq 0} = -\varepsilon_i f^1_m,$$

$$f_{m>0} = -\frac{i}{m}e^{i\frac{\sigma_0}{2}}\sum_{l=1}^{m}\frac{(-i)^l m!}{(m+l)!}l\mathrm{P}^l_m(\cos\sigma_0).$$

$$f^1_0 = \frac{1}{2}\left[(1+\varepsilon_i)(1-\sin\frac{\sigma_0}{2})-i(1-\varepsilon_i)\cos\frac{\sigma_0}{2}\right],$$

$$f^2_0 = \frac{1}{2}\left[(1+\varepsilon_i)(1+\sin\frac{\sigma_0}{2})-i(1-\varepsilon_i)\cos\frac{\sigma_0}{2}\right].$$

All together we get:

$$\hat{P} = \frac{1}{\pi^3} \int_0^{2\pi} d\sigma \int_0^{\pi} d\sigma_0 \frac{1}{\sin^3 \sigma_0} e^{\sum_{rs,imn} N^{rs}_{i,nm} a^\dagger_{irn} a^\dagger_{ism} e^{-i(m+n)\sigma}} \prod_{i/\varepsilon_i=+1} \delta(p^i_1 + p^i_2)|0\rangle$$

Small holes ($\sigma_0 \to 0$)

$$\sum_{rs,imn}{}' N^{rs}_{imn} a^\dagger_{irm} a^\dagger_{isn} \to iq\bar{y} + \frac{i}{4}\sigma_0^2 q.y'' - \frac{\sigma_0^2}{8} y'y' + \sigma_0^2 \bar{p}k - \frac{\sigma_0^2}{2}\bar{p}\bar{p} + \dots,$$

Gives:

Tachyon pole

$$\hat{P}_0 \simeq 2c_3 \left(\frac{\pi}{2}\right)^{q^2-2} \frac{1}{q^2-2} e^{iq\bar{y}} - \frac{8c_3}{q^2}\left(\frac{\pi}{2}\right)^{q^2}\left(\frac{1}{2}(k-\bar{p})^2 + \frac{1}{8}y'y' + \frac{1}{24}q^2 - \frac{1}{2} - \frac{i}{4}qy''\right) e^{iq\bar{y}}$$

$$\hat{P} \simeq -\int_{-\pi}^{\pi} d\sigma \frac{16\pi^3 c_3}{Y^4}\left((4\pi\Pi_X)^2 + Y'Y' - 4 + 4\frac{YY''}{Y^2}\right), \qquad (q^2 \to 0 \text{ pole part})$$

We reproduced the operator P in a certain (small hole) limit. There are extra terms due to the fact that we consider the bosonic string. (tachyon). Should be absent in the superstring.

There are also extra terms which do not correspond to the $q^2 \rightarrow 0$ pole. However we should take into account that the hamiltonian form the background is classical and we should have expected further corrections.

In pple. the Hamiltonian we proposed should reproduce order by order the planar diagrams (by definition).

The supersymmetric case: D3 open superstrings

We need to add fermionic degrees of freedom.
θ^A, λ_A, right moving and $\tilde{\theta}^A$, $\tilde{\lambda}_A$ left moving.
There is an SO(6)=SU(4) symmetry. The index A is in the fundamental or anti-fundamental (upper or lower).

Conditions: (preserving half the supersymmetry)

$$\left(\theta_1^A - \theta_2^A - \tilde{\theta}_1^A + \tilde{\theta}_2^A\right)|V\rangle = 0, \quad -\pi \leq \sigma \leq \pi,$$

$$\left(\lambda_{1A} + \lambda_{2A} + \tilde{\lambda}_{1A} + \tilde{\lambda}_{2A}\right)|V\rangle = 0, \quad -\pi \leq \sigma \leq \pi,$$

$$\left(\theta_1^A - \theta_2^A + \tilde{\theta}_1^A - \tilde{\theta}_2^A\right)|V\rangle = 0, \quad \sigma_0 \leq |\sigma| \leq \pi,$$

$$\left(\lambda_{1A} + \lambda_{2A} - \tilde{\lambda}_{1A} - \tilde{\lambda}_{2A}\right)|V\rangle = 0, \quad \sigma_0 \leq |\sigma| \leq \pi,$$

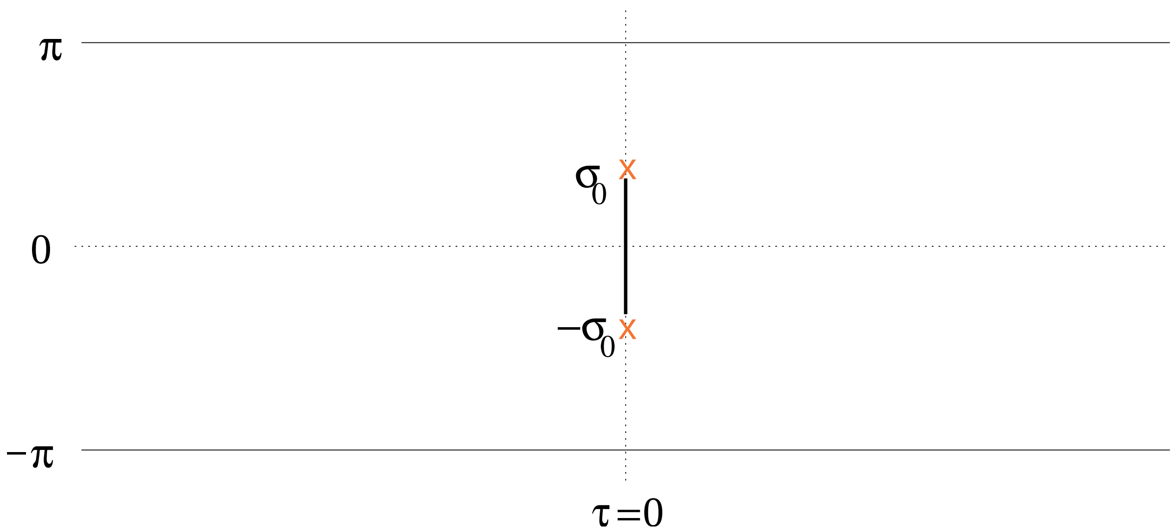
$$\left(\theta_1^A + \theta_2^A - \tilde{\theta}_1^A - \tilde{\theta}_2^A\right)|V\rangle = 0, \quad -\sigma_0 \leq \sigma \leq \sigma_0,$$

$$\left(\lambda_{1A} - \lambda_{2A} + \tilde{\lambda}_{1A} - \tilde{\lambda}_{2A}\right)|V\rangle = 0, \quad -\sigma_0 \leq \sigma \leq \sigma_0.$$

Can be solved again in terms of a vertex state:

$$|V\rangle = e^{\text{bilinear in creation ops.}}|0\rangle$$

However extra operator insertions are required:



Why?

It is known in the open string channel but we can also see it in the closed string channel

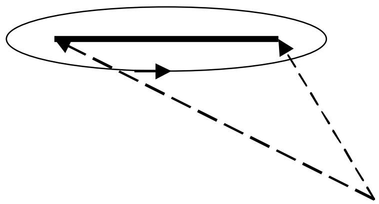
Before going into that it is useful to compute the supersymmetric algebra:

$$\{\mathbb{Q}_+^A, \mathbb{Q}_-^B\} = -2\sqrt{2}P^I\rho^{IBA},$$
$$\{\mathbb{Q}_{+A}, \mathbb{Q}_{-B}\} = 2\sqrt{2}P^I\rho^I_{BA},$$
$$\{\mathbb{Q}_{-A}, \mathbb{Q}_-^B\} = 2(H_l - H_r)\delta_A^B = -16P_\sigma\delta_A^B.$$

Notice that the Q's commute to translations in σ.

If they commute to H, since H has a term of order λ then so should the Q's implying the possible presence of a λ^2 term in H. This is actually the case in the open string channel but not here.

Operator insertions create singularities for other operators, e.g. translation operator. Here:



$$[P_\sigma, \hat{P}_S] = \oint (H_r - H_l)\hat{P}_S = -i\partial_\sigma \hat{P}_S$$

Pole singularities since slit is not invariant there

This indeed can be verified by explicit computation. However it implies by the Jacobi identity:

$$[\{\mathbb{Q}^-_A, \mathbb{Q}^{-B}\}, \hat{P}_S] + \{[\hat{P}_S, \mathbb{Q}^-_A], \mathbb{Q}^{-B}\} + \{[\mathbb{Q}^{-B}, \hat{P}_S], \mathbb{Q}^-_A\} = 0 \Rightarrow$$

$$-16\delta^B_A[P_\sigma, \hat{P}_S] + \{[\hat{P}_S, \mathbb{Q}^-_A], \mathbb{Q}^{-B}\} + \{[\mathbb{Q}^{-B}, \hat{P}_S], \mathbb{Q}^-_A\} = 0.$$

$\hat{P}_S$ is not supersymmetric!

In fact it is also not invariant under U(1) rotations in the $X_{L,R}$ directions (q=+2) as verified by explicit computation and also by small hole limit and continuity argument.

To make P invariant under supersymmetry and U(1) we insert operators at the end points:

$$\hat{P} = \int d\sigma_L d\sigma_R H_1(\sigma_L) H_1(\sigma_R) \hat{P}_S(\sigma_L, \sigma_R)$$

$$H_1 = \sqrt{\epsilon} \left\{ \Pi^L - \frac{i}{8\pi\sqrt{2}} \epsilon \, \partial_\sigma Y^I \rho^I_{CD} \Theta^C \Theta^D - \epsilon^2 \Pi^R \Theta^4 \right\}$$

where $\Theta^A = \frac{1}{\sqrt{2}} \left(\theta^A - \tilde{\theta}^A \right)$ and $\varepsilon \longrightarrow 0$.

It is convenient also to write it in normal ordered form:

$$\hat{P} = :\hat{H}\hat{P}_S: \quad \text{with}$$

$$\hat{H} = \left(Z^L + \frac{i}{\sqrt{2}}\rho^I_{AB} Z^I Y^A Y^B - 4Z^R Y^4\right)\left(\bar{Z}^L - \frac{i}{\sqrt{2}}\rho^J_{AB} \bar{Z}^I \bar{Y}^A \bar{Y}^B - 4\bar{Z}^R \bar{Y}^4\right)$$
$$+\frac{1}{8\pi^2}\frac{1}{\sin\sigma_0}\left(Y^4 + \bar{Y}^4 + \frac{1}{4}\epsilon_{ABCD} Y^A Y^B \bar{Y}^C \bar{Y}^D\right),$$

where Z^I and Y^A are linear combinations of creation operators giving the divergent parts of Y^I and Θ^A.

The matrix element of P are give the scattering amplitude for arbitrary closed string states from the D3-brane. It is useful to check for massless states. We obtain perfect agreement with known results. (Myers-Garousi, Klebanov-Hashimoto, …)

Again we can compute the limit of small holes.
The first result is that the tachyon pole cancels!.
We get:

$$16\pi^3 \hat{P} \simeq \mathbb{H} = -\frac{1}{4}\frac{1}{Y^4}\Pi^a\Pi^a - \frac{1}{2^6\pi^2}\frac{1}{Y^4}\partial_\sigma Y^I \partial_\sigma Y^I + \frac{i}{16\pi Y^4}\left(\Theta^A \partial_\sigma \bar{\Lambda}_A + \bar{\Lambda}_A \partial_\sigma \Theta^A\right)$$
$$-i\sqrt{2}\pi \frac{Y^I}{Y^6}\Pi^L \rho^{ICD}\bar{\Lambda}_C\bar{\Lambda}_D + \frac{i\sqrt{2}}{4\pi}\frac{Y^I}{Y^6}\Pi^R \rho^I_{AB}\Theta^A\Theta^B$$
$$-\frac{i}{8\pi}\rho^{IAC}\rho^J_{CB}\bar{\Lambda}_A\Theta^B \frac{1}{Y^6}\left(\partial_\sigma Y^I Y^J - \partial_\sigma Y^J Y^I\right)$$
$$+\frac{1}{4Y^6}\left(\delta^{IJ} - 6\frac{Y^I Y^J}{Y^2}\right)\rho^{IAB}\rho^J_{CD}\bar{\Lambda}_A\bar{\Lambda}_B\Theta^C\Theta^D.$$

Namely bosonic part + susy completion. We propose this to be the Hamiltonian for strings in the <u>full</u> D3-brane background in σ–gauge (including susy part).

We can take the near horizon limit. The result is:

$$H_{[AdS_5\times S^5]} = 2\pi\int d\sigma\left(\Pi_Y^2 + \frac{1}{16\pi^2}\left(\partial_\sigma X\right)^2\right) + i\int d\sigma\,\partial_\sigma\Lambda\bar{\Theta}$$
$$+32\pi^2\lambda\int d\sigma\left\{\frac{1}{Y^4}\Pi^a\Pi^a + \frac{1}{16\pi^2}\frac{1}{Y^4}\partial_\sigma Y^I\partial_\sigma Y^I - \frac{i}{4\pi Y^4}\left(\Theta^A\partial_\sigma\bar{\Lambda}_A + \bar{\Lambda}_A\partial_\sigma\Theta^A\right)\right.$$
$$+i4\sqrt{2}\pi\frac{Y^I}{Y^6}\Pi^L\rho^{ICD}\bar{\Lambda}_C\bar{\Lambda}_D - \frac{i\sqrt{2}}{\pi}\frac{Y^I}{Y^6}\Pi^R\rho^I_{AB}\Theta^A\Theta^B$$
$$+\frac{i}{2\pi}\rho^{IAC}\rho^J_{CB}\bar{\Lambda}_A\Theta^B\frac{1}{Y^6}\left(\partial_\sigma Y^I Y^J - \partial_\sigma Y^J Y^I\right)$$
$$\left.-\frac{1}{Y^6}\left(\delta^{IJ} - 6\frac{Y^IY^J}{Y^2}\right)\rho^{IAB}\rho^J_{CD}\bar{\Lambda}_A\bar{\Lambda}_B\Theta^C\Theta^D\right\}.$$

Again, bosonic + susy completion. Should be compared with Metsaev-Tseytlin. However here we did not use AdS_5xS^5, only planar open strings !!

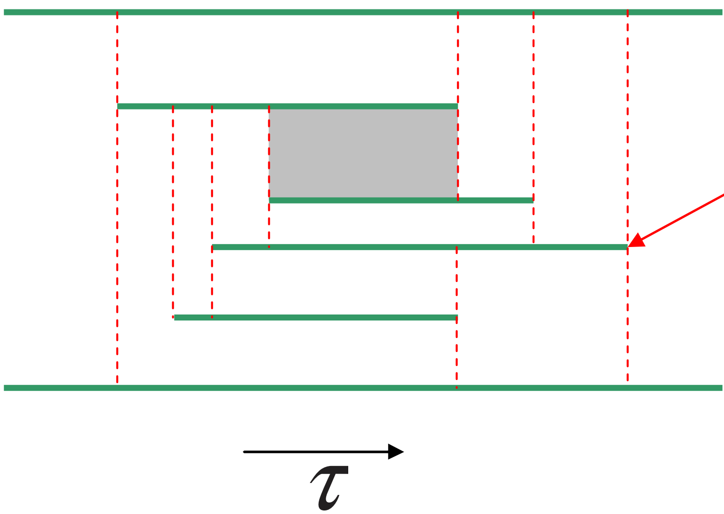
Valid in $1 << Y^2 << \sqrt{\lambda}$ (string units)
Maldacena limit $Y << 1$ (decoupling limit → field theory)

Comments on field theory

't Hooft:

propagator in light cone frame:

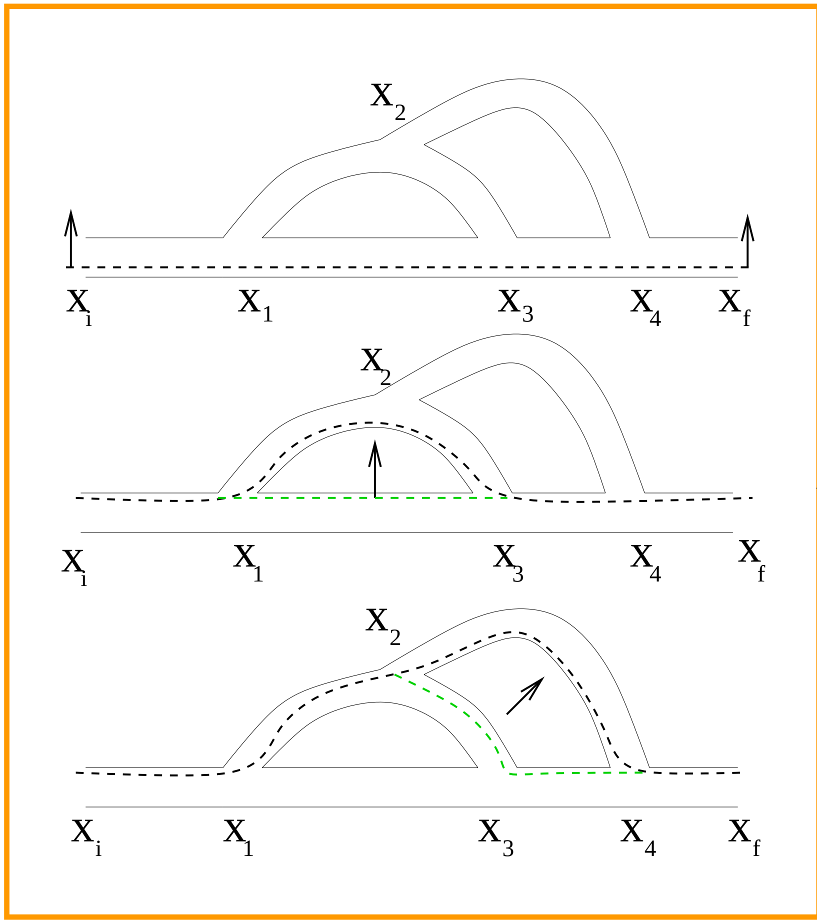
$$\frac{1}{|p^+|}e^{-\frac{p_\perp^2+m^2}{2p^+}t_0}$$



Local in τ and non local in σ. We want to flip σ ↔ τ.
But we should get a local evolution in the new τ.
Not clear if it is possible.

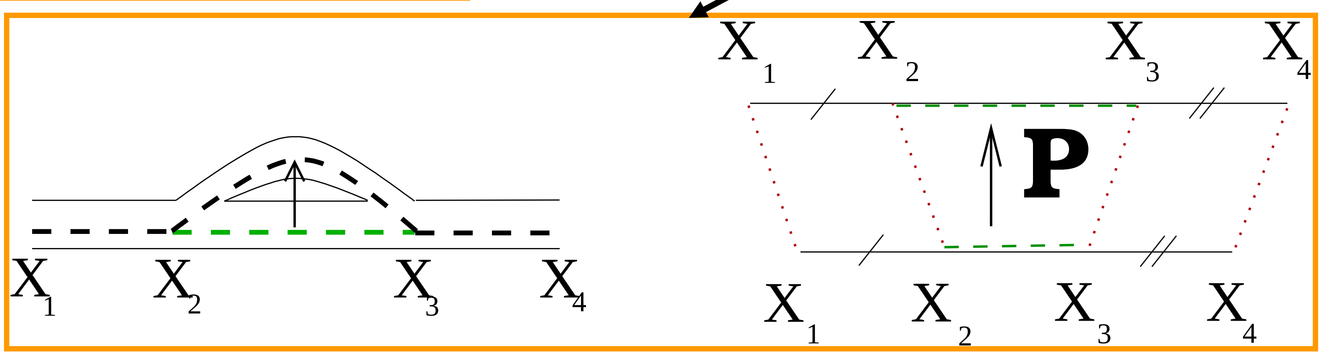
Here we identify trajectory with shape of the string

Suggestive, but needs more understanding.



Two loop example

One loop example



Conclusions

- The sum of planar diagrams is determined by an operator P acting on closed strings. It inserts a hole in the world-sheet.

- The "(σ ↔ τ) dual" closed string Hamiltonian is:

$$H_{closed} = H_0 - \lambda P$$

- For open superstring on D3 branes we obtained P explicitely. From it, after taking a limit, we obtained a Hamiltonian whose bosonic part is the one for closed strings in the correct background.

- There can be corrections to H but, nevertheless, the operator P contains important information (e.g. bkgnd.)

- Two interesting side results:
  - Closed expression for the scattering of generic closed string states from a D3-brane.
  - Hamiltonian for a closed string in the full D3-brane background including fermionic terms (in σ-gauge). Also novel form for $AdS_5 x S^5$.
- In field theory we can use a "($\sigma \leftrightarrow \tau$) duality" if we get a representation local in the new τ. In that case we can define a dual $H = H_0 - \lambda P$ that contains the information on the planar diagrams. Less ambitious than obtaining a dual string theory.