String / gauge theory duality

and ferromagnetic spin chains

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Summary

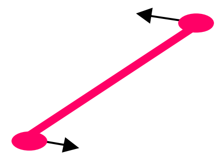
- **Introduction**

mesons

$\pi, \rho, \ldots$

String picture → Fund. strings

(Susy, 10d, Q.G.)



Quark model → QCD

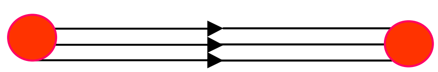
$q\,\bar{q}$

↓

Large N-limit

↓

Effective strings

q  $\bar{q}$

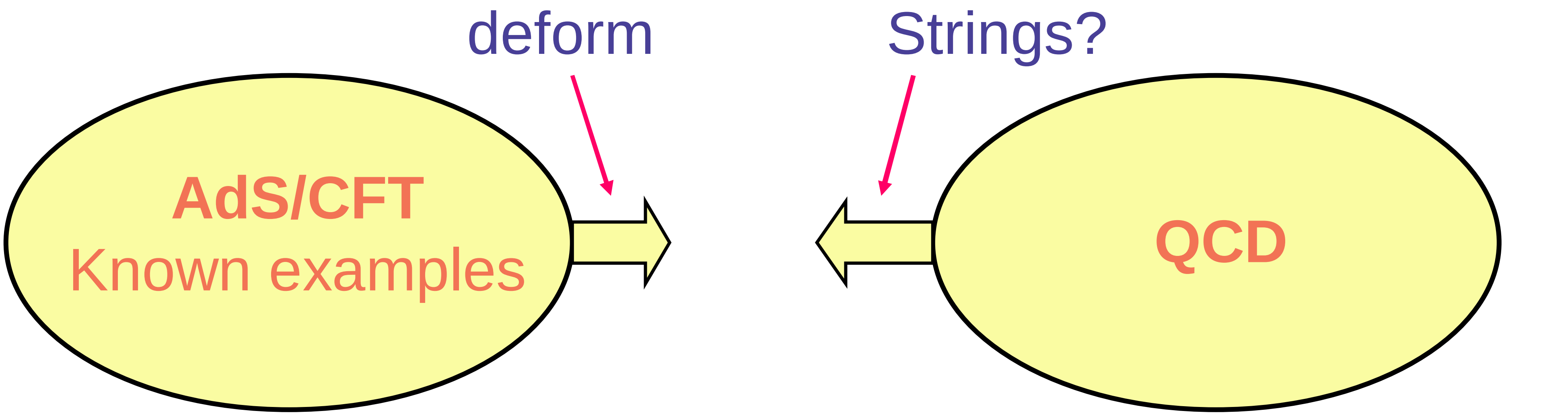
Strong coupling

AdS/CFT

$\mathcal{N} = 4$ SYM ⟷ II B on $AdS_5 x S^5$

S^5: $X_1^2+X_2^2+\ldots X_6^2 = R^2$

AdS_5: $Y_1^2+Y_2^2+\ldots -Y_5^2-Y_6^2 = -R^2$



- **Add quarks**

We get models where the spectrum of $q\bar{q}$ bound states can be computed in the strong coupling regime

- **Strings from gauge theory**

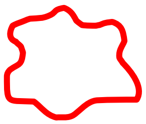
Problem: Compute scaling dimension (or energy) of states of a large number of particles in $\mathcal{N}$ =4 SYM

Equivalent to solving Heisenberg spin chain (Minahan and Zarembo).
Use effective action for spin waves: is the same as the string action AdS/CFT predicts!.

Introduction

String theory

●) Quantum field theory:
Relativistic theory of point particles

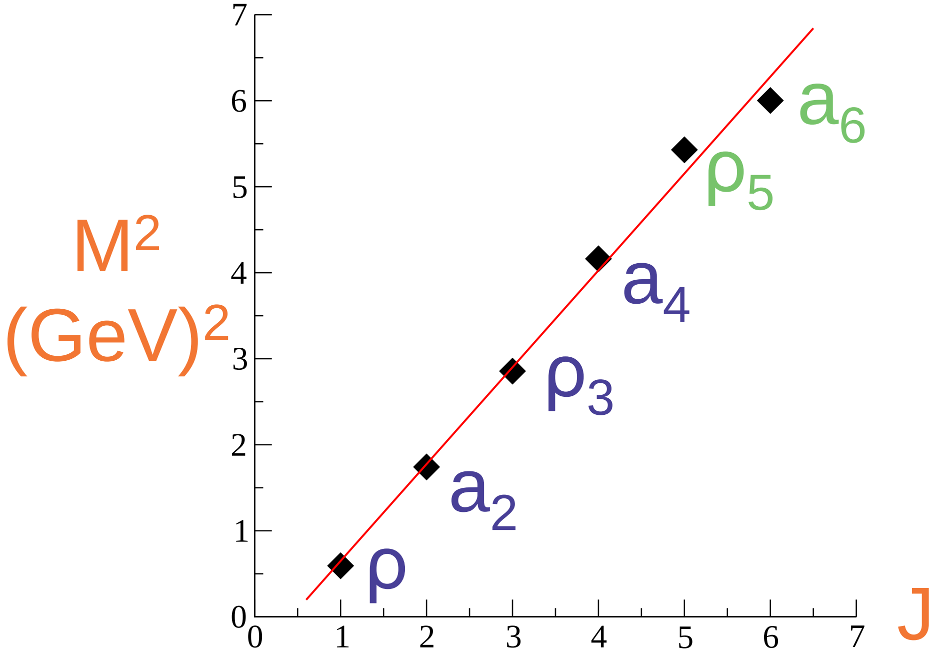
●) String theory:
Relativistic theory of extended objects: Strings 

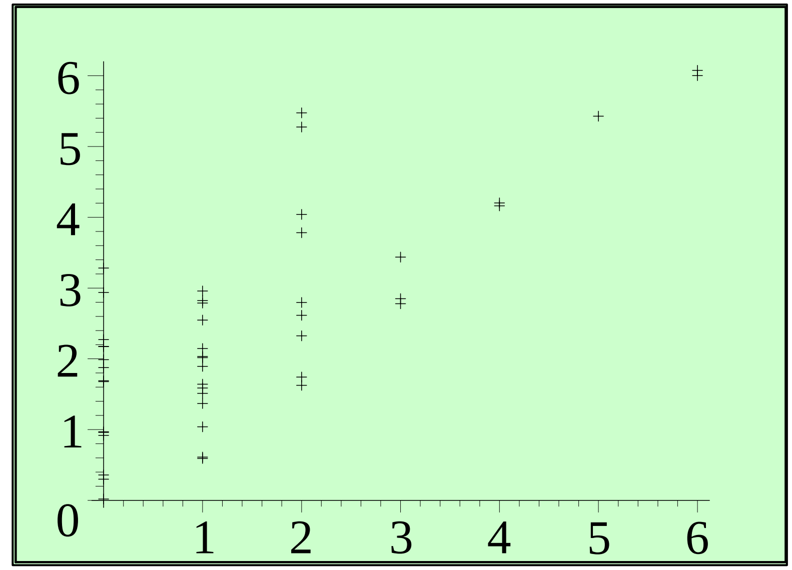
Why?

Original motivation:

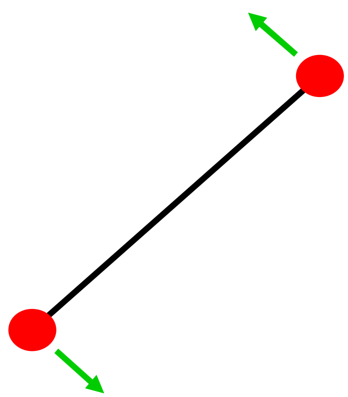
Phenomenological model for hadrons
(proton, neutron,pions, rho, etc.)

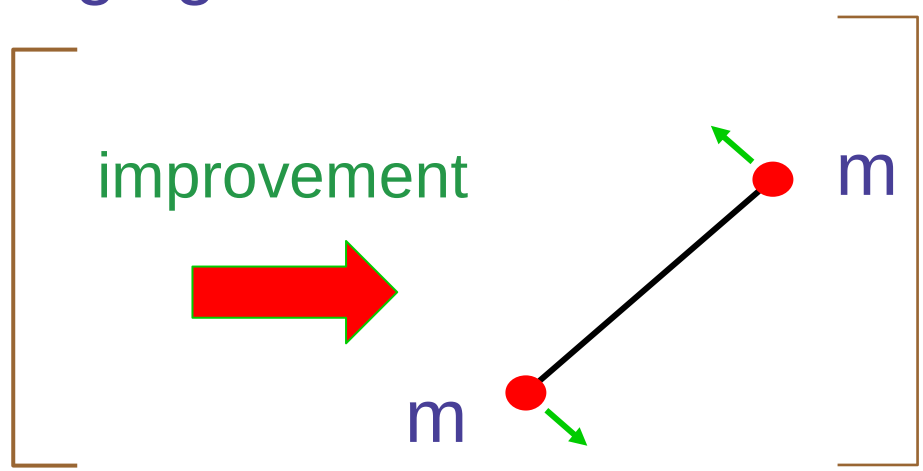
Regge trajectories





Simple model of rotating strings gives $E \approx \sqrt{J}$





Strings thought as **fundamental**

Theoretical problems

Tachyons → Taking care by supersymmetry

Quantum mechanically consistent only in 10 dim.

Unified models? Including gravity

5 types of strings

What about hadrons?

Instead: bound states of quarks. mesons: $q\bar{q}$

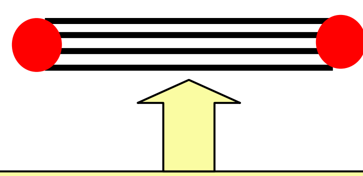
baryons: qqq

Interactions: SU(3); $q = \begin{bmatrix} - \\ - \\ - \end{bmatrix}$; $A_\mu = \begin{bmatrix} - & - & - \\ - & - & - \\ - & - & - \end{bmatrix}$

quarks gluons

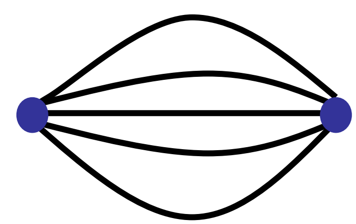
Coupling constant **small** at large energies (100 GeV) but **large** at small energies. No expansion parameter.

Confinement



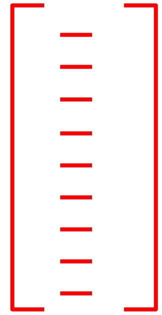
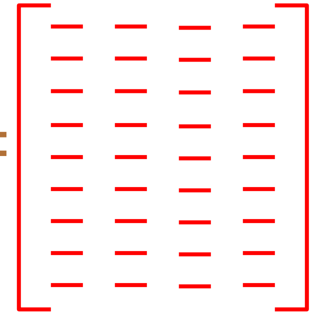
(color) electric flux=string?

V=k r



V=- k/r

Idea ('t Hooft)

Take large-N limit, q=  ; A_μ= 

N   N x N

$$N \to \infty, \quad g_{YM}^{2} N = \lambda$$ fixed ('t Hooft coupling)

1/N: perturbative parameter.

Planar diagrams dominate (sphere)

Next: $1/N^2$ corrections (torus) + $1/N^4$ (2-handles) + …

Looks like a string theory

Can be a way to derive a string descriptions of mesons

AdS/CFT correspondence (Maldacena)

Gives a precise example of the relation between strings and gauge theory.

Gauge theory	String theory
$\mathcal{N} = 4$ SYM SU(N) on R^4	IIB on $AdS_5 x S^5$
A_μ, Φ^i, Ψ^a	radius R
Operators w/ conf. dim. Δ	String states w/ $E = \frac{\Delta}{R}$

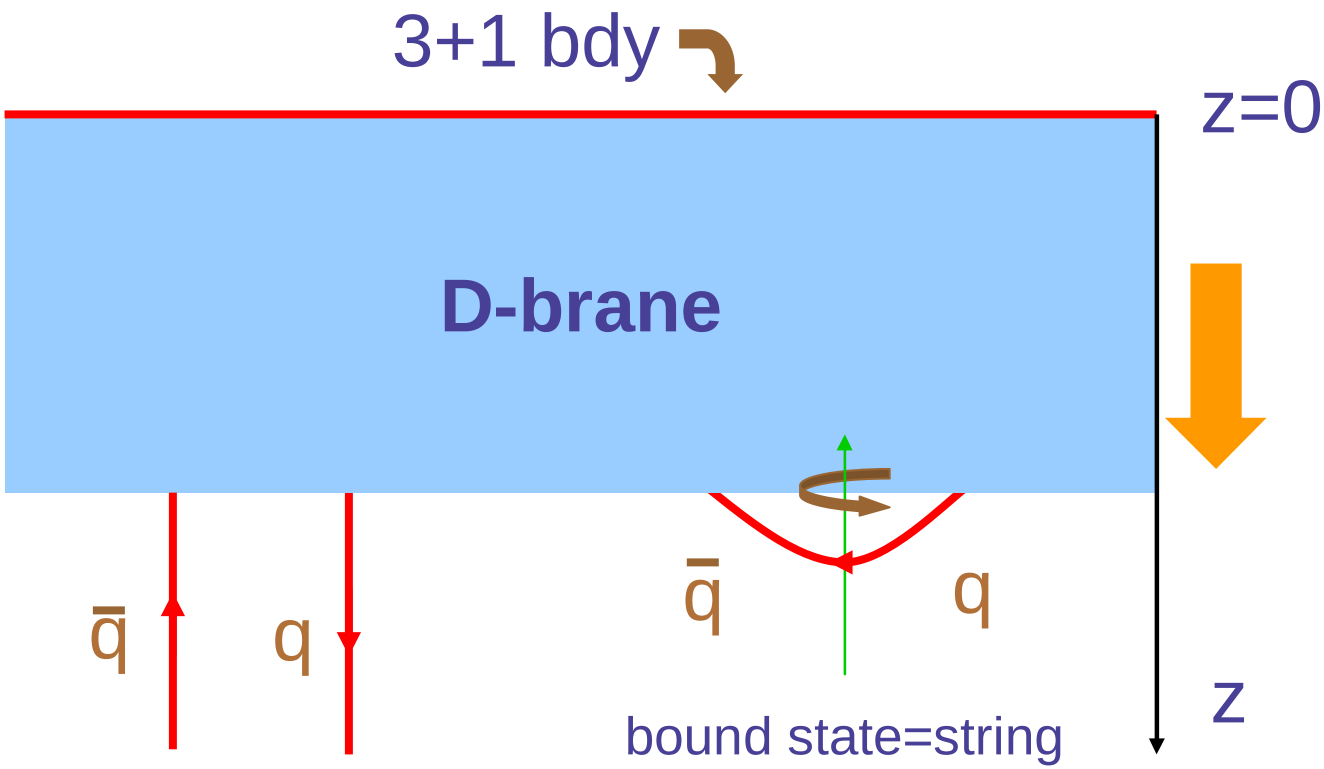
$$g_s = g_{YM}^2; \qquad R / l_s = (g_{YM}^2 N)^{1/4}$$

$N \to \infty, \lambda = g_{YM}^2 N$ fixed ➡

λ large → string th.
λ small → field th.

Mesons (w/ D. Mateos, R. Myers, D. Winters)

We need quarks (following Karch and Katz)



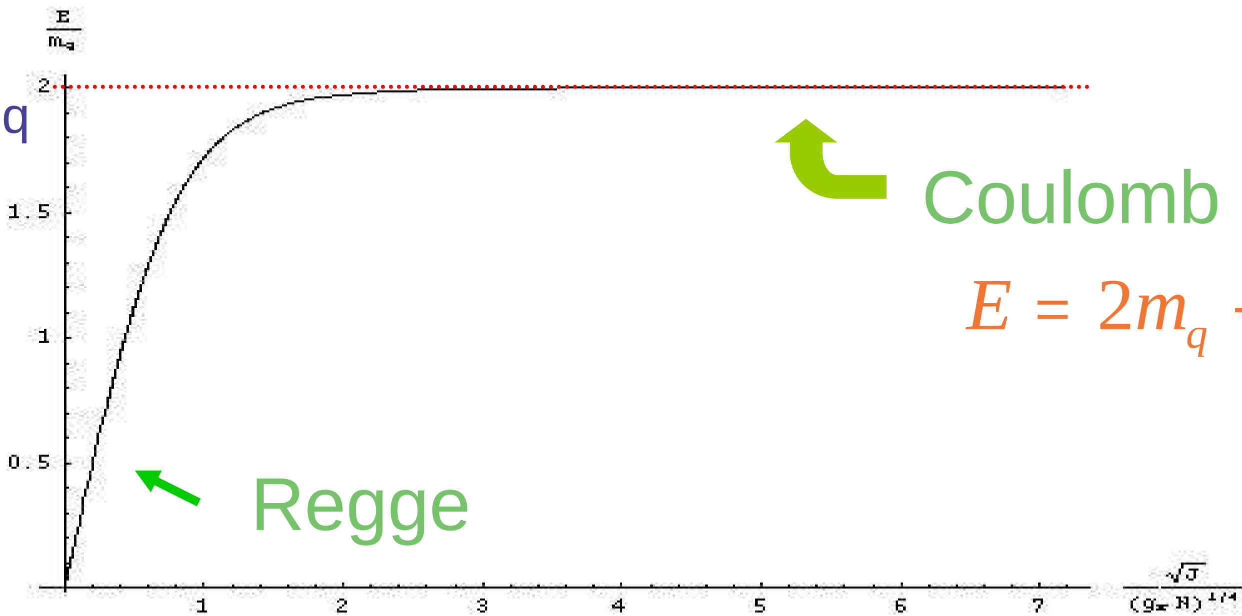
$$ds^2 = \frac{-dt^2 + d\vec{x}^2 + dz^2}{z^2}$$

So, in AdS/CFT, a meson is a string rotating in 5 dim.!

Meson spectrum

($\mathcal{N}$ = 4 is conformal → Coulomb force)

$2m_q$



Coulomb

$$E = 2m_q - \alpha / J^2$$

Regge

$$\sqrt{J} / (g_{YM}^2 N)^{1/4}$$

(numerical result)

The cases J=0, ½, 1 are special, very light , namely “tightly bound”. ($E_b \sim 2\ m_q$)

For J=0,1/2,1 we can compute the **exact** spectrum (in 't Hooft limit and at strong coupling)

2 scalars	$(M/M_0)^2 = (n+m+1)(n+m+2)$	$, m \geq 0$
1 scalar	$(M/M_0)^2 = (n+m+1)(n+m+2)$	$, m \geq 1$
1 scalar	$(M/M_0)^2 = (n+m+2)(n+m+3)$	$, m \geq 1$
1 scalar	$(M/M_0)^2 = (n+m)(n+m+1)$	$, m \geq 1$
1 vector	$(M/M_0)^2 = (n+m+1)(n+m+2)$	$, m \geq 0$
1 fermion	$(M/M_0)^2 = (n+m+1)(n+m+2)$	$, m \geq 0$
1 fermion	$(M/M_0)^2 = (n+m+2)(n+m+3)$	$, m \geq 0$

$n \geq 0$; there is a mass gap of order M_0 for $m_q \neq 0$

$$M_0 = \frac{L}{R} = \frac{m_q}{\sqrt{g_{YM}^2 N}} << m_q \quad \textit{for} \quad \sqrt{g_{YM}^2 N} >> 1$$

Confining case (w/ D. Mateos, R. Myers, D. Winters)

Add quarks to Witten's confining bkg.

- Spectrum is numerical
- For $m_q=0$ there is a massless meson Φ. ($M_\Phi=0$)

  Goldstone boson of chiral symmetry breaking

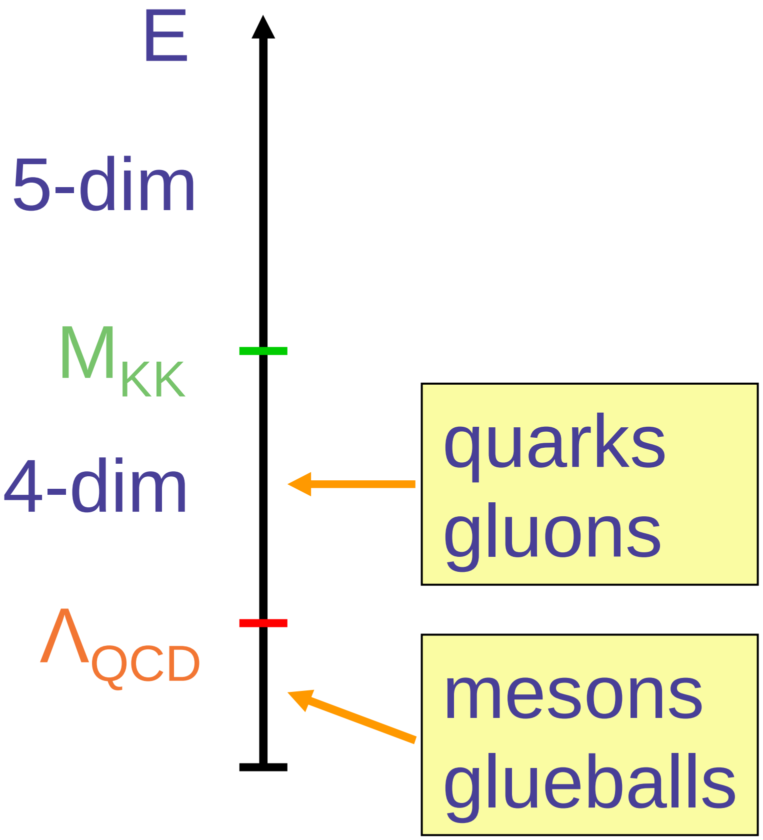
- For $m_q \neq 0$ $$M_\phi^2 = -\frac{m_q}{f_\phi^2}\langle\bar{\psi}\psi\rangle$$ GMOR- relation
- Rot. String (w/ Vaman, Pando-Zayas, Sonnenschein)

  reproduces "improved model":

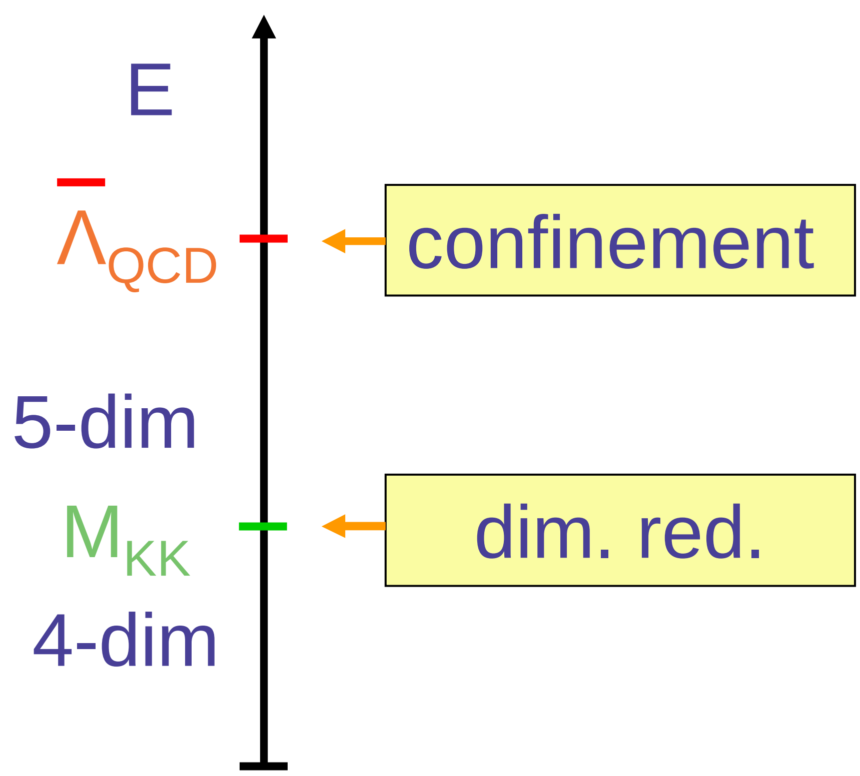
We can compute meson spectrum at strong coupling.
In the confining case results are similar to QCD.

How close are we to QCD?

Ideal sit.



In practice



Can we derive the string picture from the field theory? (PRL 93 (2004) M.K.)

Study known case: $\mathcal{N} = 4$ SYM

Take two scalars $X = \Phi_1 + i\,\Phi_2$; $Y = \Phi_3 + i\,\Phi_4$

$O = \mathbf{Tr}(XX\ldots Y..Y\ldots X)$, J_1 X's , J_2 Y's, J_1+J_2 large

Compute 1-loop conformal dimension of O , or equiv. compute energy of a bound state of J_1 particles of type X and J_2 of type Y (but on a three sphere)

$R^4 \longleftrightarrow S^3xR$

$\Delta \longleftrightarrow E$

Large number of ops. (or states). All permutations of Xs and Ys mix so we have to diag. a huge matrix.

Nice idea (Minahan-Zarembo). Relate to a phys. system

Tr(X X…Y X X Y) ⟷ | ↑ ↑…↓ ↑ ↑ ↓ ›

operator ⟷ conf. of spin chain

mixing matrix ⟷ op. on spin chain

$$H = \frac{\lambda}{4\pi^2} \sum_{j=1}^{J} \left(\frac{1}{4} - \vec{S}_j \cdot \vec{S}_{j+1} \right)$$

Ferromagnetic Heisenberg model !

Ground state (s)

$|\uparrow\uparrow\ldots\uparrow\uparrow\uparrow\uparrow\rangle \longleftrightarrow \mathrm{Tr}(X\,X\ldots X\,X\,X\,X)$

$|\downarrow\downarrow\ldots\downarrow\downarrow\downarrow\downarrow\rangle \longleftrightarrow \mathrm{Tr}(Y\,Y\ldots Y\,Y\,Y\,Y)$

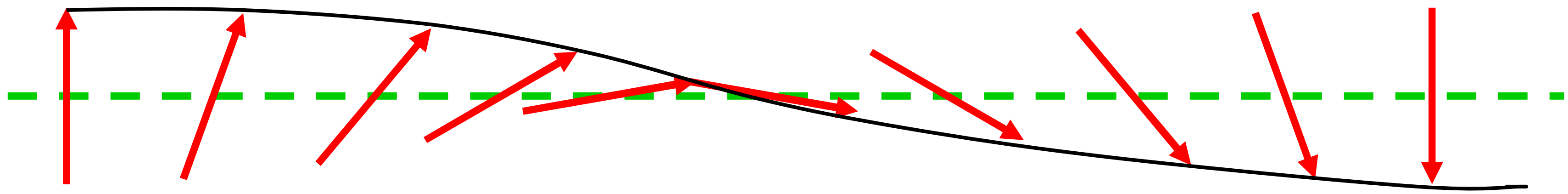
First excited states

$$|k\rangle = \sum e^{ikl}|\uparrow\uparrow\ldots\underset{l}{\downarrow}\ldots\uparrow\uparrow\rangle, \quad k = \frac{2\pi n}{J};\ (J = J_1 + J_2)$$

$$\varepsilon(k) = \frac{\lambda}{J^2}\left(-1+\cos k\right) \xrightarrow[k\to 0]{} \frac{\lambda n^2}{2J^2} \qquad \text{(BMN)}$$

More generic (low energy) states: Spin waves

Other states, e.g. with $J_1=J_2$



Spin waves of long wave-length have low energy and are described by an effective action in terms of two angles θ, φ: direction in which the spin points.

$$S_{eff.} = J\left\{ -\frac{1}{2}\int d\sigma\, d\tau \cos\theta\, \partial_\tau \phi - \right.$$

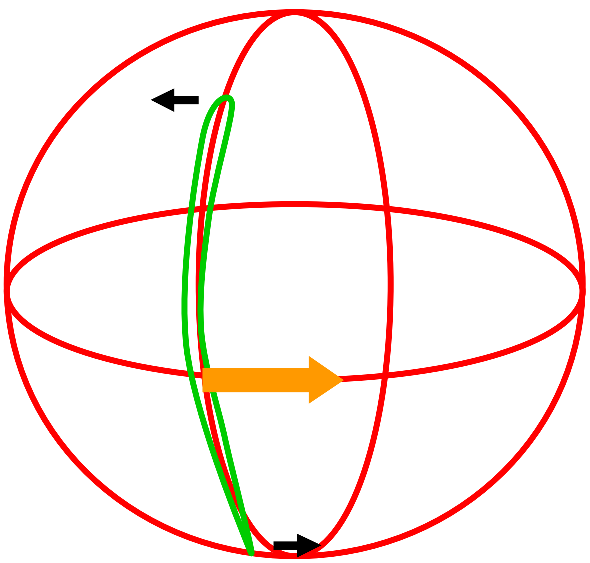
$$\left. -\frac{\lambda}{32\pi J^2}\int d\sigma\, d\tau \left[(\partial_\sigma \theta)^2 + \sin^2\theta (\partial_\sigma \phi)^2 \right] \right\}$$

Taking J large with λ/J^2 fixed: classical solutions

According to AdS/CFT there is a string description

particle: $X(t)$ string: $X(\sigma,t)$

We need S^3: $\underbrace{X_1^2+X_2^2}_{J_1}+\underbrace{X_3^2+X_4^2}_{J_2} = R^2$



CM: J_1
Rot: J_2

Action: $S[\,\theta(\sigma,t),\ \varphi(\sigma,t)\,]$, which,

for large J is: (agrees w/ f.t.)

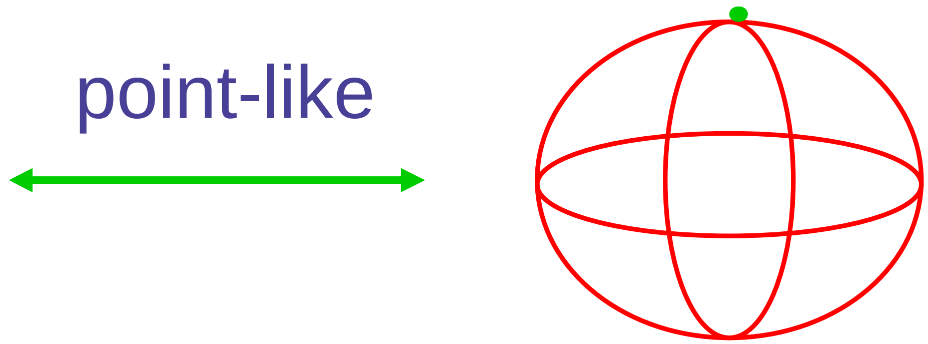
$$S_{eff.} = J\left\{-\frac{1}{2}\int d\sigma\, d\tau \left[\cos\theta\,\partial_\tau\phi - \frac{\lambda}{32\pi J^2}\left[(\partial_\sigma\theta)^2 + \sin^2\theta(\partial_\sigma\phi)^2\right]\right]\right\}$$

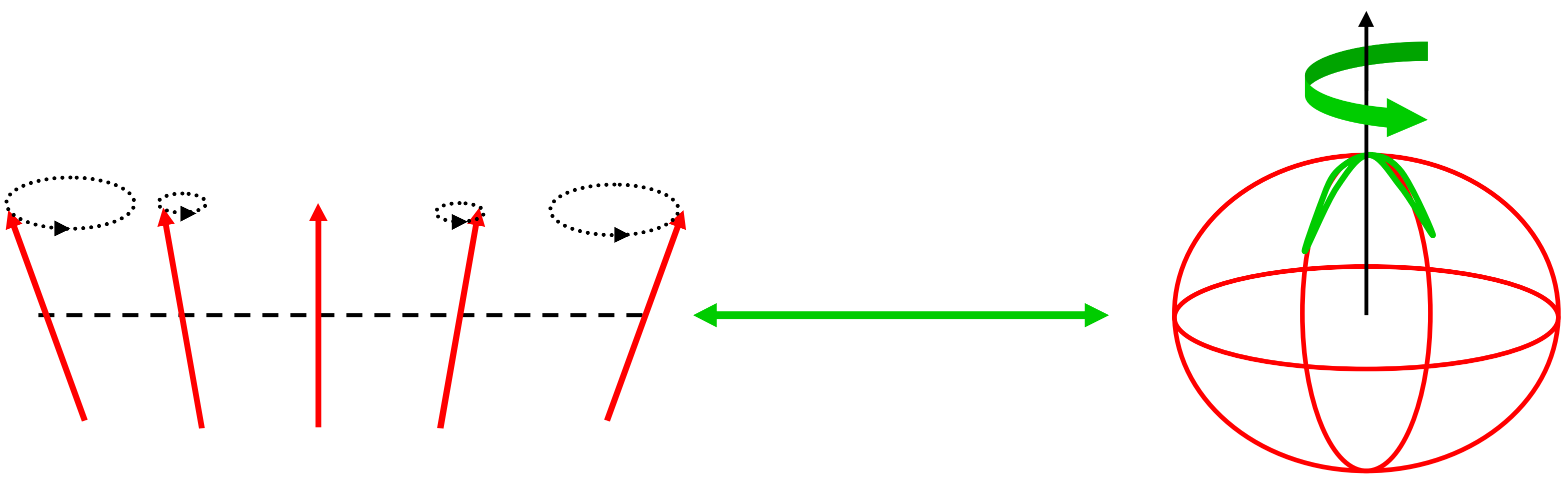
Suggests that $(\theta, \varphi) = (\theta, \varphi)$ namely that

$\langle \vec{S} \rangle$ is the position of the string

<u>**Examples**</u>

$| \uparrow \uparrow \ldots \uparrow \uparrow \uparrow \uparrow \rangle$





Strings as bound states

Fields create particles:

$$X \rightarrow |\mathbf{x}\rangle \,, \qquad Y \rightarrow |\mathbf{y}\rangle$$

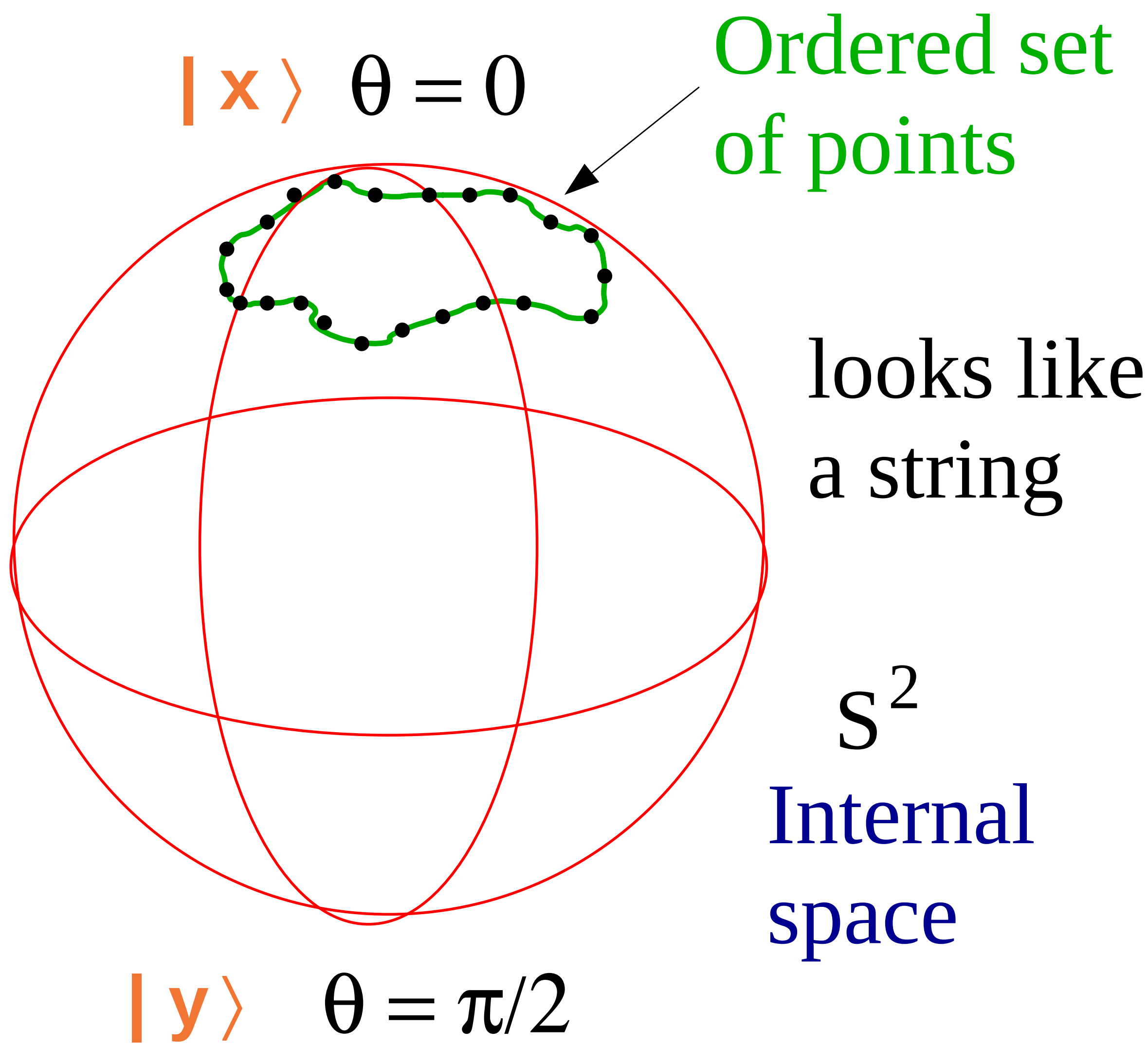
Q.M. : $|\psi\rangle = \cos(\theta/2)\exp(i\,\phi/2)\,|\mathbf{x}\rangle$

$$+ \sin(\theta/2)\exp(-i\,\phi/2)\,|\mathbf{y}\rangle$$

We consider a state with a large number of particles

$i=1\ldots J$ each in a state $v_i = |\psi(\theta_i, \phi_i)\rangle$. (Coherent state)

Can be thought as created by $O = \mathrm{Tr}\,(v_1 v_2 v_3 \ldots v_n)$



Strings are useful to describe states of a large number of particles (in the large–N limit)

Extension to higher orders in the field theory: O(λ^2)

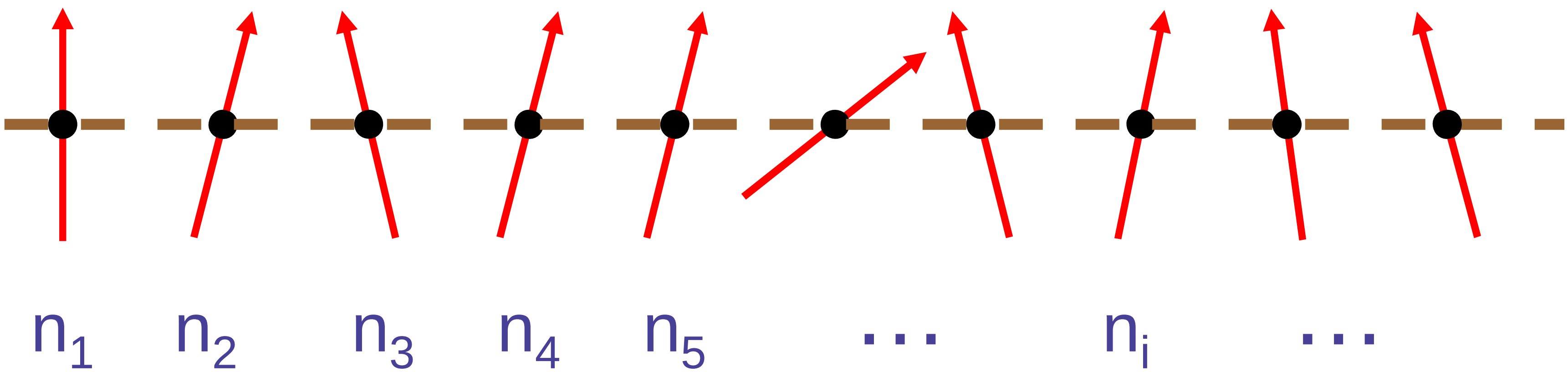
We need **H** (Beisert et al.)

$$H = \frac{\lambda}{4\pi}\sum_{j=1}^{J}\left(\frac{1}{4} - \vec{S}_j \cdot \vec{S}_{j+1}\right) +$$

$$+ \frac{\lambda^2}{128\pi^2}\left[-\frac{3}{2}J + 8\sum_{j=1}^{J}\vec{S}_j \cdot \vec{S}_{j+1} - 2\sum_{j=1}^{J}\vec{S}_j \cdot \vec{S}_{j+2}\right]$$

At next order we get second neighbors interactions

We have to define the effective action more precisely



Look for states $|\psi\rangle$ such that $\langle\psi|\vec{S}_j|\psi\rangle // n_j; \quad n_j^2 = 1$

From those, find $|\psi\rangle$ such that $\langle\psi|H|\psi\rangle = E(n_j)$=minimum

$$S = \int p\,\dot{q} - H(p,q) = -\frac{1}{2}\sum_i \int \cos\theta_i\,\partial_t\phi_i - E(n_i)$$

$$n_i = (\sin\theta_i\cos\varphi_i, \sin\theta_i\sin\varphi_i, \cos\theta_i)$$

After doing the calculation we get:
(Ryzhov, Tseytlin, M.K.)

$$S_{eff.} = -\frac{J}{2}\int d\sigma d\tau \cos\theta \partial_\tau \phi - J\int d\sigma d\tau \left\{ \frac{\lambda}{8J^2}\left(\partial_\sigma n\right)^2 \right.$$

$$\left. - \frac{\lambda^2}{32J^4}\left[\left(\partial_\sigma^2 n\right)^2 - \frac{3}{4}\left(\partial_\sigma n\right)^4\right]\right\}$$

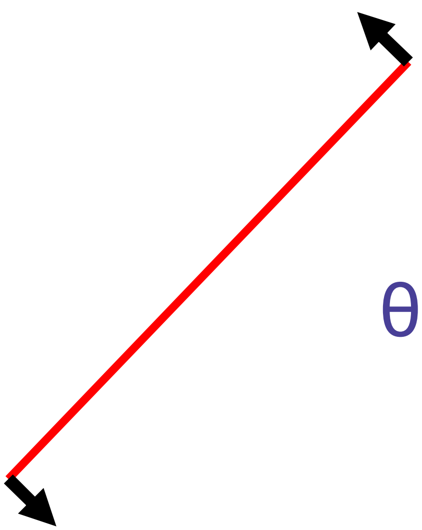
Agrees with string theory!

Rotation in AdS_5? (Gubser, Klebanov, Polyakov)

$$Y_1^2 + Y_2^2 + Y_3^2 + Y_4^2 - Y_5^2 - Y_6^2 = -R^2$$

$\sinh^2\rho;\ \Omega_{[3]}$ $\qquad$ $\cosh^2\rho;\ t$

$$ds^2 = -\cosh^2\rho\, dt^2 + d\rho^2 + \sinh^2\rho\, d\Omega_{[3]}^2$$



$\theta = \omega t$

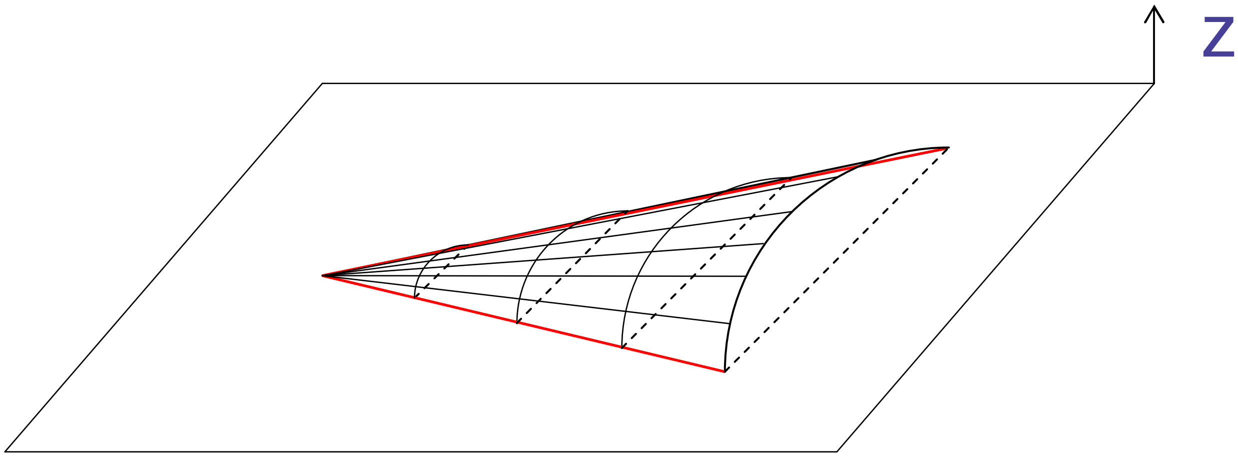
$$E \cong S + \frac{\sqrt{\lambda}}{2\pi}\ln S, \quad (S \to \infty)$$

$$O = Tr\left(\Phi\, \nabla_+^S \Phi\right), \quad x_+ = z + t$$

Verification using Wilson loops (MK, Makeenko)

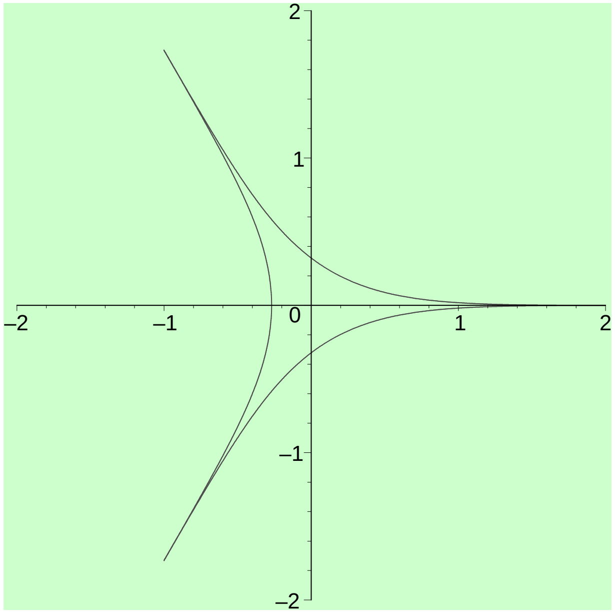
The anomalous dimensions of twist two operators can also be computed by using the **cusp anomaly** of light-like Wilson loops (Korchemsky and Marchesini).

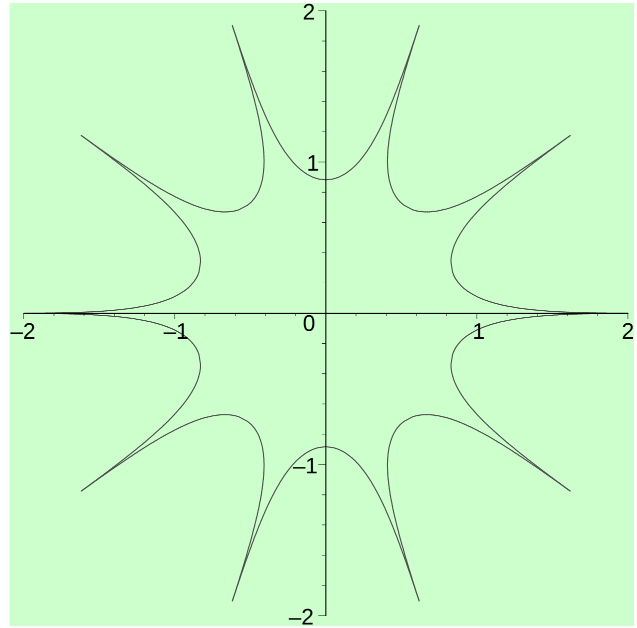
In **AdS/CFT** Wilson loops can be computed using surfaces of minimal area in AdS_5 (Maldacena, Rey, Yee)



The result **agrees** with the rotating string calculation.

Generalization to higher twist operators (MK)





$$E \cong S + \left(\frac{n}{2}\right) \frac{\sqrt{\lambda}}{2\pi} \ln S, \quad (S \to \infty)$$

$$O = Tr\left(\nabla_+^{S/n}\Phi\, \nabla_+^{S/n}\Phi\, \nabla_+^{S/n}\Phi \ldots \nabla_+^{S/n}\Phi\right)$$

$$S = \frac{\sqrt{\lambda}}{2\pi}\int dt \sum_j (\cosh 2\rho_1 - 1)\dot{\theta}_j - \frac{\sqrt{\lambda}}{8\pi}\int dt \sum_j \left\{ 4\rho_1 + \ln\left(\sin^2\left(\frac{\theta_{j+1} - \theta_j}{2}\right)\right)\right\}$$

Conclusions

AdS/CFT provides a unique possibility of analytically understanding the low energy limit of non-abelian gauge theories (confinement)

Two results:

- Computed the masses of quark / anti-quark bound states at **strong** coupling.

- Showed a way in which strings directly emerge from the gauge theory.