

The Art and Science of Experimentation in Quantum Physics

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Abstract. Taking its historical point of departure in Heisenberg's work, this article offers a view of quantum mechanics as, arguably, the first truly experimental and truly mathematical physical theory, that is, a theory concerned with *experimenting* with nature and mathematics alike. It is truly experimental because it is not, as in classical physics, merely the independent behavior of the system considered, in other words, what happens in any event, that we track, but what kind of experiments we perform that defines what happens. By the same token, the theory is also truly mathematical because, at least in the interpretation adopted here, its mathematical formalism does not stand in the service of a mathematical description of (quantum) physical processes in space and time in the way the formalism of classical physics does, but is only used to predict the outcomes of relevant experiments. It also follows that quantum theories experiment more freely with mathematics itself, since we invent predictive mathematical schemes, rather than proceed by refining mathematically our phenomenal representations of nature, which process constrains us in classical mechanics.

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1. A PHILOSOPHICAL INTRODUCTION

The argument of this article proceeds along three different but interrelated and ultimately converging tracks—physical, philosophical, and historical. “Mathematical” could be added to this list as well, especially given that I will address several mathematical questions important for quantum theory. It is only left out because it is assumed here to be part of physics; it defines all modern physics (classical, relativistic, or quantum), and it will be discussed accordingly here. While it is readily acknowledged that physics and philosophy need to be brought together in foundational and, sometimes, even technical discussions of quantum theory, it is rarely recognized that the role of history is significant in these discussions as well. There are exceptions, such as, notably, N. Bohr, who understood this significance of history for our understanding of quantum theory and who effectively used history in his thinking and work. According to A. Pais:

I must admit that in the early stages of the collaboration [with Bohr] I did not follow Bohr's line of thinking a good deal of the time. ... I failed to see the relevance of such remarks as that Schrödinger was completely shocked in 1926 when he was told of the probability interpretation of quantum mechanics, or a reference to some objection of Einstein in 1928, which apparently had no bearing whatever on the subject at hand. But it didn't take long before the fog started to lift ... Bohr would relive the struggles which it took before the content of quantum mechanics was understood and accepted ... Through steady exposure to Bohr's 'daily struggle' and his ever repeated emphasis on 'the epistemological lesson which quantum mechanics has taught us,' to use a favorite phrase of his, my understanding deepened not only of the history of physics but of physics itself. ([1], p. 249)

Revealing as this passage may be concerning Bohr's “line of thinking,” it may be even more significant in indicating how history may help us to deepen our understanding of “*physics itself*.” One might argue that, even when we do expressly engage it, the history of continuing and, it appears, interminable “daily struggles” is unavoidably part of our foundational work on and discussions of quantum theory, however unacknowledged or unconscious our reliving of these struggles may be. At this point, we may, for the most part, be reliving more recent *reincarnations* of these debates, such as those found in the problematic defined by the experiment of A. Einstein, B. Podolsky, and N. Rosen (the EPR experiment) and J. S. Bell's and related theorems. But, as my use of “reincarnation” indicates, we thus also bring into play the longer history of these debates. Besides, one can hardly

say that “the content of quantum mechanics [has been] understood and accepted” even now, or at least that there is much agreement concerning this content, since even ostensibly close views often diverge at key points.

This article aims to bring together a particular physical-philosophical perspective on quantum mechanics and a particular historical trajectory, defined by W. Heisenberg’s work, a trajectory that is both especially important for the emergence of this perspective and exceptionally helpful for understanding it. The physical-philosophical perspective in question is defined by the following considerations.

Quantum mechanics and higher-level quantum theories continue classical physics (and relativity) insofar as they, too, are experimental-mathematical sciences of nature. However, quantum theory, at least in the interpretation of the type adopted here, breaks with both classical physics and relativity in two crucial respects—causality and realism. While both of these concepts involve further nuances and qualifications (some of which I shall offer below), their more or less common (dictionary-like) definitions are sufficient for my purposes, especially since this is how both concepts usually function in classical physics or relativity (in most interpretations of either theory).

Causality is the relationship between two events or phenomena, whereby the first (the *cause*) determines the second (the *effect*). The *principle* of causality, as formulated by I. Kant, states that “If ...we experience that something happens, then we always presuppose that something else precedes it, which it *follows* in accordance with a rule,” in opposition to a mere sequence of appearances, when, in the absence of a such rule, the very concept of “following” would not apply ([2], p. 308).¹ I shall refer to the combination of this concept and the principle of causality as *classical causality*, in part in parallel with classical physics, where classical causality is essential. It is also usually assumed that a cause always precedes its effect, or at the limit is simultaneous with it, in other words, that there is no backward-in-time causality. There are exceptions, especially in the context of quantum mechanics, where the introduction of the backward-in-time causality allows one to avoid certain (to some) undesirable philosophical features of quantum mechanics, such as and in particular, a lack of realism (e.g., [3], pp. 176-210). Relativity further restricts causes to those occurring in the backward (past) light cone of the event that is seen as an effect of this cause, while no event can be a cause of any event outside the forward (future) light cone of that event. These restrictions follow from the assumption that causal influences cannot travel faster than the speed of light in a vacuum, c , that is, with an assumption of locality.² Alternative forms of causality are possible, specifically those that are more suited to quantum physics than classical causality appears to be. I shall suggest such a concept here, which, although different from classical causality in other respects, retains all of the requirements of relativity and thus is also consistent with the absence of the backward-in-time causality.³ Given, however, that most of my discussion (apart from the concluding part of this article) concerns classical causality, “causality” will, unless specified otherwise, refer to classical causality. An (ideally) causal nature of certain physical processes sometimes, although not always, enables a given physical theory, such as classical mechanics, to predict exactly the state of a given physical system at a future point, or to infer its state at any past point, once its state is established at a given point. I shall refer to this type of *predictive* capacity as “determinism,” thus distinguished from causality, which is an ontological category referring to the nature of the processes that the theory considers. Both (strict) causality and determinism apply only at the level of idealizations or models used in classical mechanics; such models, however, provide excellent approximations in handling many actual physical processes in nature, at least for sufficiently simple systems. The situation is more complicated in classical statistical mechanics and chaos theory. In these theories exact predictions are no longer possible, even at the level of idealizations or models, because of the complexity of the systems considered by these theories, even though these systems are assumed to be at bottom causal. In other words, these theories are not deterministic in the way the classical mechanics of simpler systems is.

A *realist* theory would offer an exact or approximate mapping or model of physical *reality*, that is, of the properties of the physical systems considered and their behavior, usually assumed to be independent of our interactions with these systems. This is, again, so in the case of classical mechanics, which makes its predictions concerning the systems it considers by virtue of, in the first, providing (idealized) mathematical descriptions of their

¹ It should be noted that, following D. Hume, Kant offered a rigorous philosophical critique of causality and, in particular, a far-reaching exploration of the limits within which the concept and the principle of causality, as just formulated, apply. While, however, both Hume and Kant were attentive to these limits, and were critical of those who used the idea of causality uncritically, without paying proper attention to such limits, they appear to have believed that nature is ultimately causal (although scholars do sometimes disagree on Hume’s position in this respect). In other words, in their view, the limits in question were those of human cognition. Quantum physics presents a major challenge to this type of argumentation, which defines, often jointly with realism, most philosophical or scientific views of causality.

² There are further complexities in classical physics as well, which may, however, be put aside for the purposes of this article, since they mostly amount to the spatial and temporal framing of a given classical situation (classical physics usually considers the behavior of its objects within contained, if sometimes large, spatial and temporal limits), and to barring the backward-in-time causality.

³ I see the backward-in-time causality as highly undesirable, and in the absence of, thus far, any experimental evidence for it or for retroaction in time, I don’t find arguments entertaining either possibility compelling (e.g. [3], p. 188). The absence of realism, restoring which often motivates recourse to the backward-in-time causality or nonlocality, worries me far less, for the reasons explained this article.

properties and behavior. More generally, realism presupposes an independent architecture of reality (which may be temporal and is often assumed to be causal or otherwise analogous to that found in classical physics) governing this behavior, even if this architecture cannot be mapped by a theory, however partially or approximately. Both classical physics and relativity are generally assumed to fulfill the requirements of realism just stated, just as they do those of causality, at least, again, at the level of idealized models and in most interpretations of both theories.⁴ It is worth noting that, under the assumption of the independent architecture of reality, whether our limitations concerning a possible access to the ultimate reality or causality of nature are temporary (i.e. defined by the present state of physical knowledge) or permanent, these limitations are essentially practical. They become fundamental only if this type of assumption is suspended altogether, as it is in the present interpretation of quantum mechanics, which, in this respect, follows both Heisenberg (in his early work on quantum mechanics) and Bohr, who was, arguably, the first to rigorously develop this type of epistemology of quantum mechanics.

Now, unlike those of classical mechanics, quantum-mechanical predictions are in general probabilistic. These predictions, however, properly correspond, or *respond*, to the nature of quantum phenomena themselves, since it is an experimentally well-established fact that the identically prepared experiments (in terms of the state of our measuring instruments) in general lead to different outcomes. This is, it is worth stressing, not a matter of the mechanical complexity of the system considered, analogous to that found in classical statistical physics or chaos theory, since our predictions are unavoidably probabilistic even in the case of the simplest possible quantum system, such as single elementary particles. Accordingly, given the present state of experimental evidence, no other type of predictions is possible, even in the case of ontologically causal quantum theories (as concerns the ultimate behavior of quantum objects themselves), such as Bohmian mechanics, which, however, is nonlocal, as opposed to the standard quantum mechanics, at least in most interpretations, including the present one. (There are dissenting views on this point, advanced especially on the basis of Bell's and related theorems, by, I would argue, misconceiving their content and import.) Indeed, our predictions concerning quantum phenomena are irreducibly probabilistic insofar as, unlike in the classical case, we cannot in principle improve on them beyond a certain point, defined by Planck's constant, h , which also and correlatively enters the uncertainty relations. Hence, it is impossible to idealize the situation so as to make our predictions (ideally) exact in the way we can in classical mechanics. It is true that the underlying quantum processes responsible for this situation at the level of observed phenomena could, in principle, be (idealized as) causal, and some argue that such is in fact the case or, at least, that there are no reasons to assume otherwise. However, as the history (by now nearly a century long) of the interpretation of quantum mechanics tells us, there are also compelling reasons to think that such may not be the case, and the interpretation of quantum mechanics adopted here responds to this possibility. That is, there are reasons to question the possibility of the underlying causality of quantum processes, or to begin with, the possibility of assigning to them any specific independent architecture, even if one assumes that it is not possible to describe this architecture even partially or approximately. Indeed, in the latter eventuality, the impossibility of ascribing causality to these processes becomes an automatic consequence. For, taking a cue from Erwin Schrödinger's observation, if one cannot assume that quantum processes have any definable architecture, especially of the type found in classical mechanics, one could hardly assume that these processes are causal ([4], p. 154).

The situation, specifically in juxtaposition to classical mechanics, is correlative to the uncertainty relations, $\Delta q \Delta p \cong h$. The causal and realist description of classical mechanics assumes (at least, again, by way of idealization) and depends on the simultaneous exact determination of both conjugate quantities, such as the position and the momentum of an object, at any point, while the uncertainty relations prevents this type of determination in quantum physics, even ideally. Both this situation itself (and hence quantum mechanics) and the uncertainty relations must be given the corresponding interpretation, if one is to follow, as I do here, the radical epistemology based on suspending or even precluding the assumption that quantum processes can be assigned an ontologically specifiable dynamics, especially, again, by analogy with classical mechanics. These processes are of course assumed to occur and quantum objects to exist; indeed it is the existence of these objects and the (inaccessible) nature of quantum processes that are, in this interpretation, responsible for this situation. It is just that nothing can be said and perhaps even meaningfully thought about them. In sum, quantum mechanics predicts as much as we can observe, even though and possibly because it does not describe the behavior of the ultimate objects, quantum objects (such as electrons, photons, or quarks), and processes that are responsible for these outcomes. But then, this behavior as such, as an independent behavior, is not observable either. Nobody has ever seen a moving electron or photon, but only

⁴ As in the case of causality, establishing the ways in which realism works and the limits within which it applies in these theories involves further complexities, especially in general relativity, whose realist nature, assumed by Einstein (although he was aware of and pondered these complexities), has been questioned sometimes. I shall, however, put the subject aside here, since it requires a separate discussion and does not affect my main argument.

observed the traces of the interactions between them and measuring instruments. We infer the existence of quantum objects on the basis of the peculiar effects of their behavior on the world we can and do observe.

Classical mechanics and relativity both mathematically describe the objects and processes they consider and predict the outcome of the relevant experiments, and (apart, again, from chaos-theoretical or other complex systems) predict them exactly, at least, again, in considering sufficiently simple systems and at the level of the idealized models they use. Keeping this last qualification in mind, it may be said that these theories predict because they describe, and, for sufficiently simple systems, they predict exactly because they describe causally. By contrast, quantum mechanics is able to properly predict the outcomes of quantum experiments without being able to describe quantum objects and processes, which situation also appears to be correlative to the fact that these predictions are probabilistic in nature, even in principle and at the level of idealized models and even for the simplest systems considered.

This lack of descriptive capacity of quantum mechanics may be and has often been, most famously by Einstein, viewed as a disadvantage, indeed a disabling disadvantage, which for some, especially, again, Einstein, made a search for a classical-like and preferably causal alternative imperative. One can understand this discontent. One might, however, also see the situation differently. In particular, taking advantage of and bringing together both main meanings of the word “experiment” (as a test and as an attempt at innovative creation), one might argue, as I do here, that, while, obviously, not without help from nature, quantum mechanics is the first physical theory that is both, and jointly and interactively, *fundamentally* experimental and *fundamentally* mathematical.

It is fundamentally experimental because it is not, as in classical physics, merely the independent behavior of the systems considered, in other words, what happens in any event, that we track, by however ingenious experiments, but what kinds of experiments we perform, how we *experiment* with nature, that defines what happens.⁵ Thus, in the double-slit experiment, the two alternative set-ups of the experiment, whether we, respectively, can (by way of using one experimental device or another) or cannot know, even in principle, through which slit each particle, say, an electron, passes, we obtain two different outcomes of the statistical distributions of the traces on the screen (with which each particle collides). Correspondingly, our probabilistic predictions concerning each outcome will be different as well (quantum mechanics provides them in either set-up). Or, thus also giving a rigorous physical and philosophical meaning to the uncertainty relations, we can set up our apparatus so as to measure and correspondingly predict exactly (in idealized cases) either the position or the momentum of a given quantum object, but never both together. Either case requires a separate experiment, incompatible with the other, rather than merely representing an arbitrary selection of either type of measurement within the same physical situation, by tracking either one of its aspects or the other in the way we do this in classical mechanics. There, this is possible because we can, at least in principle, measure and assign simultaneously both quantities within the same experimental arrangement. In quantum physics we cannot. As will be seen, this view of the quantum situation also allows one to introduce a concept of causality that is applicable in quantum physics, including in the present interpretation, which manifestly precludes the application of classical causality. This concept is more active as concerns our capacity to influence events, since it makes us, rather than only physical processes independent of our intervention, influence and even define, “cause,” events and phenomena. Things become more subtly causal, as it were, even though classical causality—the classical concept and, especially, the classical principle of causality (“if ... something happens, then we always presuppose that something else precedes it, which it *follows* in accordance with a rule”)—can no longer be maintained.

By the same token, quantum mechanics is also fundamentally mathematical because the mathematical formalism of the theory is not in the service of tracking, by way of a mathematical description, what would have happened anyhow, but is in the service of predictions defined by our experiments. This formalism is able to predict correctly the experimental data in question without offering any description at all of the physical processes responsible for these data. As a result, beginning with and, as I shall argue, specifically in Heisenberg’s discovery of the theory, quantum mechanics, in this interpretation, established the radically new relationships between mathematics and physics, vis-à-vis those, essentially physically descriptive in character, found in classical physics and relativity. The mathematics of quantum mechanics describes nothing, except (mathematically) itself; it is only connected to the outcomes of actual or possible measurements (we need the former to be able, by using the formalism, to predict the latter). In other words, in this interpretation, it is strictly a theory of predictions concerning the outcomes of possible future experiments on the basis of previously performed experiments. As such, it is always about the future and not about the past. I shall return to this crucial point below, merely noting for the moment that time-reversibility or the backward-in-time causality of physical processes—or, rather, their description (there is no actual physical evidence thus far that any actual physical processes are time-reversible)—are easily, automatically

⁵ I am indebted to G. M. D’Ariano on this point. See also his contribution to this volume.

avoided in the present interpretation. In classical physics the appeals to time-reversibility or the backward-in-time causality are sometimes justified by the fact that the equations of classical physics are time reversible, as are those of relativity. There, however, time-reversibility is more difficult to justify, apart from esoteric conjectural assumptions, such as the existence of tachions, particles that travel only faster than light. One might, in principle, see (and many do) the time-dependent Schrödinger's equation as, mathematically, time-reversible, although this reversal involves a complex conjugation of variables. This fact already poses certain difficulties (since, in principle, such a procedure implies that the resulting equation refers to a different "time-reversed" quantum system than the original one), customarily disregarded by those who argue the case. In any event, in the present view, Schrödinger's equation does not offer a physical description of quantum processes but only, in each case, a probabilistically predictive algorithm concerning the outcome of future experiments on the basis of the data obtained in previously performed experiments. In short, it functions in the same way Heisenberg's formalism does in Heisenberg's original approach to matrix quantum mechanics, rather than along the lines of Schrödinger's classical-like program for wave quantum mechanics.

It also follows that, as we thus experiment with nature by using mathematics and technology, we experiment with mathematics itself, in any event more so than in classical physics, since we invent predictively effective mathematical schemes of whatever kind they might be, rather than proceed by refining mathematically our phenomenal representations of nature, which process limits us in classical physics. In fact, the experimentally mathematical character of quantum mechanics was discovered first in Heisenberg's original paper introducing his (matrix) version of quantum mechanics in 1925 ([5]), although this aspect of his discovery became apparent only in retrospect and is still underappreciated, if recognized at all. It is, one might also argue, Heisenberg's second great paper on quantum mechanics in 1927 ([6]), which introduced the uncertainty relations, that revealed the *physically* experimental character of quantum theory, as explained above, although, it appears, again, without quite realizing this on Heisenberg's part at the time. Bohr was, arguably, the first to realize this, even if without putting it in this way. This realization took a while as well and a long debate with Einstein on the foundations of quantum theory, culminating in the exchanges concerning the EPR argument and related arguments by Einstein ([7], [8]).

This brings me to the historical trajectory of my argument. It was, I argue, Heisenberg's physical and philosophical thinking that established the mathematically and then physically experimental character of quantum mechanics as here defined in his two papers introducing, respectively, quantum mechanics itself and the uncertainty relations. The first paper discovered or invented, *constructed*, quantum mechanics in its matrix form by experimenting and discovering a mathematical scheme that was entirely new in physics, although, mathematically, the scheme in fact amounted, as Max Born was the first to realize, to (infinite-dimensional) matrix algebra, developed in mathematics much earlier. The second paper, which introduced the uncertainty relations, opened the way to defining quantum mechanics as the physics of experimentation in the present sense. In some respects, Heisenberg's first paper moves in this direction as well, since it conceived of quantum mechanics as a strictly predictive theory rather than a theory also describing the independent behavior of quantum systems on the model of classical mechanics. Schrödinger aimed, with great determination, to restore this classical-like project in his program for his wave mechanics, developed shortly thereafter, although, given the mathematical equivalence of both formalisms, his famous equation can be interpreted in accordance with the present view of quantum theory. His determination, however, reminds us that the classical project or, as Schrödinger called it later, "classical ideal" was by no means lacking in appeal at the time, having its greatest champion in Einstein, nor has it lost its appeal now ([4], p. 152). Indeed, in this particular battle, Einstein appears to have won. A sizable and indeed dominant majority of physicists and philosophers hold similar philosophical views, as reality and causality continue to hold their grip on our thought and aims in physics and beyond. In any event, it was Heisenberg's paper on the uncertainty relations, written, it is worth noting, in part to combat Schrödinger's program, that revealed the potential of quantum physics as a form of experimentation in the present sense, at least, again, in retrospect. It took a while and Bohr's sustained effort to realize this aspect of the uncertainty relations, and, as I said, Heisenberg himself did not quite see them along these lines at the time.

Quantum theory makes us *know* less than we used to think, on the model of classical physics, it is possible to know. On the other hand, we can *do* more, indeed for the first time we can do something in *defining* the world by our experiments, and *our experiments cannot avoid doing so*. This last qualification is crucial because some of our classical experiments *may* sometimes also change the world, if our interference is sufficient to significantly disturb the classical configuration involved. Quantum experiments always change the world; they *make* us experiment with it by means of our technology. They make physics unavoidably artistic, mathematically and technologically. The ancient Greek word *tekhnē* also meant art. Quantum physics may be seen as mathematical-experimental *tekhnē* in the ancient Greek sense.

2. MATHEMATICAL EXPERIMENTATION AND PROBABILITY

The character of Heisenberg's conception of quantum mechanics as an instance, the inaugural instance, of the mathematical and physical experimentation in the present sense, becomes immediately apparent if one considers it in juxtaposition to Bohr's semi-classical theory of the hydrogen atom introduced in 1913. Bohr's theory described the orbital motion of electrons on classical lines but also, famously, postulated discontinuous quantum jumps between such orbits. Each such jump was accompanied by either an emission or absorption of the radiation quantum, $h\nu$, corresponding to the difference between the corresponding energy levels, in accordance with Planck's hypothesis. Thus, only a discrete set of orbits ("stationary states") was allocated to electrons. Bohr also postulated a minimal energy level, from which electrons would not radiate. These, at the time, radical postulates allowed Bohr and his followers to account for the atomic spectra. These postulates were of course manifestly in conflict with both the classical mechanics of motion and the classical electrodynamics of radiation. Philosophically, however, the theory was still shadowed by the spirit or ghost of classical physics insofar as it assumed the underlying architecture of atomic reality, even though this architecture was classical only in the case of stationary states, while any attempt to describe the mechanism of transitions between them was suspended (hence, the term "semi-classical"). Heisenberg's approach was entirely different: it dispensed altogether with the classical spirit of relating the theory to the behavior of physical objects in space and time, as Bohr immediately realized ([9], v. 1, p. 48).

Heisenberg's decision to do so was not a matter of making a revolutionary move for its own sake. It was instead compelled by the persistent failure of semi-classical methods to account comprehensively for the experimental data in question, a decade of Herculean efforts, including by Heisenberg himself. For, although successful, sometimes spectacularly successful, in some respects, by the time of Heisenberg's discovery, Bohr's theory had also proven to be deficient and even helpless in dealing with much pressing experimental data, especially that pertaining to the behavior of more complex atoms, although the hydrogen atom was, by then, found to be beyond the theory's explanatory power as well. In particular, it became clear that one could no longer apply the concept of classical orbits even to electrons' behavior in stationary states, but could only consistently speak of energy levels corresponding to them. In mathematical terms, the old quantum theory tried to handle the situation by suitably modifying, ad hoc, the equations of classical physics for each specific situation, while retaining classical variables and, by the same token, to the degree possible, the descriptive approach of classical mechanics.

Heisenberg, by contrast, made a truly radical move on all three counts—physical, mathematical, and epistemological—and by proceeding along all these lines at once and linking them together. In physical and mathematical terms, Heisenberg formally retained the equations of classical mechanics, while introducing the entirely new type of variables, essentially (infinite-dimensional) matrix variables, to which these equations applied, the change that he famously defined as "new kinematic." Although matrices had been known in mathematics itself for some time before 1925, Heisenberg was famously unaware of this fact: he reinvented matrix algebra in the process of his discovery of quantum mechanics. In epistemological terms, he abandoned the idea that the equations of the theory should describe the quantum processes as those of motion in space and time (in this respect the term "kinematic" was misleading). He realized that, if one used a different type of variables than those used in classical physics or even relativity, it was possible to predict observable quantum phenomena (specifically, spectra) without describing the quantum objects and processes responsible for the appearance of these phenomena.

To follow his procedure as presented in his famous paper introducing quantum mechanics, Heisenberg starts by formally adopting the classical equations of motion in Fourier's representation. This step is prompted by what may be seen as a mathematical form of Bohr's correspondence principle. As indicated above, the correspondence principle more or less tells us that in the regions where classical physics can be used the predictions of quantum and classical theory should coincide, and the classical equations, applied to the standard classical physical variables, work well for the large quantum numbers (when electrons are far from the nucleus). Importantly, as Bohr often stressed and as Heisenberg explains in his uncertainty-relations paper, this does not mean that the physical processes themselves in question may be seen as classical and hence that classical equations correctly describe them, but only that one can use these equations to make correct predictions in these regions ([6], pp. 72-76). However, classical equations (with classical variables) do not work at all for small quantum numbers because they do not satisfy Bohr's rules for frequencies and the Rydberg-Ritz combination rules, which fact led Bohr to his 1913 theory of the hydrogen atom.

At the outset of his paper announcing his discovery of quantum mechanics, Heisenberg states: "[I]n quantum theory it has not been possible to associate the electron with a point in space, considered as a function of time, by means of observable quantities. However, even in quantum theory it is possible to ascribe to an electron the emission of radiation" ([5], p. 263). Heisenberg then says: "In order to characterize this radiation we first need the frequencies which appear as functions of two variables. In quantum theory these functions are in the form:

$$v(n, n - \alpha) = I/h \{W(n) - W(n - \alpha)\} \quad (1)$$

and in classical theory in the form:

$$v(n, \alpha) = \alpha v(n) = \alpha/h(dW/dn) \quad ([5], \text{p. 263}).$$

This difference leads to a difference between classical and quantum theories as concerns the combination relations for frequencies, which correspond to the Rydberg-Ritz combination rules. However, “in order to complete the description of radiation [in accordance with the Fourier representation of kinematic formulas] it is necessary to have not only frequencies but also the amplitudes” ([5], p. 263). The crucial point here is that, in Heisenberg’s theory and in quantum mechanics since then, these “amplitudes” are no longer amplitudes of any physical, such as orbital, motions, which makes the name “amplitude” itself an artificial, symbolic term. Instead, quantum amplitudes were to be linked to the probabilities of transitions between stationary states; in other words, they are what we now call probability amplitudes. “The amplitudes may be treated as complex vectors, each determined by six independent components, and they determine both the polarization and the phase. As the amplitudes are also functions of the two variables n and α , the corresponding part of the radiation is given by the following expressions:

Quantum-theoretical:

$$\text{Re}\{A(n, n - \alpha)e^{i_{\omega}(n, n - \alpha)t}\}$$

Classical:

$$\text{Re}\{A_{\alpha}(n)e^{i_{\omega}(n)t}\},$$

Heisenberg now proceeds by introducing a new theory around the problem that appears to be insurmountable and is insurmountable within the old theory: “[T]he phase contained in A would seem to be devoid of physical significance in quantum theory, since in this theory frequencies are in general not commensurable with their harmonics” ([5], pp. 263-264). The solution is reached by a complete change of the perspective on the situation. Most of all, the new theory offers the possibility of rigorous predictions of the outcomes of the experiments, even if at the cost of abandoning the physical description of the ultimate objects considered, which is no longer seen as a problem but instead as a way to the solution. Heisenberg now says: “However, we shall see presently that also in quantum theory the phase had a definitive significance which is *analogous* to its significance in quantum theory” ([5], p. 264; emphasis added). “Analogous” could only mean here that the way it functions mathematically is analogous to the way the classical phase functions mathematically in classical theory (or analogous in accordance with this form of the correspondence principle), for physically there is no analogy. As Heisenberg explains, if one considers “a given quantity $x(t)$ [a coordinate as a function of time] in classical theory, this can be regarded as represented by a set of quantities of the form

$$A_{\alpha}(n)e^{i_{\omega}(n)t},$$

which, depending upon whether the motion is periodic or not, can be combined into a sum or integral which represents $x(t)$:

$$x(n, t) = \sum_{\alpha} A_{\alpha}(n) e^{i_{\omega}(n)t}$$

or

$$x(n, t) = \int_{-\infty}^{+\infty} A_{\alpha}(n) e^{i_{\omega}(n)t} d\alpha \quad ([5], \text{p. 264}).$$

Heisenberg is now ready to make his most decisive and most extraordinary move, which will revolutionize the nature of theoretical physics. He first notes that “a similar combination of the corresponding quantum-theoretical

quantities seems to be impossible in a unique manner and therefore not meaningful, in view of the equal weight of the variables n and $n - \alpha$.” “However,” he says, “one might readily regard the ensemble of quantities $A(n, n - \alpha)e^{i(n, n - \alpha)t}$ [an infinite square matrix] as a representation of the quantity $x(t)$ ” ([5], p. 264). The arrangement of the data into square tables is a brilliant and, within his scheme, natural way to connect the relationships (transitions) between two stationary states, and it is already a great concept. However, it does not by itself establish an *algebra* of these arrangements, for which one needs to find the rigorous rules for adding and multiplying these elements—rules without which Heisenberg cannot use his new variables in the equations of the new mechanics. To produce a quantum-theoretical interpretation of the equations of motion, as applied to these new variables, Heisenberg needs to be able to construct the powers of such quantities, beginning with $x(t)^2$. The answer in classical theory is of course obvious and, for the reasons just explained, obviously unworkable in quantum theory. Now, “in quantum theory,” Heisenberg proposes, “it seems that the simplest and most natural assumption would be to replace classical [Fourier] equations ... by

$$B(n, n - \beta)e^{i(n, n - \beta)t} = \sum_{\alpha} A(n, n - \alpha)A(n - \alpha, n - \beta)e^{i(n, n - \beta)t}$$

or

$$= \int_{-\infty}^{+\infty} A(n, n - \alpha)A(n - \alpha, n - \beta)e^{i(n, n - \beta)t} d\alpha \quad ([5], p. 265).$$

This is the main postulate, the (matrix) multiplication postulate, of Heisenberg’s theory, “and in fact this type of combination is an *almost* necessary consequence of the frequency combination rules” [equation (1) cited above] ([5], p. 265; emphasis added). “Almost” is an important word here, since the justification or derivation is not strictly mathematical, but it corresponds to the observable phenomena and numerical data. This combination of the particular arrangement of the data and the (re)invention through physics of an algebra of multiplying his new variables is his great invention. This multiplication is of course in general noncommutative, and the scheme essentially amounts to the Hilbert space formalism, with Heisenberg’s matrices serving as operators.

In sum, while the classical equations themselves of classical motions are retained, the variables and rules of mathematically rigorously manipulating them are replaced, as are by necessity the rules on relating the resulting equations to the experiments. These relations are no longer based on describing the physical behavior of quantum objects, but only concerns the probabilistic outcome of the corresponding experiments. One can also look at the situation as follows. Heisenberg had to suspend, to “forget,” classical physics or the old quantum theory to arrive at his new mathematical scheme. His physics was defined by finding this predictive mathematics from the available data. Of course, he had to reinvent the wheel of (in effect) Hilbert space formalism, and the already available equations of classical mechanics, equations of motion, and the correspondence principle were exceptionally helpful. The fact, however, that this mathematics already existed was a matter of coincidence, and certain new aspects of matrix algebra were introduced in the course of developing the fully fledged form of matrix quantum mechanics by Born, Jordan, Heisenberg, and others. The main point here is the deeply mathematical—experimentally mathematical or mathematically experimental—character of Heisenberg’s approach to quantum theory, now defined by the absence of any descriptive function of the formalism as concerns the behavior of quantum objects. He invented his matrix variables as part of his predictive machinery for quantum data, as against the descriptive machinery of classical mechanics, from which he adopted the *formal structure* of his equations.

Indeed, highly unvisualizable in its nature, this (Hilbert-space) type of mathematics is not suited for the kind of description of physical behavior (a description that ultimately derives from our phenomenal representation of nature) that defined the models of classical physics or even relativity, although the latter already has significant complexities in this regard. That we can, nevertheless, use these mathematical concepts to make proper predictions concerning the outcome of quantum experiments may be enigmatic or miraculous, insofar as we have no descriptive or, for that matter, other physical reason why it works, except perhaps for the fact that complex conjugation is a natural enough mechanism for going from amplitudes to probabilities. This mathematics does work predictively, however. It may be noted that the way this mathematics was put to work by Heisenberg is in part (there were also more direct physical reasons) why his approach did not encounter the kind of problems that haunted that of Schrödinger, who initially tried to bring his mathematics in accord with the intuitions and representations of classical (wave) physics. One might say that quantum mechanics *benefits* from this split between our phenomenal world and the world of

modern mathematics, given that the mathematics used in classical physics for the purposes of its descriptive idealization does not appear to offer effective, if any, algorithms for predicting quantum phenomena.

The approach of the type adopted or, again, invented by Heisenberg essentially changes the practice of theoretical physics. I would argue that, beginning with the work of Heisenberg and then that of Dirac, who was especially impressed and arguably learned most from Heisenberg's paper at the time, this type of approach largely took over this practice, even though the philosophical attitudes or beliefs of the practitioners themselves may be different, for example, more descriptively oriented. That is, the practice of theoretical physics is transformed into inventing and working with the mathematical apparatus of the theory (while building upon the preceding mathematical architecture) to make this apparatus enable correct predictions, rather than trying to develop an idealized mathematical description of the physical processes considered. This type of theoretical physics is a kind of experimental mathematics, or in any event, is experimentation with mathematics, predictively linked to our experimentation with our experimental technology in quantum physics—an invention, *creation* of new configurations of experimental technology—rather than, as in classical physics, tracking what would have happened independently of our experimental intervention in any event.

3. PHYSICS OF EXPERIMENTATION: THE UNCERTAINTY RELATIONS, COMPLEMENTARITY, AND CAUSALITY

The observable effects or traces of the “behavior” of quantum objects, as manifest in the paradigmatic quantum experiments, such as the double-slit experiment, are remarkable. They are nothing like anything we have ever encountered or even imagined in any domain before we encountered the quantum imagination of nature itself. Other common locutions include strange, puzzling, mysterious (and sometimes mystical), and incomprehensible, which is not surprising since we may not be able to even imagine how quantum phenomena are possible. Thus, for example, how do particles “know,” individually or (which may indeed be even more disconcerting) collectively, that both slits are open and no counters are installed or, conversely, that counters are installed to check which slits particles pass through, and modify their behavior accordingly? Attempts to conceive of this behavior in terms of physical attributes of quantum objects themselves appear to lead to unacceptable or at least highly problematic consequences. Among such consequences are logical contradictions; the behavior of quantum objects based on such difficult assumptions as attributing volition or personification to nature in allowing quantum objects individual or collective “choices;” or the spatial or temporal nonlocality of quantum phenomena (and hence the incompatibility of these phenomena with relativity).⁶

On the other hand, one can consistently and locally account for the situation by suspending any account of the independent quantum behavior, and instead follow Bohr's logic of complementarity. This logic is grounded in the fact that these two set-ups and, hence, the two types of phenomena occurring in the double-slit or related experiments, or in the two alternative measurements and, correspondingly, predictions (that of the position and that of the momentum of a given quantum object) are always mutually exclusive. At the same time, however, we always have a free choice of which set-up to arrange or which measurement to perform. Bohr appears to have been the first to take advantage of this fact. It allowed him to contend that the features of quantum phenomena exhibited in the double-slit experiments or other key quantum experiments need not be seen as paradoxical. As he says: “[I]t is only the circumstance that we are presented with a choice of *either* tracing the path of a particle *or* observing interference *effects*, which allows us to escape from the paradoxical necessity of concluding that the behavior of an electron or a photon should depend on the presence of a slit in the diaphragm through which it could be proven not to pass” ([9], v. 2: 46; emphasis added).

This argumentation can be suitably adjusted to apply to such more recent developments as Bell's and the Kochen-Specker theorems (and their extensions) and related experimental findings, which are, admittedly, under debate as concerns their epistemological meaning and implications. These developments are beyond my scope here, as are the debates surrounding them, which, as noted from the outset, continue the apparently interminable debates concerning quantum phenomena and quantum theory.⁷ I would like instead to revisit from the perspective of complementarity, Heisenberg's second great discovery, made in 1927, that of the uncertainty relations. This

⁶ As I said, the spatial (and sometimes temporal) nonlocality is found acceptable or is even assumed by some physicists and philosophers, which is, in my view, problematic and is often inadequately justified, in particular, as I said, on the basis of misconceptions concerning the EPR experiments and Bell's and related theorems. Unfortunately, I cannot address the subject here or offer a more rigorous criticism of these views. I have, however, considered the situation in detail in ([10], pp. 268-278).

⁷ For a discussion of these and related subjects, in particular, the EPR experiment and the Bohr-Einstein exchange concerning it, from the present perspective, see [10, pp. 237-278].

discovery was decisive for Bohr's invention of the concept of complementarity and arguably led to this invention, although it took Bohr a while, about a decade, and some "help" from Einstein to develop this concept in the form that concerns me here (e.g. [9], v. 2, p. 40). Einstein, however, never came to terms with the idea of complementarity itself, undoubtedly to Bohr's chagrin ([11], p. 674).

Complementarity is defined by (a) a mutual exclusivity of certain phenomena, entities, or conceptions; and yet (b) the possibility of applying each one of them separately at any given point, and (c) the necessity of using all of them at different moments for a comprehensive account of the totality of phenomena that we must consider. Parts (b) and (c) of this definition are just as important as part (a); and to miss or disregard them, as is often done, is to miss much of the import of Bohr's conception. Arguably the most significant examples of complementarity in this sense are those of the position and the momentum measurements, and of the space-time coordination and the application of momentum or energy conservation laws (there are, thus, two complementarities here), which are correlative to Heisenberg's uncertainty relations (e.g., [8], v. 3, p. 5). Because, as the uncertainty relations themselves, complementary phenomena are characteristic of quantum, vis-à-vis classical, physics, the concept guided Bohr's overall quantum-theoretical thinking and came to ground, and the term complementarity to designate, his interpretation of jointly both concerning quantum phenomena and quantum mechanics. As I said, the present interpretation largely follows Bohr's interpretation epistemologically, as concerns causality and realism or, as the case may be, the lack thereof. However, his emphasis on the irreducible role of measuring instrument in defining quantum phenomena notwithstanding, Bohr does not appear to see quantum physics in terms of physical and mathematical experimentation in the way the present article does.

While, however, Heisenberg's discovery of the uncertainty relations had a crucial role in Bohr's invention of complementarity, Bohr deserves credit for the interpretation and, vis-à-vis Heisenberg's view in his paper, re-interpretation of the uncertainty relations that his concept of complementarity and his interpretation of quantum mechanics *eventually*, albeit only eventually, enabled. For, under the impact of several intervening developments, most especially Schrödinger's version of quantum mechanics and certain ideas of Dirac ([12]), Heisenberg retreated from the radical epistemological position that shaped his discovery of quantum mechanics. So in fact did Bohr in developing his initial conception of complementarity under the impact of the same developments and Heisenberg's uncertainty relations paper itself. Ultimately, Bohr rethought both complementarity and, via complementarity, the uncertainty relations in more radical epistemological terms, in effect returning to Heisenberg's and his own thinking at the time of Heisenberg's discovery of quantum mechanics in 1925.

According to Heisenberg in his uncertainty relations paper: "*All concepts which can be used in classical theory for the description of a mechanical system can also be defined exactly in atomic processes in analogy to the classical concepts.*" The experiments which provide such a definition themselves suffer from an indeterminacy introduced purely by the observational procedures that we use when we ask of them the simultaneous determination of two canonically conjugate quantities. The magnitude of this indeterminacy is given by [the uncertainty] relation $[\Delta q \Delta p \approx h]$ ([6], p. 68; Heisenberg's emphasis). In other words, the disturbance introduced by measurement leads to the impossibility, reflected in the uncertainty relations, of establishing both variables simultaneously. Thus, this disturbance also makes it impossible to maintain the causal connections between quantum events in the way this is done in classical physics, where causality is possible because we can, at least in principle, properly define both variables at any point. Nevertheless, in contrast to his view in his paper introducing quantum mechanics or Bohr's view in response to that paper, Heisenberg at the very least suggests here that it is possible, at least partially (subject to the uncertainty relations), to speak of the quantum-level physical processes themselves, in other words, of the quantum-level reality, even if not of causality.

Both Bohr and Dirac at that time took an even more classically oriented view by bringing back classical causality as well into quantum physics. They maintained that the independent (undisturbed) quantum behavior is causal, while the lack of causality and determinism manifested in quantum phenomena, although irreducible (thus justifying the nature of quantum mechanics as an irreducibly probabilistic theory), is due to the disturbance introduced by observation. Although *triggered* by Schrödinger's wave theory (Schrödinger himself believed that one could, ideally, avoid indeterminism altogether), the idea appears to originate, via the transformation theory, with Dirac, who introduced it sometime in 1926 while at Bohr's Institute in Copenhagen ([12]). Although, I would argue, difficult to sustain, this view of quantum behavior has been pervasive in foundational arguments concerning quantum theory.⁸

Heisenberg, by contrast, is careful not to speak of the causality of the undisturbed behavior of quantum systems; indeed he is resolute in speaking against it later in the paper. His formulation just cited may or may not imply causality at this level, since it only says that "*all concepts which can be used in classical theory for the*

⁸ I discuss the subject extensively in [10, pp. 202-218].

description of a mechanical system can also be defined exactly in atomic processes in analogy to the classical concepts." On the one hand, such a definition—that is, that of “*all concepts*” involved—is what assures causality in classical physics. On the other hand, even though he speaks of “disturbance,” Heisenberg does not say that such concepts can be defined simultaneously for an undisturbed system. In closing his article, he regards “the presumption[s] that behind the perceived statistical world [of quantum observations] there still hides a ‘real’ world in which causality holds” as “fruitless and senseless ... speculations” ([6], p. 83). Heisenberg obviously rejects, just as Bohr and Dirac do, causality at the level of observation in view of the uncertainty relations, which establish “the final failure of causality” ([6], p. 83). This strong formulation appears to imply that Heisenberg also rejects that causality applies at the quantum level. Indeed, he adds: “[quantum] physics ought to describe only the *correlation* of observations” ([4], p. 83; emphasis added).

And yet, Heisenberg’s paper also appeals to the idea of “disturbing” the *independent* behavior of quantum objects by an act of observation in his γ -ray-microscope thought experiment. While Bohr, famously, corrected some of Heisenberg’s argumentation even before the paper was published, he at this point did not critique the ingredients under discussion here, but instead adopted them in his own discussion of the experiment and, as explained above, took an even more classical view of causality. To his credit, however, he quickly realized the difficulties of sustaining this view. Eventually Bohr rejected the use of the term “disturbance” as implying that there is some in principle describable or even conceivable (let alone causal) independent quantum behavior, which is then disturbed by observation. He spoke instead of our *interfering* with quantum objects and thus affecting their subsequent behavior—and, hence, our probabilistic predictions concerning future experiments—or changing our expectations concerning the outcome of these experiments, as Bayesian quantum-information theorists would have it. Again, nothing could any longer be said about the independent behavior of quantum objects themselves. That one cannot under these conditions speak of any quantum-level (classical) causality, again, follows automatically. As I said, Bohr moves to this type of view (it took, again, a while to crystallize it) under the impact of his exchanges with Einstein during the decade following the introduction of the uncertainty relations and complementarity (both in 1927), especially the exchanges concerning the EPR-type experiments. By the time of his reply to EPR, he speaks of “a *final renunciation* of the classical ideal of causality” ([8], p. 697; emphasis added) and eventually of complementarity as “a rational *generalization* of the ... ideal of causality” ([9], v. 2, p. 41; emphasis added). What kind of generalization, then? Before I address this question, however, I would like to revisit the physics of the uncertainty relations from this more radical epistemological perspective. Asher Peres offers a helpful interpretation of Heisenberg’s famous formula consistent with this perspective:

The only correct interpretation of [an uncertainty relation $\Delta x \Delta p \approx h$, where x is a coordinate and p the momentum in the same direction] is the following: If the *same* preparation procedure is repeated many times, and is followed either by a measurement of x , or by a measurement of p , the various results obtained for x and for p have standard deviations, Δx and Δp , whose product cannot be less than $[h]$. There never is any question here that a measurement of x “disturbs” the value of p and vice-versa, as is sometimes claimed. These measurements are indeed incompatible, but they are performed on *different [quantum objects]* (all of which were identically prepared) and therefore these measurements cannot disturb each other in any way. An uncertainty relation [$\Delta x \Delta p \approx h$] [or the corresponding representation in the formalism of quantum mechanics] only reflects the intrinsic randomness of the outcomes of quantum tests. ([13], p. 93)

While it may not be “the *only* correct interpretation,” as Peres claims, it is both rigorous and consistent. As Peres also observes, in accordance with the present view, “an uncertainty relation [...] is not a statement about the accuracy of our measuring instruments. On the contrary, its derivation assumes the existence of *perfect* instruments (the experimental errors due to common laboratory hardware are usually much larger than quantum uncertainty)” ([13], p. 93). Bohr corroborates this view in a striking sentence that also brings in the role of measuring instruments, which define *quantum phenomena*, as different from quantum *objects*, a key point of his ultimate interpretation of quantum phenomena and quantum mechanics. He says, “[I]n this context, we are of course not concerned with a restriction as to the accuracy of measurement, but with a limitation of the well-defined application of space-time concepts and dynamical conservation laws, entailed by the necessary distinction between measuring instruments and atomic objects” ([9], v. 3, p. 5).

Thus interpreted, physically, mathematically, and epistemologically, the uncertainty relations and complementarity establish quantum physics as the physics of experimentation in the present sense, and, which is part of my historical argument here, they were the first law of quantum mechanics that revealed the possibility and perhaps necessity of this view of quantum physics. The mathematically experimental nature of quantum mechanics was, as we have seen, established by Heisenberg in his first paper on quantum mechanics. It is true that the formulations just cited may be seen as primarily establishing, on experimental grounds, the inevitability of the

probabilistic or statistical nature of quantum predictions and, hence, the corresponding character of any proper theory of quantum phenomena, and thus also the adequacy of quantum mechanics to this phenomena. I would argue, however, that these formulations also make apparent the major and possibly unavoidable role of experimentation and of the freedom of experimentation in quantum physics. Thus, to return to Peres's formulation, if "there never is any question here that a measurement of x "disturbs" the value of p and vice-versa," it is *because* "these measurements are incompatible" and "are performed on *different [quantum objects]* (all of which were identically prepared)," in other words because of complementarity. It is accordingly, our decision, defined by our freedom of experimentation, that, in each case, determines what happens, rather than, as in classical physics, reveals what happens in nature apart from us in any event. More accurately, one should speak of what *only* happens in nature apart from us, since it is also nature, independent of us, that is responsible for anything that happens in our experiments and experimentations, classical and quantum. However, unlike in classical physics, in quantum physics, at least in the present interpretation, what happens at the ultimate level cannot be revealed, and, by the same token, whatever is revealed in the *idealization* of classical physics is not how nature ultimately works. Indeed, the irreducibly probabilistic character of quantum theory may be seen as correlative to this type of techno-physical and techno-mathematical experimentation, because we cannot rely on either tracking independent physical processes (physics) nor on mathematically describing or representing them, even ideally (mathematics).

It is hardly surprising that it is difficult to retain classical causality, especially, the classical principle of causality under these conditions.⁹ Determinism is clearly excluded outright, given that the identically prepared experiments in general lead to different outcomes. On the other hand, it would appear possible to say that a given event of measurement, say, that of the position, at time t_1 , enabling our prediction concerning a possible future measurement of the position of this object, say, at time t_2 , is a cause of the event of such a subsequent measurement. This assessment appears possible in a given individual case even if our prediction is probabilistic, and especially if it is exact, as it is in some idealized cases, such as those of the EPR situation. That is, it is possible, if this later measurement is performed, which is a crucial qualification and, as is easily seen, an obstacle here. For, it is always possible to perform an alternative measurement at t_2 , that of the momentum, and, in view of the uncertainty relations, this measurement would unavoidably preclude the possibility of establishing the physical state of the system as predicted on the basis of the initial measurement. In classical mechanics, both the position and the momentum of an object can always be simultaneously measured and, at least ideally and in principle, assigned to the object considered, and assigned independently. In quantum physics, establishing a causal connection (defined by such a simultaneous assignment) between two measurements cannot be guaranteed or indeed maintained, even ideally or in principle. Hence, at least in the present interpretation, it is, again, the experiment that we perform and thus the corresponding setting-up of our experimental apparatus at a given point that establishes what happens and not or, again, not only the independent behavior of the quantum object or system under consideration. Once the alternative (momentum) measurement is performed, the initial (position) measurement is no longer the cause of what happens. Indeed, the event defined by this alternative measurement can no longer be assigned an ascertainable cause in an earlier event. Its only, strictly localized or discrete "cause" is our particular experimental set-up, as opposed to the continuous causal chain of events characterizing the behavior of classical objects in classical mechanics.

In fact, it is not possible to ascertain in any individual case that any given preparation (say, that enabling an emission of an electron or a photon in the double-slit experiment) will actually lead to an event we expect to happen. There is always a non-zero probability that we will not observe such an event. Nor is it possible to be certain that a given individual event (say, a given trace on the screen) that we do observe is a result of the preparation that was expected to lead to this event. No determinate connection, either defined by a general law or otherwise, of a given quantum event to any preceding event can ever be guaranteed, if established at all, which circumstance automatically disables the classical *principle* of causality: "if ...we experience that something happens, then we always presuppose that something else precedes it, which it *follows* in accordance with a rule." Unlike in classical mechanics, the correlations of such events are strictly statistical, and it is, again, in general impossible to repeat any given experiment identically at the outcome and thus establish (classical) causal correlations. The possibility of such a repetition, at least, again, ideally and in principle, is a defining feature of classical physics, correlative to the applicability of classical causality or, to begin with, the concept of reality in classical mechanics, at least, again, at the level of idealized models (e.g., [10], pp. 72-76, 252-261). In view of the impossibility of this type of repetition

⁹ Attempts to do so have been persistent throughout the history of quantum mechanics, beginning with Schrödinger's wave mechanics, and they continue to proliferate. I would argue, however, that their success is limited, if any, and they often carry a heavy and, to the present author, unwarranted cost, such as nonlocality (as in Bohmian theories) or retroaction in time and thus adding the backward-in-time causality to classical causality (e.g., [3], pp. 176-210).

and other reasons just outlined (some of them correlative to this impossibility), models of this type do not appear workable in quantum theory. Different types of models or idealizations are possible, however, such as the one, offered here, defined by the essential role of our technological experimentation.

Is it possible to introduce some concept of causality, even if, it follows, in the absence of the *classical* principle of causality, within this model? Yes, it is possible; and ironically, it is possible to do so by using some of those features of the model that essentially distinguish it from the idealization of classical mechanics and the form of causality found there. Indeed, while this concept of causality, “quantum causality,” is different from the classical concept of causality, it may be seen as “a rational *generalization* of the very ideal of causality,” via or even as complementarity, thus justifying Bohr’s contention cited above ([9], p. 36). The essential difference between this concept and classical causality remains crucial, especially by virtue of the absence of the (classical) principle of causality in quantum causality. Indeed, to my knowledge, as a concept of *causality*, quantum causality is different from any concept of causality hitherto available. I shall now outline the conditions defining this concept and its architecture as a concept.

The essential situation of quantum physics is defined by the fact that we can perform a particular (ideally exact) measurement, say, that of the position, on a given quantum object, S , at a given time, t_1 , and, by using quantum mechanics (or possibly some alternative theory), make the corresponding prediction of its possible future position. In the present interpretation, any such knowledge, actual or possible, only amounts to that of the effect of quantum objects and their behavior on measuring instruments, and the measurable quantities in question must be defined accordingly. This, however, can easily be done: the position in question is defined by a spot registered in some portion of the apparatus used for this (position) type of measurement, and the momentum by a change in the momentum in a suitable part of the apparatus used for the alternative (momentum) type of measurement. The two corresponding arrangements are mutually exclusive, in accordance with Bohr’s concept of complementarity and correlatively to the uncertainty relations, in turn interpreted in accordance with this concept. As I explained, in general our quantum predictions are probabilistic and, in the case of determining a future position of a given object, they concern possible regions where the object can be found. They can, however, also be exact in certain idealized cases, such as those of the EPR measurement, because each such prediction is enabled by a measurement on the other object of the EPR pair. It can be argued, as it was by Bohr, that the latter fact does not change the essential epistemological aspects of the situation, specifically the irreducible role of measuring instruments in determining all quantum variables, although Einstein, never persuaded by Bohr’s argument, thought otherwise.¹⁰

Now, unlike in classical mechanics, no subsequent measurement on S , say, at the time t_2 , of any kind, either that of the position or that of the momentum, can give us any additional information concerning either the position or the momentum of S at t_1 . For example and in particular, if the initial measurement in question is that of the position, it is never possible to know, as it is in classical physics, the momentum associated with S at t_1 . In this respect the situation may indeed be seen as causal in the following sense, which defines the concept of quantum causality as understood here. The initial measurement irreducibly constrains our *possible* knowledge concerning quantum objects in a specific and rigorous way, in view of the uncertainty relations. We cannot in principle improve on our knowledge of the past physical state of the object, any more than on our possible knowledge of its future physical state, in the way we can in classical mechanics by more precise or more extensive subsequent (repeated or alternative) measurements on the same object. The measurement of the momentum at t_2 would of course annul our initial prediction and thus disable any knowledge whatsoever we can get concerning the position of S at t_2 , including, importantly, in the EPR case, to which the present argument concerning quantum causality applies in general, both for continuous and discrete variables.¹¹ This fact amplifies the complementary nature of the relationships between position and momentum variables in quantum theory, and is part of the physical content of the

¹⁰ For a discussion of this situation from the present perspective, see, again ([10], pp. 237-278).

¹¹ Cf., a compelling recent article by M. Pawłowski, *et al.* ([14]). The article introduces the principle of “information causality,” via the EPR-type experiments for discrete variables. The principle is shared by classical and quantum physics, and is an extension of “no-signaling” (locality) principle. The principle states that, if Alice, who performs an experiment on one particle of an EPR pair, communicates m bits of information, obtained in this experiment, to Bob, the total information, obtainable by Bob, concerning the experiment performed by Alice cannot be greater than m . (If $m = 0$, this of course merely amounts to locality.) There are conceptual parallels and connections between this principle and quantum causality in the present sense. In both cases, at stake (under the conditions of locality and in the absence of the backward-in-time causality) are the limits upon how much information associated with a past event, such as that of the measurement performed by Alice, one could obtain from a later event, such as a possible measurement performed by Bob, that is connected to or correlated with the first event. However, while the principle of information causality is respected for both classical and quantum physics, some of the key informational aspects of classical physics as defined by classical causality are not shared by quantum causality, insofar as, as here explained, classical physics does allow us to obtain new information concerning past events, which is not possible in quantum physics. The respective roles and possible connections between “information causality” and “quantum causality” specifically in the EPR-type situations merit further attention, but the subject cannot be pursued here.

uncertainty relations as explained above. This complementary nature is established by the original measurement of the position of S at t_1 , which disables any information concerning the momentum of S at t_1 or at any future point, unless we perform the alternative momentum measurement at a later point, t_2 , which, again, disables any possibility of knowing its position at t_2 . The irreducible relation between the first and the second measurement may, again, be seen as *causal* insofar as it is fundamentally determined by the initial measurement. By the same token, however, this relation is fundamentally different from classical causality—first, insofar as this relation is generally probabilistic and, secondly and correlatively, insofar as it could be experimentally disabled by the alternative second measurement. This difference manifests the nature of quantum experimentation as defined here, since it arises because what happens at any point is defined only by what kind of experiment we perform, rather than by a pre-given independent physical event or process in the way it is in classical mechanics. It is obvious that quantum causality as defined here not only avoids any backwards-in-time causality (which, again, is seen here as highly undesirable), but is *strongly* time-asymmetric, future-oriented.

In classical mechanics, the predictive relationships between measurement and prediction just discussed (a given measurement enables and defines our predictions concerning the future behavior of an object), found in both classical and quantum causality, reduce themselves to classical causality. This is because the role of measuring instruments could, in principle, be neglected or compensated for, and both the position and the momentum could be measured simultaneously (no uncertainty relations) and thus, ideally, assigned simultaneously to the system under consideration. Hence, the concept of quantum causality also allows one to see complementarity as “a rational *generalization* of the very ideal of causality,” as against the form of this ideal reflected in classical causality, in accordance with Bohr’s assessment, never really explained by him.

It is noteworthy that Bohr himself, who was aware of the essential nature of the relationships between quantum phenomena defining the present concept of quantum causality, was hesitant to speak of causality in this connection, and preferred to see these relationships in terms of complementarity, as a generalization of the ideal of causality. Perhaps, he was right to be cautious and to stick to his new term, complementarity. It is very difficult to divorce an old term, a term with a long history of use, such as causality, from its history and from many undesirable aspects of the corresponding concept that this history may carry along, and in the case of quantum physics one might well want to leave behind most of the baggage of classical causality. Most, but perhaps not all; and it is equally noteworthy that Bohr could not avoid an appeal to causality either. The reason may well be that, while quantum physics may not be classically causal, it may not be altogether without some form of causality, that is, something that is related to certain features of what we define as causality in classical physics. The preceding discussion suggests that quantum physics may well require and, philosophically, lead to a deeper concept or ideal corresponding to these features than those concepts of causality that have been hitherto developed in classical physics or relativity, or in philosophy. This may be because, by giving us and our experimentation a greater role in defining events, nature may also reveal to us its deeper workings, including those that define it apart from our experimentation with it, even though we cannot describe, know, or conceive of these features in accordance with the ideal of classical physics. But we can still do physics and can do it as well as ever, even better than ever, by experimenting, physically and mathematically, with the impact of these hidden, perhaps forever hidden, workings of nature on what we can think, know, and describe.

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