## Bernhard Riemann

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Mathematics played a major role in Gilles Deleuze's thought, beginning with his engagement with calculus and Gottfried Leibniz, who was also a major philosophical influence on Deleuze. Bernhard Riemann may, however, be the most significant mathematical influence on Deleuze. especially in his later works, such as the Cinema books, and in his collaborations with Félix Guattari. The conjunction of Riemann's mathematics and Deleuze's philosophy is a remarkable event in the history of twentieth-century philosophy, and it has major implications for our understanding of the relationships between mathematics and Deleuze's thought, and between mathematics and philosophy in general. Riemann's thought, however, is also part of the philosophical, and not simply mathematical, lineage of Deleuze's thought. Born from philosophy with the pre-Socratics, mathematics has a great philosophical potential, even though this potential is not always utilised in the disciplinary practice of mathematics. Riemann's work represents one of the greatest cases of exploring this potential and creating it, to begin with, in part by fusing philosophical ideas, such as those extending from post-Kantian philosophy, with his mathematical thinking. Deleuze, I would argue, takes advantage of both Riemann's mathematical and philosophical concepts in building his own philosophical concepts. Thus, the relationship between Riemann and Deleuze not only represents a remarkable conjunction of mathematics and philosophy but also establishes a philosophical friendship, as Deleuze and Guattari see it in What is Philosophy? (WP 4-5, 9-10).

### MATHEMATICS AND PHILOSOPHY IN RIEMANN

Bernhard Riemann (1826–66) was one of the greatest mathematicians of the nineteenth century and one of the greatest mathematicians who ever lived. His work rivals and sometimes outshines even that of such legendary figures as Sir Isaac Newton, Karl Friedrich Gauss (Riemann's teacher) and Evariste Galois, before him, and

Henri Poincaré and David Hilbert after him (often listed, along with Riemann, as the greatest mathematicians of the modern era). Riemann's ideas, moreover, had arguably the greatest impact (even compared to those of Poincaré and Hilbert, his main competitors in this respect) on mathematics in the twentieth and twenty-first centuries. Riemann also made a significant philosophical contribution, perhaps comparable to that of other mathematicians such as Pythagoras and Euclid, whose ideas have had, and continue to exert, a powerful philosophical impact. In particular, we can see the import of Riemann's non-Euclideanism on Deleuze. This claim concerning Riemann's philosophical contribution is somewhat unorthodox and requires qualification.

Although his extraordinary mathematical capabilities became apparent early on, Riemann, who was born to a Lutheran pastor's family, was initially trained in philology and theology. Later in his life he became well versed in post-Kantian German philosophy. These theological and philosophical (and earlier philological) interests had their impact on his mathematical ideas. Riemann, however, was not a philosopher, unlike, say, Descartes and Leibniz who, by and large, practised philosophy and mathematics as separate fields of inquiry, although their thought was shaped by a complex traffic between both fields. Riemann's philosophical concepts were developed primarily through his mathematical concepts. This may of course also be said of Descartes' and Leibniz's mathematical concepts, or of those of other mathematicians such as those listed above. But Riemann's capacity for developing and utilising this philosophical potential of mathematics is especially remarkable and his significance for Deleuze in this respect is unmatched - although Leibniz, Galois, Niels Henrik Abel, and Karl Weierstrass make similar contributions to Deleuze's work.

In his short mathematical career (he died of tuberculosis at the age of forty), Riemann made fundamental contributions to most areas of modern mathematics - algebra, analysis, geometry, topology and number theory. It would be impossible to do justice to these contributions even from the more limited philosophical perspective of this essay. One might argue, however, that from this perspective and as concerns his significance for Deleuze's philosophy Riemann's greatest contributions are, first, his concept of spatiality, and, secondly, his capacity of combining different fields in approaching problems apparently belonging to a single field. What I call non-Euclideanism, mathematical or philosophical, is conceived on the model of Riemann's thought and practice, as defined by these two phenomena.

Riemann's concept of spatiality as manifoldness allows one to define certain spaces as patchwork-like assemblages of local spaces, without, in general, the overall space possessing the same type of structure as these local subspaces do, while the latter may differ from each other as well. These qualities give a Riemannian space heterogeneity, which is, however, interconnected by virtue of the overlapping between local spaces. In particular, these local spaces can be considered as infinitesimally Euclidean, while the overall space is, in general, not a Euclidean space. The overall space may be given a global determination. In particular, it may be given an overall metrical structure, determined by the formula for measuring the distance between points that varies locally and that, infinitesimally (that is, when the two points in question are close to each other), converts itself into the formula for measuring distances in the Euclidean space. Such a space may have a constant curvature, as for example in the case of a two-dimensional sphere, which is a Riemannian manifold, or it can be a space as variable curvature, similar to a rolling-hill landscape. The Euclidean space of a given dimension, such as the twodimensional plane or three-dimensional space as we ordinarily perceive them, would be trivial cases of manifoldness in which both the local spaces involved and the overall spaces are Euclidean. Modern mathematics considers spaces, whether Euclidean or Riemannian, of any number of dimensions, including the infinite-dimensional spaces, and Riemann considered such spaces as well.

The second main component of non-Euclideanism is defined by the theoretical practice - mathematical or philosophical - of combining different fields in approaching objects defined or problems formulated within, and apparently belonging to, a single field. Riemann's concept of a manifold was developed by bringing together algebra, analysis and geometry, and thus by means of a multiple or manifold - heterogeneous yet interactive - theoretical practice that he deployed and expanded throughout his work. Riemann's multi-field approach to mathematical problems exemplifies the rise of a new type of mathematical practice, defined by the multiply interactive and yet heterogeneous workings of different mathematical fields - geometry, topology, algebra, analysis, and so forth - in dealing with a single concept or problem, without there necessarily being a wholeness or oneness governing this multiplicity. One can thus easily perceive shared features in the 'space' of practice and in Riemann's concept of spatiality as manifoldness, and certain aspects of Riemann's thinking are manifest in both. While it would be difficult to simply map Riemann's concept of spatiality onto his practice, this type of spatial thinking and this type of practice often go hand in hand and variously overlap, and hence can partially map each other, in non-Euclidean thinking, whether mathematical, such as that of Riemann, or philosophical, such as that of Deleuze.

Thus understood, mathematical non-Euclideanism extends far beyond the ideas that led to the alternative geometries with which the term 'non-Euclidean' originated, important as their discovery in early 1800s was in this context. Riemann discovered one type of such geometries - those of positive curvature. There are also those of negative curvature, and Euclidean geometry itself has zero curvature, that is, is flat. Riemann's concept of the manifold allowed him to encompass both Euclidean and non-Euclidean geometry within a single more general concept, which also enabled it to serve as the mathematical basis for Einstein's non-Newtonian theory of gravity. known as general relativity. One finds certain ingredients of non-Euclideanian plural practice in ancient Greek mathematics, specifically in the relationships between arithmetic and geometry. Indeed, the unresolved complexity of these relationships has continued to haunt mathematics ever since, with algebra having eventually supplanted arithmetic, and Riemann's thought and his concept of manifolds reflect this complexity. Nevertheless, the eruptive emergence of plural mathematics on a large scale in the early 1800s, roughly at the time of Gauss (Riemann's teacher and precursor in this respect as well), was one of the most significant developments in the history of mathematics. One finds this mathematics at work throughout the nineteenth century and then, with ever increasing effectiveness, in the twentieth and twenty-first centuries.1

Riemann's thought is among the greatest early manifestations of non-Euclideanism not only in mathematics but also in philosophy, using the term philosophy in Deleuze and Guattari's sense of the invention of new concepts, or even concepts 'that are always new' (WP 5). This sense is also defined by a different *concept* of the philosophical concept itself. A philosophical concept is not an entity established by a generalisation from particulars or 'any general or abstract idea' (WP 11–12, 24) but a multi-layered conglomerate entity: 'there are no simple concepts. Every concept has components and is defined by them. It therefore has a combination [*chiffre*]. It is a multiplicity [manifold(ness)?] . . . There is no concept with only one component' (WP 16). Each concept is a multi-component conglomeration of concepts (in their conventional senses), figures, metaphors, and so forth,

which form a unity or have, as Deleuze's own concepts often do, a more heterogeneous, if interactive, architecture that is not unifiable. The architecture of Deleuze and Guattari's concept of the concept is itself defined in part by Riemannian spatiality as manifoldness by linking the very invention of philosophical concepts to a spatial and, in part, Riemannian concept - the plane of immanence - thus making the space of functioning of a given concept a Riemannian space. This concept of a concept is traceable to Deleuze's earlier texts, and the activity of creating concepts may be seen as defining Deleuze's work. Equally significantly, each concept is also seen as a problem, another hallmark of Deleuze's philosophy. From Difference and Repetition to What is Philosophy?, philosophical thinking is seen, on a mathematical model, as problematic (thinking defined by posing problems) rather than theorematic (thinking proceeding by deriving propositions from axioms according to proscribed rules, in the manner of Euclid's Elements, rather than by posing problems). Difference and Repetition appeals to Abel's and Galois' mathematical or, again, mathematical-philosophical practice as paradigmatic examples, and states that 'Ideas are essentially "problematic", while 'conversely, problems are ideas' (DR 168).

Certain forms of mathematical thought, such as that of Riemann, may be seen in Deleuze and Guattari's philosophical terms. That is, one can extend to mathematical thinking, as Deleuze and Guattari in effect do, their definition of philosophical thinking and of philosophical concepts themselves, even though, as I shall discuss presently, they are also right to stress the disciplinary difference between mathematics and philosophy (WP 117–18). According to Deleuze:

There are two sorts of scientific concepts. Even though they get mixed up in particular cases. There are concepts that are exact in nature, quantitative, defined by equations, and whose very meaning lies in their exactness: a philosopher or writer can use these only metaphorically, and that's quite wrong, because they belong to exact science. But there are also essentially inexact yet completely rigorous concepts that scientists can't do without, which belong equally to scientists, philosophers, and artists. They have to be made rigorous in a way that's not directly scientific, so that when a scientist manages to do this he becomes a philosopher, an artist, too. This sort of concept's not unspecific because something's missing but because of its nature and content. (N 29, translation modified)

Thus, a *philosophical* concept corresponding to a mathematical or scientific object could also be discovered by mathematics and science,

now working as philosophy on Deleuze and Guattari's definition. Thus, as they contend, 'when an object - a geometrical space, for example - is scientifically constructed by functions, its philosophical concept, which is by no means given in the function, must still be discovered' (WP 117). On the other hand, it is a complex question where and how, and in what order of invention, among mathematics, physics and philosophy, a given philosophical concept of space, say, Euclidean or Riemannian, has emerged. In particular, Riemann may be seen as primarily responsible not only for many key mathematical (geometrical and topological) features of his concept of space as manifold but also for many of its key philosophical aspects, even though both Leibniz before him and Einstein after him contributed significantly on both scores. Deleuze's appeal to the numerical or quantitative nature of scientific concepts may have been made with the question of mathematical versus philosophical spatiality, and Riemann as well as Bergson, in mind. Deleuze and Guattari juxtapose the (qualitative) concept of distance and the (quantitative) concept of magnitude, related to the juxtaposition (due to Pierre Boulez) of the smooth and the striated spaces in A Thousand Plateaus (TP 483-4). Similarly to Deleuze and Guattari's use of Riemann's concept of manifoldness, Bergson's duration may be seen as, in part, a distillation of an inexact, qualitative concept from Riemann's 'metric manifoldness or the manifoldness of magnitude' (TP 483; translation modified).2

Hence, Deleuze is both cautious concerning the use of mathematics and science in philosophy, and yet also defends its use. As he says in Cinema 2 – which, like Cinema 1 (guided by Bergson's philosophy), uses the idea of Riemannian spaces, to which this statement also refers:

We realize the danger of citing scientific propositions outside their own sphere. It is the danger of arbitrary metaphor or of forced application. But perhaps these dangers are averted if we restrict ourselves to taking from scientific operators a particular conceptualizable character which itself refers to non-scientific areas, and converge with science without applying it or making it [simply] a metaphor. (TI 129)

#### PHILOSOPHY AND MATHEMATICS IN DELEUZE AND GUATTARI

In What is Philosophy? Deleuze and Guattari define thought in terms of its confrontation with chaos, a great enemy and a great friend of thought and its indispensable ally in its yet greater struggle against

opinion, doxa (WP 201–2). Mathematics or science, philosophy and art, are particular forms of thought in this confrontation (WP 118, 201–18). Chaos itself is given a particular concept as well:

Chaos is defined not so much by its disorder as by the infinite speed with which every form taking shape in it vanishes. It is a void that is not a nothingness but a *virtual*, containing all possible *particles* and drawing out all possible forms, which spring up only to disappear immediately, without consistency or reference, without consequence. (WP 118)

The difference between philosophical and scientific, including mathematical, thinking, as confrontation with chaos, is defined by their determination in terms of, respectively, concepts and functions. (Mathematically, functions rigorously relate numbers or other entities to each other according to specified rules.) According to Deleuze and Guattari:

The object of science is not concepts but rather functions that are presented as propositions in discursive systems. The elements of functions are called functives. A scientific notion is defined not by concepts but by functions or propositions. This is a very complex idea with many aspects, as can be seen already from the use to which it is put by mathematics and biology respectively. Nevertheless, it is this idea of the function that enables the sciences to reflect and communicate. Science does not need philosophy for these tasks. On the other hand, when an object - a geometrical space, for example - is scientifically constructed by functions, its philosophical concept, which is by no means given in the function, must still be discovered. Furthermore, a concept may take as its components the functives of any possible function without thereby having the least scientific value, but with the aim marking the differences in kind between concepts and functions. . . . Philosophy wants to know how to retain the infinite speed while gaining consistency, by giving the virtual a consistency specific to it. The philosophical sieve, as a plane of immanence that cuts though chaos, selects infinite movements of thought and is filled with concepts formed like consistent particles going as fast as thought. Science approaches chaos in a completely different, almost opposite way: it relinquishes the infinite, infinite speed, in order to gain a reference able to actualize the virtual. By retaining the infinite, philosophy gives consistency to the virtual through concepts; by relinquishing the infinite, science gives a reference to the virtual, which actualizes it through functions. Philosophy proceeds with a plane of immanence and consistency; science with a plane of reference. In the case of science it is like a freeze-frame. It is a fantastic slowing down. (WP 117-18; translation slightly modified)

Philosophy's thought, thus, tries to hold to a concept that traverses a plane of immanence with the infinite speed of thought and to give this plane consistency. The plane of immanence is itself a complex multi-component philosophical concept (WP 35–61). The main point here is that, in contrast to philosophy, science 'freezes' chaos in slow motion or freeze-frames – sometimes, especially in physics, literally photographs the physical processes considered. By so doing science creates a plane of reference or co-ordination that it requires as science.

While, however, the differences between philosophy and science, or between either and art, appear to be irreducible, the interaction between them appears to be unavoidable as well. Thus, philosophy's thought may sometimes *hold* to a virtual concept by slowing-down or freeze-framing it. Conversely, science sometimes proceeds with the philosophical infinite speed on (and by creating) the plane of immanence, in order to create a *philosophical* concept corresponding to a mathematical or scientific object or in order to create this object. Philosophy and science appear to need each other, as Deleuze and Guattari say in closing their discussion of the difference between philosophy and science in *What is Philosophy?* According to them:

If philosophy has a fundamental need for the science that is contemporary with it, this is because science constantly intersects with the possibility of concepts and because concepts necessarily involve allusions to science that are neither examples nor applications, nor even reflections. Conversely, are there functions – properly scientific functions – of concepts? This amounts to asking whether science is, as we believe, equally and intensely in need of philosophy. But only scientists can answer that question. (WP 162)

The answer, I would argue, would be positive, at least if one asks good scientists. Deleuze and Guattari suggest as much in closing their book, by noting that science at least 'tries' to create 'functions of concepts, as Lautman demonstrates for mathematics insofar as the latter actualizes virtual concepts' (WP 217).

#### MANIFOLDS IN RIEMANN AND DELEUZE AND GUATTARI

According to Riemann, in his habilitation lecture 'On the Hypotheses which Lie at the Bases of Geometry', which introduced the ideas of manifold and Riemannian geometry:

The concepts of magnitude are only possible where there is an antecedent general concept which admits of different specializations. According as there exists among these specializations a continuous path from one to another or not, they form a continuous or discrete manifoldness [Mannigfaltigkeit]; the individual specializations are called in the first case points, in the second case elements, of the manifoldness. Concepts whose specializations form a discrete manifoldness are so common that at least in the cultivated languages any things being given it is always possible to find a concept in which they are included. . . . On the other hand, so few and far between are the occasions for forming concepts whose specializations make up a continuous manifoldness, that the only simple concepts whose specializations form a multiply extended manifoldness are the positions of perceived objects and colours. More frequent occasions for the creation and development of these concepts occur first in the higher mathematics.3

Riemann thus defines manifolds not in terms of ontologically pregiven assemblies, 'sets', of points and relations between them, but in terms of concepts. Each concept has a particular mode of determination, such as a discrete versus a continuous manifold, whose elements. such as points, are related through this determination. Thus, beyond giving an essential priority to thinking and specifically to thinking in concepts over calculational or algorithmic approaches - to the point of, in this case, containing only one (!) formula in the whole lecture - Riemann's mathematical thinking is structurally conceptual, which brings it close to philosophical thinking in Deleuze and Guattari's sense. It is based on specifically determined concepts, as against the set-theoretical mathematics that followed him or the mathematics of formulas that preceded him.4 Continuous and discrete manifolds are given different conceptual determinations, and thus become, in effect, different concepts, the point noted by Deleuze and Guattari (TP 32). It is significant, and adds to the conceptual difference between two types of manifoldness, that Riemann speaks of 'points' only in the case of continuous manifolds, and in the case of discrete manifolds uses the term 'elements', for the simplest constitutive entities comprising manifolds. This is astute, since, phenomenally, points qua points only appear as such in relation to some continuous space, ambient or background, present or implied, such as a line or a plane. Riemann primarily pursues the conception of space as a continuous manifold, for which the modern mathematical usage of the term is primarily reserved as well.5 A manifold is, as I said, defined as a conglomerate of local spaces, which can be infinitesimally mapped by a (flat) Euclidean or Cartesian map without allowing for a global Euclidean map or a single co-ordinate system for the whole, except in the case of the Euclidean space itself. In other words, every point has a small neighbourhood that can be treated as Euclidean, while the manifold as a whole in general cannot.

As noted above, one of the starting points of Riemann's reflection on space was the possibility of non-Euclidean geometry, which also led him to a particular new type of non-Euclidean geometry, that of positive curvature. This also means that there are no parallel shortest or, as they are called, geodesic lines crossing any point external to a given geodesic. In Euclidean geometry, where geodesics are straight lines, there is only one such parallel line, but in non-Euclidean geometry of negative curvature or the hyperbolic geometry of Gauss, Johann Bolyai and Nikolai I. Lobachevsky - the first non-Euclidean geometry discovered - there are infinitely many such lines. Riemannian geometry encompasses all of these as special cases. Significant as the discovery of non-Euclidean geometry was for the history of mathematics and intellectual history, it was also in retrospect, as Hermann Weyl argued, 'a somewhat accidental point of departure' for Riemann's radical rethinking of the nature of spatiality.6 Riemannian geometry is that of (continuous) manifoldness, an approach that makes both Euclidean and non-Euclidean spaces only particular cases of this more general understanding of space. Weyl speaks of Riemannian geometry as 'a true geometry': 'This theory . . . is a true geometry, a doctrine of space itself and not merely like Euclid, and almost everything else that has been done under the name of geometry, a doctrine of the configurations that are possible in space." Deleuze and Guattari agree and take the point further by also crediting Riemann with the creation of a new philosophical conceptuality: 'It was a decisive event when the mathematician Riemann uprooted the multiple [manifold] from its predicate state and made in to a noun, 'manifold' [multiplicité]' (TP 482-3; translation modified). They also acknowledge the role of discrete manifolds in Riemann, and the significance of still other spaces, such as porous spaces, in mathematics and elsewhere. Citing Lautman, they describe, Riemannian or Riemann spaces as (continuous) manifolds as follows:

'Riemann spaces are devoid of any kind of homogeneity. Each is characterized by the form of the expression that defines the square of the distance between two infinitely proximate points. . . . It follows that two neighboring observers in a Riemann space can locate the points

in their immediate neighborhood but cannot locate their spaces in relation to each other without a new convention. Each vicinity is therefore like a shred of Euclidean space, but the linkage between one vicinity and the next is not defined and can be effected in an infinite number of ways. Riemann space at its most general thus presents itself as an amorphous collection of pieces that are juxtaposed but not attached to each other.' It is possible to define this multiplicity without any reference to a metrical system, in terms of the conditions of frequency, or rather accumulation, of a set of neighborhoods: these conditions are entirely different from those determining metric spaces and their breaks (even though a relation between the two kinds of space necessarily results). In short, if we follow Lautman's fine description, Riemannian space is pure patchwork. It has connections, or tactile relations. It has rhythmic values not found elsewhere, even though they can be translated into a metric space. Heterogeneous, in continuous variation, it is a smooth space, insofar as smooth space is amorphous and not homogeneous. We can thus define two positive characteristics of smooth space in general: when there are determinations that are part of one another and pertain to enveloped distances or ordered differences, independent of magnitude; when, independent of metrics, determinations arise that cannot be part of one another but are connected by processes of frequency or accumulation. These are the two aspects of the nomos of smooth space. (TP 485: translation modified)

The cartographical terminology and conceptuality, crucial to Deleuze (and Foucault, whom Deleuze discusses from this perspective in Foucault), are not accidental and have their own history. Gauss arrived at his ideas, extended by Riemann, through his work in land surveying. The spatial architecture here outlined can be generalised to spaces that are not manifolds, that is, to spaces that are defined as patchworks of local spaces that are not infinitesimally Euclidean. These local spaces could be, in the language of Cinema 1, 'any spaces whatever'. This architecture is, however, inherent in Riemannian manifolds, from which it was in part developed historically, since manifolds are in the first place topological (non-metrical), rather than only geometrical (metrical) spaces. The function of Riemannian spaces as smooth spaces (in Deleuze and Guattari's sense) is defined by their topology, by their (in Boulez's and Deleuze and Guattari's language) 'rhythmic' properties, rather than by their geometry or their 'metric' (this language is also mathematical) properties (TP 485). In contrast to geometry (geo-metry), which has to do with measurement, topology disregards measurement and scale, and deals only with the structure of space qua space and with the essential shapes of figures. Insofar as one deforms a given figure continuously (that is, insofar as one does not separate points previously connected and, conversely, does not connect points previously separated) the resulting figure is considered the same. Thus, all spheres, of whatever size and however deformed (say, into pear-like shapes), are topologically equivalent. They are, however, topologically distinct from tori. Spheres and tori cannot be converted into each other without disjoining their connected points or joining the disconnected ones. The holes in tori make this impossible. Such qualitative topological properties can be related to certain algebraic and numerical properties associated with these spaces, which topology indeed must do as a mathematical discipline, unlike philosophy, where this is not necessary, as Bergson's or Deleuze's qualitative use of Riemann's ideas shows. Anticipated by Leibniz, these ideas were gradually developed in the nineteenth century by - in addition to Riemann - Gauss, Poincaré and others, establishing topology as a mathematical discipline by the twentieth century.

Topological spaces need not have any metric structure or striation, either global or local. Global Euclidean/Cartesian striations are not found in Riemannian spaces (apart, again, from special cases, such as those of Euclidean spaces), while local ones are allowed but not required. This is why Deleuze and Guattari say above that Riemannian space 'has rhythmic values not found elsewhere, even though they can be translated into a metric space' and hence that 'a relation between the two kinds of space necessarily results' in Riemannian space (TP 485). When we consider the discussion of space in A Thousand Plateaus, we can see that, smooth (nomadic) spaces almost inevitably give rise to local striations (reterritorialisation), even as they simultaneously arise from them (deterritorialisation) - in other words, they again lead to Riemannian spaces as both smooth and (locally) striated. The nomos of the smooth space (as against the logos of the striated space) is defined by the rhythmic interplay of connectivities between neighbourhoods, which defines topological spaces in general rather than Riemannian spaces, defined by local Euclidean striations. Accordingly, it appears that the underlying mathematical model of 'Riemann space at its most general' and, by extension, of smooth space in Deleuze and Guattari, is a general topological space, which, however, underlies any Riemannian space.

Deleuze takes advantage of these ideas throughout his work. The Cinema books are built in part upon Riemannian spatiality, via Bergson, whose ideas were indebted to Riemann. Cinema 2 offers spectacular examples of this 'Riemannianism' as against Euclideanism: 'Riemannian spaces in Bresson, . . . topological spaces of Resnais' (TI 129). It also explores the far-reaching implications – aesthetic, philosophical and cultural, including political – of Riemannianism.

# Manifoldness and Materialism in Riemann and in Deleuze and Guattari

Riemann's radical rethinking of spatiality offers an extension of Gauss's ideas concerning the internal geometry of curved surfaces. that is, a geometry independent of the ambient (three-dimensional) Euclidean space where such curved spaces could be placed. This view of space also allows one to extend Leibniz's ideas concerning the relational nature of all spatiality. The actual space is now no longer seen as a given, ambient (flat) Euclidean space or, in Weyl's words, a 'residential flat' (flat is a fitting pun here), where, phenomenally, geometrical figures or, physically, material things are placed.8 Instead it emerges as a (continuous) manifold, whose structure, such as curvature, would be determined internally, mathematically or materially (for example, by gravity, as in Einstein's general relativity theory, based on Riemannian mathematics), rather than in relation to an ambient space, Euclidean or not. From this point of view, the concept of empty space might be entertained mathematically or phenomenally, but, as Leibniz grasped, it is difficult to apply this concept to the physical world. According to Leibniz, space cannot be seen as a primordial ambient given, as a container of material bodies and the background arena of physical processes, along the lines of Newton's concept of absolute space in his *Principia* – the most influential and, in many respects, defining form of Euclideanism in all of modernity. Einstein gave a rigorous physical meaning to these ideas and extended them by arguing that space, or time, are not given but arise, are the effects of our instruments, such as rods and clocks, and, one might add, of our perceptual and conceptual interactions with those instruments. Space is thus possible as a phenomenon (or a concept) by virtue of two factors. The first is the presence of matter and technology, such as rods and clocks (or natural objects that function in this role). The second is the role of our perceptual phenomenal machinery, a role that one might argue to be the primary condition of the possibility of space, along with time, which machinery is still due to the materiality of our bodies.

Riemann offers extraordinary intimations of Einstein's theory, based on his ideas discussed here. According to Weyl:

Riemann rejects the opinion that has prevailed up to his own time, namely, than the metrical structure of space is fixed and [is] inherently independent of the physical phenomena for which it serves as a background, and that the real content takes possession of it as of residential flats. He asserts, on the contrary, that space in itself is nothing more than a three-dimensional manifold devoid of any form; it acquires a definite form only through the advent of the material content filling it and determining its metric relations.<sup>9</sup>

It would be more accurate (and closer to Riemann) to say that space may be given phenomenally at most as a three-dimensional manifold, as a kind of free smooth space with possible striations. Physically, it may be and, in Riemann's and Einstein's or Leibniz's view, could only be, co-extensive with matter. Weyl adds: 'Looking back from the stage to which Einstein brought us, we now recognize that these ideas can give rise to a valid [physical] theory only after time has been added as a fourth dimension to the three-space dimensions.'10 The gravitational field determines the manifold in question and its in general variable curvature. The reverse fact, that the gravitational field shapes space and shapes it as a Riemannian manifold, remains crucial, however. Different spaces become subject to investigation in their own terms, on equal footing, rather than in relation to an ambient or otherwise uniquely primary space. This view radically transforms our philosophy of space and matter, and of their relationships, by leading to a horizontal rather than vertical (hierarchical) science of space as 'a typology and topology of manifolds', which Deleuze and Guattari associate with the end of dialectic and extend to spaces that are philosophical, aesthetic, cultural, or political (TP 483; translation modified).

Deleuze and Guattari's 'physical model' of the smooth and the striated converts this transformation into a grand conceptual and historical conjunction of physics and political economy, and of both with geometry (TP 490). The technological model – specifically that of textile technology, a 'weaving' model (from Plato on) – is seen in these terms as well, in part given that the origins of the capitalist economy and labour can be especially traced to textile manufacturing in Florence, to the 'space', smooth and striated, of the Renaissance. The Renaissance (if one can still speak of one) was also a Renaissance of geometry in mathematics, science, philosophy

and art; and 'perspective', a great Renaissance striation, is only one of its aspects. The overall situation can be traced back to Galileo or to the ancient Greek mathematicians, specifically Archimedes, and to the role of geometry and physics as state, major sciences of (and in) striated spaces and as nomad, minor sciences of (and in) smooth spaces, and their interactions (TP 362). Both Galileo and Archimedes were military engineers (as was Leonardo), and Newton became a powerful state figure, the president of the Mint, thus moving from mathematics to money. From ancient Greece onwards, 'Geometry lies at the crossroads of a physics problem and an affair of the State' (TP 489). The terms of this sentence are transposable: 'Physics lies at the crossroads of a geometry problem and an affair of the State.'

Gaspar Monge, a key representative of state mathematics in A Thousand Plateaus, was instrumental in setting up, in the late eighteenth century, the famous École Politechnique as a state Institution (in either sense), where the most rigorous training in pure mathematics was combined with equally rigorous training in applied sciences and engineering. A major role in this programme was given to the new discipline of differential geometry, which combined geometry and calculus. Calculus, especially in the work of Newton and Leibniz, can, as both a major and a minor science, be considered from this perspective. Differential geometry, however, became a minor science in Gauss's work, eventually leading to Riemann's geometry and then to Einstein's physics. The nineteenth century brought physics and geometry into a new conjunction, under equally revolutionary developments of both the politico-economic history of capitalism and of the social and economic sciences, from Adam Smith onward.

The same type of matrix, interactively Riemannian and materialist, defines a vertiginous landscape, from brain to politics, that emerges in Cinema 2 and towards the end of What is Philosophy? The Riemannianism of What is Philosophy? is more implicit, and yet equally powerful. The philosophised concept of Riemannian spaces appears by name at a crucial juncture, that of the interference of mathematics and philosophy (WP 217). The space of such interference is defined by and manifest in the ultimate dynamics of thought as a confrontation with chaos and, through that confrontation, 'extracted from chaos', the shadow of a political world yet to come, in which even philosophy, art and science may dissolve, while still leaving space for thought itself as a confrontation with chaos (WP

216–18). 'In this submersion [of brain into chaos] it seems that there is extracted from chaos the shadow of the "people to come" in the form that art, but also philosophy and science, summon forth: masspeople, world-people, brain-people, chaos-people' (WP 218). The same type of intersection of the brain, thought, chaos and a 'people to come' defines the closing chapters of *Cinema 2*, especially Chapter 8, 'Cinema, Body and Brain, Thought' (TI 189–224).

I can only briefly sketch here the Riemannian dimensions of these extraordinary pages of both books. Roughly, at stake here are the complex - heterogeneously interactive and interactively heterogeneous - relationships not only between neighbourhoods in a Riemannian space, but also between such spaces themselves. Our mathematics and physics on the one hand, and our neuroscience on the other, tell us that, to the degree that the processes that define nature and life, and our brains (neural networks), can be mapped, they are likely to be mapped in terms of Riemannian spaces, and of the interplay of the smooth and the striated within them. The same mapping needs to be deployed when we approach our politics and culture. It is not only a matter of mirroring such Riemannian manifoldness from inanimate nature to life to bodies to brains to thought to culture to politics, but also and primarily that of contiguous relations that manifoldly connect these manifolds. This is a new kind of 'landscape architecture', the architecture of many landscapes, in which these spaces coexist and horizontally interact, without necessarily mirroring each other.

Leibniz's monadology could be viewed from this perspective as well, and Deleuze and Guattari juxtapose 'monads' to 'the unitary Subject of Euclidean space' (TP 574, n. 27). This monadology must, however, become nomadology in the new, post-Riemannian Baroque, as against the old, Leibnizian Baroque. Leibniz's monads ultimately interact with each other only through their interaction with the world, whose overall interactive architecture is, in the Leibnizian Baroque, containable in and converging upon a harmony, fully available to, or calculable by, only God (see FLB 26). The divergent harmonies of the new Baroque retain the fold, made manifold, but convert monadology into nomadology, which contains but is not reducible to monadology (FLB 137). The chapter in A Thousand Plateaus, 'The Smooth and the Striated', may also be read in terms of this link between 'Riemannology' and nomadology in various models of the smooth and the striated - especially dramatically in the musical and the aesthetic models. The first is exemplified by the work of Boulez, who introduced the language of 'the smooth and the striated' and who is also a key figure of the new Baroque in *The Fold*; the second by the work of Cézanne and the painters who came after him (TP 477–8, 493–4). This conversion of Leibniz's monadology into Riemann's nomadology is expressly linked to Riemannian space, against Euclidean space:

All of these points already relate to Riemannian space, with its essential relation to 'monads' (as opposed to the unitary Subject of Euclidean space). . . . Although the 'monads' are no longer thought to be closed upon themselves, and are postulated to entertain direct step-by-step local [Riemannian] relations, the purely monadological point of view proves inadequate and should be superseded by a 'nomadology' (the identity of striated spaces versus the realism of smooth space). (TP 573–4)

We can now readily perceive why Deleuze and Guattari see Riemann's mathematics of manifolds as implying a kind of horizontal rather than vertical, hierarchical science of space as 'a [nondialectical] typology and topology of manifolds' (TP 483; translation modified). This view suggests a new – horizontal – space of science itself, or a new space of thought and different ways to think, either within a given discipline, such as mathematics or philosophy, or (but this is now the same) between and among disciplines. We can think of spaces or landscapes of thought and culture in an interactively heterogeneous way – in terms of distinct and varied but actually and potentially interactive maps, arranged and related horizontally rather than vertically or hierarchically. Anticipated by Riemann's practice of mathematics through the interactions of different fields – topology, geometry, algebra, analysis, and so forth – this practise defines non-Euclideanism, mathematical and philosophical, such as that of Deleuze.

It is difficult to avoid the conclusion that the passage on Riemannian space cited above also describes the chapter 'The Smooth and the Striated', with its different but, again, interactive models – the technological, the musical, the maritime, the mathematical, the physical, the aesthetic (nomad-art), etc. – and A Thousand Plateaus as a whole. I list only those expressly named by Deleuze and Guattari, whose analysis implies many other possible models, a thousand models. In part these different models are necessary to establish certain general or shared aspects of (more) abstract concepts of the smooth and the striated (TP 475). Most crucial, however, is that these models enable an exploration of various aspects of each type of space and of the

relationships between them, and of spaces, heterogeneously interactive assemblages, of such spaces, manifolds of manifolds (TP 475).

Remarkably, this type of concept was introduced by Riemann in considering the families of the so-called Riemannian surfaces (such as tori). This type of object, known as 'moduli spaces', is one of the most extraordinary conceptions in modern mathematics; it was, for example, instrumental in proving Fermat's last theorem, by Andrew Wiles, one of the greatest achievements of contemporary mathematics. This concept, however, cannot be only mathematical, or only mathematical and philosophical. It is something more than either or both. Mathematics proves itself to be more like thought and life (which is more complex than thought) than thought and life prove to be like mathematics – that is, mathematics understood, as it has been all to often, as an abstraction from the richness, the manifoldness, of life. The idea of a manifold of manifolds is a product of thought as a confrontation with chaos and part of a shadow of the future – of things, thoughts, and the people to come.

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#### Notes

- I have considered the subject in more detail in The Knowable and the Unknowable: Modern Science, Nonclassical Thought, and the 'Two Cultures' (Ann Arbor: University of Michigan Press, 2002), pp. 126-36, 266-8, and nn. 24-6).
- The English translation by Brian Massumi uses 'multiplicity' to render the French 'multiciplité'. The English mathematical term is manifold, which also preserves the 'fold' of Riemann's Mannigfaltigkeit.
- 3. Bernhardt Riemann, 'On the Hypotheses which lie at the Bases of Geometry', trans. W. K. Clifford, *Nature* 8 (1873), section #1; translation modified. The lecture, given in 1854, was published posthumously in 1868. The English translation I cite is available at http://www.maths.tcd.ie/pub/HistMath/People/Riemann/Geom/WKCGeom.html.
- Cf. D. Laugwitz, Bernhard Riemann: Turning Points in the Conception of Mathematics, trans. A. Shenitzer (Boston: Birkhäuser, 1999), pp. 303-7, which, however, takes a more conventional view of Riemann's conceptual mathematics.
- Technically, Riemann considered the so-called differential manifolds, meaning that one can define differential calculus on them.
- H. Weyl, Space Time Matter, trans. Henry L. Brose (New York: Dover, 1952 [1918]), p. 92.
- 7. Weyl, Space Time Matter, p. 102.
- 8. Weyl, Space Time Matter, p. 98.

- 9. Weyl, Space Time Matter, p. 98.
- Weyl, Space Time Matter, p. 101. The resulting spaces are significant in the context of the question of temporality in Bergson and in Deleuze, especially in The Logic of Sense.