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Arkady Plotnitsky

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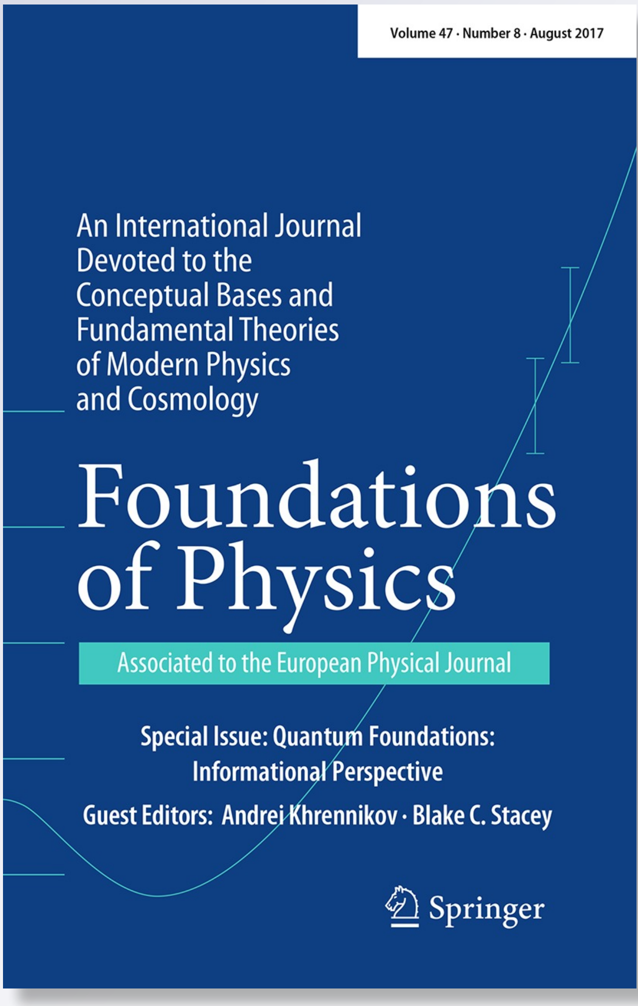

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On the Character of Quantum Law: Complementarity, Entanglement, and Information

Arkady Plotnitsky[1]

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Abstract This article considers the relationships between the character of physical law in quantum theory and Bohr's concept of complementarity, under the assumption of the unrepresentable and possibly inconceivable nature of quantum objects and processes, an assumption that may be seen as the most radical departure from realism currently available. Complementarity, the article argues, is a reflection of the fact that, as against classical physics or relativity, the behavior of quantum objects of the same type, say, all electrons, is not governed by the same *physical law* in all contexts, specifically in complementary contexts. On the other hand, the *mathematical formalism* of quantum mechanics offers correct probabilistic or statistical predictions (no other predictions are possible on experimental grounds) in all contexts, here, again, under the assumption that quantum objects themselves and their behavior are beyond representation or even conception. Bohr, in this connection, spoke of "an entirely new situation as regards the description of physical phenomena that, the notion of *complementarity* aims at characterizing." The article also considers the relationships among complementarity, entanglement, and quantum information, by basing these relationships on this understanding of complementarity.

Keywords Complementarity · Entanglement · Measuring instruments · Quantum information · Reality

"The deep truth is imageless." —P. B. Shelley, Prometheus Unbound (Act II.iv.116)

✉ Arkady Plotnitsky
plotnits@purdue.edu

1 Theory and Cultural Studies Program, Purdue University, West Lafayette, IN 47907, USA

1 Introduction: "An Entirely New Situation as Regards the Description of Physical Phenomena"

My title is a paraphrase of the title of Richard Feynman's book, *The Character of Physical Law* [1]. Indeed, it is more than merely a paraphrase, because this article argues that Niels Bohr's concept of complementarity reflects the character of physical law in quantum physics as different from that of physical law in classical physics or relativity, even to the point of appearing, in Bohr's words, to be in conflict "with the basic principles of science." Bohr argued that the concept of complementarity enables one to resolve this conflict. As he said: "it is only the mutual exclusion of any two experimental procedures, permitting the unambiguous definition of complementary physical quantities, which provides room for new physical laws the coexistence of which might at first sight appear irreconcilable with the basic principles of science. It is just this entirely new situation as regards the description of physical phenomena that the notion of *complementarity* aims at characterizing" [2, p. 700].

Feynman's discussion of quantum mechanics in his book is also known for his famous and much emulated pronouncement to the effect that "nobody understands quantum mechanics" [1, p. 129]. (Feynman refers as much to quantum phenomena as to quantum mechanics.) While I do not think that Feynman's statement is literally true, it contains a deeper truth, which is perhaps more in accord with Bohr's than with Feynman's view: Our *understanding* of quantum phenomena and quantum mechanics may need to involve something, a *deeper reality* of nature, that is beyond understanding or even thought itself, something that is literally unthinkable—*un*-thinkable. "The deep truth" of quantum physics, the truth of that which makes quantum phenomena possible, is "imageless," as Percy Bysshe Shelley said in his great poem, *Prometheus Unbound* (Act II.iv.116). Although Bohr often spoke of the behavior of quantum objects as beyond "visualization" and thus as being imageless in its literal sense, this behavior is, for Bohr, beyond representation of any kind and possibly (it is not clear whether, unlike the interpretation adopted here, that of Bohr would go that far) beyond conception, is unthinkable. This view is, arguably, the most radical departure from realism currently available, a claim that and, in the first place, the term "realism" require further explanation to be offered below. The absence of causality (a concept explained below as well) is an automatic consequence of this view. This makes the recourse to probability, found in quantum theory, unavoidable for fundamental reasons, in contrast to such classical theories as classical statistical physics or chaos and complexity theories where this recourse is due to the mechanical complexity of the systems considered, systems that are at bottom causal and in fact are governed by classical mechanics. To cite E. Schrödinger, "if a classical state does not exist at any moment, it can hardly change causally" [3, p. 154].

With this interpretation of quantum phenomena and quantum mechanics, accompanied by his concept of complementarity, which "aims at characterizing" this situation, Bohr initiated "the spirit of Copenhagen," as Heisenberg called it [4, p. iv]. This spirit was well in place by the time of Schrödinger's comment (made in 1935), which, in contrast to Heisenberg, disparagingly characterized it as "the doctrine born of distress" [3, p. 154].[1] Complementarity is a concept defined by

(a) a mutual exclusivity of certain phenomena, entities, or conceptions; and yet
(b) the possibility of considering each one of them separately at any given point, and
(c) the necessity of considering all of them at different moments for a comprehensive account of the totality of phenomena that one must consider in quantum physics.

Complementarity may be seen as a reflection and even a conceptual representation of the fact that, in a radical departure from classical physics or relativity, the behavior of quantum objects of the same type, say, electrons, is not governed, individually or collectively, by the same "physical law," especially a representational physical law, in all possible contexts, and specifically in complementary contexts.[2] Complementarity has rarely, if ever, been considered from this angle, which is only implicit, if unavoidable, in Bohr's argumentation concerning the concept. I would contend, however, that this perspective is not only helpful in clarifying the character of the concept of complementarity or, reciprocally, of the physical law in quantum physics, but is also essential to both and their relationships. Speaking of "*physical* law" in this connection requires caution, because in Bohr's or the present interpretation there is no physical law representing this behavior. I mean by this statement that the behavior of quantum objects leads to mutually incompatible observable, and thus physical, effects in complementary set-ups or contexts, the incompatibility not found in classical physics or relativity. On the other hand, the mathematical formalism of quantum mechanics offers correct probabilistic or statistical predictions (no other predictions are possible) of quantum phenomena *in all contexts*, in interpretations in the spirit of Copenhagen, under the assumption that quantum objects themselves and their behavior is beyond representation or even conception. There is no story to be told about, and no concept to be formed of how these phenomena emerge and why it is possible to predict them by means of quantum mechanics.

According to Bohr: "the purpose of such a technical term [complementarity] is … constantly to remind us of the difficulties which, as already mentioned, arise from the fact that all our ordinary verbal expressions bear the stamp of our customary forms of perception, from the point of view of which the existence of the quantum of action [Planck's constant, h] is an irrationality. Indeed, in consequence of this state of affairs, even words like 'to be' and 'to know' lose their unambiguous meaning" [5, v. 1, pp. 19–20]. That need not mean that there is anything irrational in quantum mechanics itself, which Bohr (who is sometimes misunderstood on this point) always saw as a *rational* theory, a characterization persistent in his work on quantum mechanics and complementarity. It is, however, a rational theory of something that is "irrational" in the sense of being inaccessible to rational (theoretical) thinking or even thinking in

1 I distinguish "the spirit of Copenhagen" from "the Copenhagen interpretation," because there is no single such interpretation, as even Bohr had different versions of his interpretation. Many (but not all) of these interpretations do share some of the key features of Bohr's interpretation(s). The interpretation adopted here follows that of Bohr (in its ultimate version, in place by 1930s), but with certain possible differences, which I indicate as I proceed.

2 Eventually, Bohr came to use the term "complementarity" to designate his overall interpretation of, importantly, both quantum phenomena and quantum mechanics. For clarity, however, I shall speak of Bohr's interpretation, and by complementarity refer to the concept or principle of complementarity.

general, and thus also a theory that must relate to the unthinkable. Bohr is known to have replied to Harald Høffding's question "Where can the photon be said to be?" with "To be, to be, what does it mean to be?" (cited in [6, p. 131]). In Bohr's view, one cannot say anything about quantum objects, such as a photon, apart from the effects of their interaction with measuring instruments or the classical world, where such questions as "Where can something be said to be?" can be answered or even meaningfully asked. We still tell stories about what kind of measurement we can perform, which mathematics we can use in predicting the outcome of experiments: in short these are narratives about what we do and not about how material entities, the ultimate constituents of nature, behave. We could not do science otherwise. One could also say that quantum phenomena compel us to rethink the concept of physical law in, to return to Bohr's expression, "an entirely new situation as regards the description of physical phenomena, that the notion of *complementarity* aims at characterizing" [2, p. 700].[3]

2 Concepts, Theories, Models and the Nature of Physical Reality

For near a century now, since the publication of John von Neumann's seminal *The Mathematical Foundation of Quantum Mechanics* [7], quantum mechanics and higher-level quantum theories (such as quantum electrodynamics and quantum field theory) are commonly defined in terms of Hilbert-space formalism. While there are other versions, such as those of C*-algebras and, more recently, category theory (thus far all, generally, equivalent mathematically), the Hilbert-space formalism remains dominant. However, the key mathematical features of quantum mechanics encoded in the Hilbert space formalism were not initially assumed, but were derived, beginning with Heisenberg's discovery of the theory, from certain physical features of quantum phenomena and principles, physical and mathematical, arising from these features.[4] The role of principles was crucial for Heisenberg, as is confirmed by the title of his first book, *The Physical Principles of the Quantum Theory,* based in his 1929 lectures at the University of Chicago [4]. I shall examine the physical principles grounding Heisenberg's derivation and Bohr's interpretation of quantum mechanics in the next section. First, however, I shall, in this section, explain my key concepts, as they will be understood here, for they can be understood otherwise.

I begin with the concept of concept itself, rarely adequately considered in physical or even philosophical literature. I shall understand a physical theory as an organized assemblage of concepts (in the present sense), associated with physical objects or phenomena, defined by physical experiments or their equivalent in nature. Objects and phenomena are not the same even in classical mechanics, as I. Kant realized, although they, say, planets moving around the Sun, and our phenomenal representation of them, could be treated there as the same for all practical purposes. This is no longer possible in the case of quantum objects and quantum phenomena (defined by the

[3] This and the following sections in part builds on an earlier study [8], but only in part: although there is some repetition, the present argument is different, especially by virtue of rethinking complementarity in terms of the character of quantum law.

[4] Schrödinger's derivation of quantum mechanics was primarily based on different, classical-like, principles, but could not avoid bringing in quantum ones, against his own grain, as discussed in [8, pp. 84–98].

effects of the interactions between quantum objects and measuring instruments), at least in interpretations in the spirit of Copenhagen. The present understanding of concepts follows Deleuze and Guattari, who, however, apply their definition only to philosophical, in contradistinction to mathematical or scientific, concepts, which does not appear to me justified [9, pp. 15–34].

A concept in this sense is not only a generalization from particulars (which is commonly assumed to define concepts) or merely a general or abstract idea, although a concept may contain such generalizations and abstract, specifically mathematical, ideas. A concept is a multicomponent entity, defined by the *organization* of its components, which may be general or particular, and some of these components are concepts in turn. It is the relational organization of these components that defines a concept. Consider the concept of "tree," even as it is used in our daily life. On the one hand, it is a single generalization of all (or most) particular trees. On the other hand, what makes this concept that of "tree" is the implied presence of further elements, components, or sub-concepts, such as "branch," "root," "leaf," and so forth, and the relationships among them. The concept of tree acquires further features and components, indeed becomes a different concept, in biology. This is characteristic of scientific use of concepts or just terms derived from the concepts of daily life but defined otherwise, arguably especially in quantum theory. As Bohr noted, in referring to his interpretation, "words like 'phenomena' and 'observations,' just as 'attributes' and 'measurements,' are used [here] in a way hardly compatible with common language and practical definition" [5, v. 2, pp. 63–64]. The situation is even more pronounced in the case of complementarity, a word that does not appear to have been used as *a noun* before Bohr (as opposed to the adjective "complementary") and was introduced by Bohr to designate a new concept. As he said in the passage cited earlier: "In the last resort an artificial word like 'complementarity' which does not belong to our daily concepts serves only briefly to remind us of the epistemological situation here encountered, which at least in physics is of an entirely novel character" [10, p. 87]. Complementarity is a specific physical concept and must, when considered in connection with quantum mechanics (it could be extended to and given a different specificity in other fields), be understood in its specificity.

Simple, single-component, concepts are rare, if possible at all in rigorous terms, as opposed to appearing as such because their multi-component structure is provisionally cut off. In practice, there is always a cut off in delineating a concept, which results from assuming some of the components of this concept to be primitive entities whose structure is assumed or not specified. These primitive concepts could, however, be specified by an alternative delineation, which would lead to a new overall concept, containing a new set of primitive (unspecified) components. The history of a given concept, and every concept has a history, is a history of such successive specifications and changes in specifications.

The same type of process defines a history of a given theory, which may be, and here will be, defined as an organized assemblage of concepts, again, possibly modified in the course of its history. The history of a given theory is also accompanied and shaped by the history of its interpretations. Thus, the history of quantum mechanics has been that of a seemingly uncontainable proliferation of its interpretations, multiplying even within each type.[5] A theory relates to certain manifolds of phenomena or (they are, again, generally not the same) objects, which form the "reality" considered by this theory. All modern, post-Galilean, physical theories establish such relations by means of the mathematical models they contain. I define a mathematical model as a mathematical structure or set of mathematical structures that enables any type of relation to the phenomena or objects considered. These relations between a model or the theory using it may be descriptive or representational, and derive their predictive capacity from their representational nature, as in the case of models used in classical mechanics, or they may be strictly predictive, without being representational, as in quantum mechanics, at least, again, if interpreted in the spirit of Copenhagen. The requirement of using *mathematical models* may be seen as a principle, the mathematization principle, or "the M principle," arguably, the single defining principle of all modern physics, from Galileo on. A theory always involves an interpretation of the model or models it uses by giving a physical meaning to them, for example, by establishing the way in which its models relate to the observed phenomena or objects considered, say, in representational as well as predictive terms as opposed to only predictive terms, bypassing representation. (Any viable physical model or theory has to have a predictive capacity.) For simplicity, however, I shall also speak of the corresponding interpretation of the theory containing a model, interpreted by the theory, although, rigorously, a different interpretation defines a different theory.

In defining "principles," I follow Einstein's distinction between "constructive" and "principle" theories ([8, pp. 35–50], [12]). It implies two contrasting, although in practice sometimes intermixed, types of theories. "Constructive theories" aim "to build up a picture of the more complex phenomena out of the materials of a relatively simple formal scheme from which they start out" [12, p. 228]. Einstein's example of a constructive theory in classical physics is the kinetic theory of gases, which "seeks to reduce mechanical, thermal, and diffusional processes to movements of molecules—i.e., to build them up out of the hypothesis of molecular motion," described by the laws of classical mechanics [12, p. 28]. By contrast, principle theories "employ the analytic, not the synthetic, method. The elements which form their basis and starting point are not hypothetically constructed but empirically discovered ones, general characteristics of natural processes, principles that give rise to mathematically formulated criteria which the separate processes or the theoretical representations of them have to satisfy" [12, p. 228]. Thermodynamics, Einstein's example of a classical principle theory (parallel to the kinetic theory of gases as a constructive theory), is a principle theory because it "seeks by analytical means to deduce necessary conditions, which separate events have to satisfy, from the universally experienced fact that perpetual motion is impossible" [12, p. 228]. Einstein's special and general relativity, which occasioned his reflections, are principle theories, although they also have constructive dimensions.

[5] It is not possible to survey these interpretations here. Just as does the Copenhagen interpretation, each rubric, on by now a long list (e.g., the many-worlds, consistent-histories, modal, relational, transcendental-pragmatist, and so forth) contains different versions. The literature dealing with the subject is immense. Standard reference sources, such as *Wikipedia* ("Interpretations of Quantum Mechanics" [11]), would list the most prominent such rubrics.

I adopt Einstein's definition of principles as "empirically discovered, general characteristics of natural processes, … that give rise to mathematically formulated criteria which the separate processes or the theoretical representations of them have to satisfy," but with the following qualification, likely to have been accepted by Einstein. Rather than "empirically discovered," principles are formulated, *constructed*, on the basis of empirically discovered evidence. It would, for example, be difficult to see "the impossibility of perpetual motion" as anything other than a principle thus formulated or constructed.

Constructive theories tend to be, and are often aimed to be, realist theories, more specifically, *representational* realist theories: they represent, usually causally, the corresponding objects and their behavior by mathematical models, assumed to idealize how nature works at the simpler, or deeper, level thus constructed by a theory. Thus, both classical mechanics (used in dealing with small classical systems) and classical statistical mechanics (used, as in the kinetic theory of gases, in dealing, statistically, with large classical systems, whose individual constituents are, however, assumed to be described by classical mechanics) are both realist theories. This characterization will serve this article as the definition of "representational realist theories," the type of realist theories that I shall primarily discuss here. There also "realist theories" that are not representational, because they do not offer this type of representation, but only presuppose a possibility, at least in principle, of a conception of the ultimate reality underlying the observable phenomena considered by such a theory. By "reality" itself I refer to that which actually exists or assumed to exist, without necessarily making a claim, as realist theories do, concerning the character of this existence. I shall explain the concepts of reality and realism in more detail presently. The assumption of realism of either type is abandoned or even precluded in nonrealist interpretations of quantum phenomena and quantum mechanics, following the spirit of Copenhagen, beginning with Bohr's interpretation. In such interpretations, the mathematical model of quantum mechanics, defined by its mathematical formalism, becomes a strictly probabilistically or statistically predictive model (as against representational models of the type used in classical mechanics or relativity), without representing the quantum-level reality, or even precluding a possible conception of this reality. Although realist interpretations of the quantum-mechanical formalism has been offered, its representational capacities and the effectiveness of such interpretations, or conversely, nonrealist interpretations, have been and continues to be debated.[6] The probabilistic or statistical character of quantum predictions must, however, be maintained by realist interpretations of these theories or alternative theories (such as Bohmian theories) of quantum phenomena, in order to conform with what is observed in quantum experiments, concerning which only probabilistic or statistical predictions are possible. This is because the repetition

[6] While the present argument advocates an overall nonrealist position in the spirit of Copenhagen, it is not a critique of realism, still a generally preferred philosophical view. One must also be mindful of additional complexities involved. One might, for example, argue against a realist interpretation of the wave function, specifically, as a continuous entity, and maintain a realist view of quantum mechanics, as referring to a discrete ultimate ontology. I refer here to Rovelli's elegant article [13]. In the present view, following Bohr, no ontology, either continuous or discontinuous, can again be assigned at the ultimate level (of quantum objects and processes), but only an ultimately discrete ontology at the level of quantum phenomena, defined by what is observed in measuring instruments.

of identically prepared experiments in general leads to different outcomes, a difference that, unlike in classical physics, cannot be diminished beyond a certain limit (defined by Planck's constant, h) by improving the capacity of our measuring instruments, the fact manifested in the uncertainty relations, which would apply even if we had perfect instruments. This situation leads to the quantum probability or (depending on an interpretation) statistics principle, *the QP/QS principle*, defining, alongside and correlatively to the quantum discreteness principle, *the QD principle*, all quantum theory, from Planck on.

Realist models are representational models, models that at least assume the possibility, at least ideally or in principle, of representing objects or phenomena they consider. Because I shall be primarily concerned, apart from quantum theory, with these type of models, the terms realist models and realism will henceforth be used in this sense, unless qualified otherwise. As I indicated, one can define another type of realism. This realism refers to theories that would presuppose an independent architecture (which may be temporal) of reality governing the behavior of the ultimate objects these theories consider, while allowing that this architecture cannot be represented, even ideally, either at a given moment in history or perhaps ever, but if so, only due to practical limitations. In the first of these two eventualities, a theory that is merely predictive may be accepted for lack of a realist alternative, but under the assumption or with the hope that a future theory will do better, in particular by virtue of being a realist theory of the representational type. Einstein adopted this attitude toward quantum mechanics, which he expected to be eventually replaced by a realist theory, ideally, a field theory of a classical-like type, on the model of Maxwell's electrodynamics. Einstein's general relativity followed this program, as did his subsequent work on the unified field theory, and he saw the development of such a theory as imperative in dealing with the ultimate constitution of nature. Even in the second eventuality, the ultimate nature of reality is customarily deemed to be conceivable on realist models of classical physics, possibly adjusting them to accommodate new phenomena, such as electromagnetism, and new concepts, such as field, classical or even quantum, or more recently automata.

What, then, unites both conceptions of realism and thus defines realism most generally is the assumption that an *architecture* of reality, rather than only reality itself, exists independently of our interactions with it, or at least that the concept of architecture or structure would apply to reality. The latter presupposition defines the so-called structural realism, although structural realism, generally, makes stronger claims concerning this structure. In sum, realism is defined by the assumption that the ultimate constitution of nature possesses properties or attributes and the structured relationships among them that may be either (a) known in one degree or another and, hence, represented, at least ideally, by a theory or model, or (b) unknown or even unknowable. The difference between (a) and (b) is that between the two types of realism here considered, representational and nonrepresentational. Nonrealist representations, in the spirit of Copenhagen, of quantum phenomena and quantum mechanics abandon both types of realism.

One could, in principle, also see a claim concerning merely the existence or reality of something to which a theory can relate in any way, even without representing it, and thus the present view of quantum reality, as a form of realism. This use of the term realism is sometimes found in advocating interpretations of quantum mechanics that are nonrealist in the present sense, that is, those that do not claim that quantum-mechanical formalism represents, even ideally, the behavior of quantum objects and processes, but only serves to predict the outcome of quantum experiments. However, placing this view of reality outside—and thus divorcing reality from—realism is in accord with a more common use of the term realism, and is advantageous in considering interpretations of quantum phenomena and quantum mechanics in the spirit of Copenhagen, which renounces or even preclude realism of either type.

While abandoning realism, again, of either type, nonrealist interpretations in the spirit of Copenhagen assume the concept of *reality*, by which I mean that which exists or is assumed to exist, without any claim concerning the *character* of this existence, claims that define realist theories. I understand existence as a capacity to have effects on the world with which we interact, the world that has such effects upon itself. In physics, the primary reality considered is that of nature or matter (including that of fields). It is generally, but not always (although exceptions are rare), assumed to exist independently of our interaction with it, and to have existed when we did not exist and to continue to exist when we will no longer exist. This assumption is upheld in nonrealist interpretations of quantum mechanics, but now in the absence of a representation or even conception of the character of this existence, for example, as either discrete or continuous. If, however, *realism* presupposes a representation or at least a possible conception of reality, which may be called the realism principle or *the R principle*, this concept of *reality* is that of "reality *without* realism," again, of either type [8, 14]. The assumption of this concept of reality is a principle, *the RWR principle*. The existence or *reality* of quantum objects, thus placed beyond representation or (in the present view, if not that of Bohr) even conception, is inferred from the totality of effects, including the complementary character of some of them, they have on our world, specifically on experimental technology. Nobody has ever observed, at least thus far, electron or photon as such, in motion or at rest, although photons of course only exist in motion. It is only possible to observe traces of their interactions with measuring instruments. In nonrealist interpretations, the reconstitution of the independent behavior of quantum objects in space and time from these traces is, in Bohr's defining words, "*in principle* excluded" [5, v. 2, p. 62]. This makes the absence of causality automatic, but allows one to predict in probabilistic or statistical terms the outcomes of quantum experiments observed in measuring instruments. This scheme is more radical than that of Kant's noumena or things in themselves, vis-à-vis phenomena or appearances formed in our minds. This is because, according to Kant, while noumena are unknowable, they are still in principle conceivable, especially by what he calls "Reason" [*Vernunft*] a higher faculty than "Understanding" [*Verstand*], rather than, as here, placed beyond conception, beyond thought ([8, pp. 19–21], [15, p. 115]). Kant's scheme is realist in the present definition: it conforms to realism of nonrepresentational type.[7]

A principle theory could be realist or not, by, in the first case, acquiring constructive dimensions. Constructive theories are, as I explained, by definition realist and are usually causal. They may and customarily do involve principles, such as the equivalence principle in general relativity, or the principle of causality, dominant throughout the

[7] See, Jaeger's helpful analysis of the concept of quantum objects from a realist perspective [16].

history of modern physics, until quantum mechanics put it into question. This principle, as defined, for example, by Kant (this type of definition has been commonly used since), states that, if an event takes place, it has a cause of which it is an effect [15, pp. 305, 308]. I shall refer to this form of causality as *classical* causality.[8] Such causal influences are also commonly, although not always, assumed to propagate from past or present towards future.[9] The distinction between constructive and principle theories is, thus, not unconditional. There is, however, an asymmetry between them: while a constructive theory always involves principles, a principle theory need not involve constructive strata at the ultimate level considered and thus need not be realist at this level.

If interpreted along nonrealist, RWR-principle-based, lines, quantum mechanics is a principle theory by definition, because it is not possible to constructively configure the ultimate entities, quantum objects, due to which quantum phenomena arise, unless one sees quantum objects as *constructed* by quantum theory as irreducibly *unconstructible*. According to Bohr, thus defining his position in accord with the RWR principle, "in quantum mechanics we are not dealing with an arbitrary renunciation of a more detailed analysis of atomic phenomena, but with a recognition that such an analysis is *in principle* excluded," beyond a certain point [5, v.2, p. 62]. The RWR principle maintains both the existence, *reality*, of quantum objects and processes, and the impossibility of representing or even conceiving of the nature of this reality, for example, again, as either discrete or continuous, the impossibility of realism, at least as things stand now. Einstein hoped that nature will eventually allows us to do better as concerns realist capacities of our fundamental theories. Bohr, by contrast, thought that nature *might not* allow us to do better in dealing with quantum phenomena, which is not the same as that it *will not*. The question, which defined the Bohr-Einstein debate, remains open and continues to be intensely debated.

Finally, an appeal to fundamental principles need not, and here does not, imply that there is some Platonist permanence to them. Principles change as our experimental findings and our theories change, and we cannot always anticipate or control these

[8] I distinguish causality, which is an ontological category, describing reality, from determinism, which is an epistemological category, describing our ability to predict the state of a system (ideally) exactly at any moment of time once we know its state at a given moment of time. Determinism is sometimes used in the same sense as causality is used here, and in the case of classical mechanics (which deals with single objects or a small number of objects), causality and determinism coincide. Once a classical system is large, one can no longer predict its causal behavior exactly, which is one of my reasons for distinguishing causality and determinism. Nonrealist interpretations of quantum mechanics automatically preclude not only determinism but also causality, and only allow for probabilistic or statistical predictions even in dealing with individual quantum objects. I shall explain the sense in which such predictions concerning individual quantum events could be statistical below.

[9] This requirement is strengthened by special relativity, which restricts causes to those occurring in the backward (past) light cone of the event that is seen as an effect of this cause, while no event can be a cause of any event outside the forward (future) light cone of that event. These restrictions follow from the assumption that causal influences cannot travel faster than the speed of light in a vacuum, c. Principle theories do not require classical causality, which becomes difficult to assume in quantum physics, without violating special relativity or more generally locality, defined by the assumption that all physical influences are local. What may be called "the relativistic causality," the prohibition of physical influences towards the past, may be maintained, as it is in quantum mechanics or quantum field theory, in the absence of classical causality. Relativity itself is, again, classically causal.

changes. The principles of quantum mechanics replaced, within a new scope, some among the main principles of classical physics, which continue to remain operative in classical physics and sometimes extend to quantum theory. Such changes can also occur within the same physical scope, as in the case of general relativity theory versus Newton's theory of gravity. Some principles of quantum theory have changed as well, both in view of extending the scope of quantum theory to quantum field theory and within the scope of quantum mechanics, as in quantum information theory. The QD and QP/QS principles have remained in place throughout the history of quantum theory, from Planck on. But that does not mean that they may not be abandoned at some point.

3 Quantum Mechanics as a Principle Theory: From Bohr to Heisenberg

The RWR principle only emerged in 1930s in the Bohr-Einstein debate. Heisenberg, in his initial approach to quantum mechanics, merely abandoned the project of describing the motion of electrons because he thought that such a description was unachievable at the time, rather than "*in principle* excluded" [5, v. 2, p. 62]. His approach was guided by the combination of a certain *proto*-RWR principle and the QD and QP/QS principles (adopted from the old quantum theory). Heisenberg's proto-RWR principle automatically abandons classical causality, even if not, in principle, precludes it, as would the RWR principle. Bohr's initial comment on Heisenberg's discovery in 1925 described the situation as follows: "In contrast to ordinary mechanics, the new quantum mechanics does not deal with a space–time description of the motion of atomic particles. It operates with manifolds of quantities [matrices] which replace the harmonic oscillating components of the motion and symbolize the possibilities of transitions between stationary states in conformity with the correspondence principle. These quantities satisfy certain relations which take the place of the mechanical equations of motion and the quantization rules [of the old quantum theory]" [5, v. 1, p. 48]. In addition to:

(1) the proto-RWR principle, according to which, "quantum mechanics does not deal with a space–time description of the motion of atomic particles,"

Heisenberg's approach to quantum mechanics and then Bohr's interpretation adopted the following principles, in essence found already in Bohr 1913 atomic theory (with Bohr's complementarity principle added in 1927):

(2) the principle of discreteness or the QD principle, according to which all observed quantum phenomena are individual and discrete in relation to each other (which is not the same the atomic discreteness of quantum objects themselves);
(3) the principle of the probabilistic or statistical nature of quantum predictions, the QP/QS principle, even (in contrast to classical statistical physics) in the case of primitive or elemental quantum processes, which nature also reflects a special, nonadditive, nature of quantum probabilities and rules, such as Born's rule, for deriving them, and
(4) the correspondence principle, which, as initially understood by Bohr, required that the predictions of quantum theory must coincide with those of classical mechanics in the classical limit, but which was given by Heisenberg a more rigorous form and was made into "the mathematical correspondence principle," requiring that

the equations of quantum mechanics convert into those of classical mechanics at the classical limit.

The QD principle originated in Bohr's 1913 atomic theory, based on "quantum postulates," pertaining to the discrete behavior ("quantum jumps") of electrons in atoms.[10] According to Heisenberg's paper introducing quantum mechanics: "In order to characterize this radiation we first need the frequencies which appear as functions of two variables. In quantum theory these functions are in the form [18, p. 263]:

$$v(n, n-\alpha) = 1/h\{W(n) - W(n-\alpha)\}''$$

The QD and QP/QS principles are, thus, correlative. Quantum discreteness, again, central to all quantum theory, was eventually recast by Bohr in term of his concept of "phenomenon," defined in terms of what is observed in measuring instruments under the impact of quantum objects, in contradistinction to quantum objects themselves, which could no be observed or represented [5, v. 2, p. 64]. Quantum phenomena are irreducibly discrete in relation to each other, and one cannot assume that there continuous processes that connected them, especially causally, even in dealing with elemental individual processes and events, rather than with complex mechanical systems, such as those considered in classical statistical physics or in chaos theory. In nonrealist interpretations, quantum mechanics, again, only estimates the probabilities or statistics of the outcomes of discrete future events and tell us nothing about what happens between them. It is worth noting that it does not describe the data observed and hence quantum phenomena either. These data, along with and as part the manifested behavior of measuring instruments, are described by classical physics, which, however, cannot predict these data.

The postulate that mathematically expressed the QP/QS principle was the formula for the probability amplitudes cum Born's rule. Heisenberg only formulated this type of rule in the case of electrons' behavior in the hydrogen atom, rather than, as Born did, as universally applicable in quantum mechanics. Born's rule is, as I said, not inherent in the formalism but is added to it: it is *postulated*, a fact reflected in the names of equivalent conceptions, such as von Neumann's projection postulate or Lüder's postulate.

The correspondence principle played an essential role in the discovery of quantum mechanics (as matrix mechanics) by Heisenberg, who, in contrast to the previous, more ad hoc, use of it by Bohr and others, gave it a rigorous mathematical form, made it the mathematical correspondence principle. In this form, the principle required that both the equations of quantum mechanics (which were formally those of classical mechanics) and the variables used (which were different) convert into those of classical mechanics at the classical limit, a conversion automatic in the case of equations but

[10] Although they do capture quantum discreteness as a defining principle of quantum theory, these postulates should not be confused with Bohr's quantum postulate, introduced, along with complementarity, in the so-called Como lecture, in 1927. This postulate "attributes to *any atomic process* [rather than only to quantum jumps] an essential discontinuity, or rather individuality, … and is symbolized by Planck's quantum of action" [5, v. 1, p. 53]; emphasis added). For an illuminating discussion of Bohr's 1913 postulates, see Folse's article [17].

not the case of variables. The processes themselves, however, are still assumed to be quantum.

Heisenberg's discovery of quantum mechanics was a remarkable achievement, ranked among the greatest achievements in the history physics. The detailed discussion of his derivation of quantum mechanics from the principles formulated above is beyond my scope here.[11] Several key points of his thinking should, however, be noted. Heisenberg's new quantum variables, to which the equations of quantum mechanics (again, formally, the same as those of classical mechanics), were infinite unbounded matrices with complex elements, in effect operators in a Hilbert space over complex numbers, which were noncommutative. This mathematical concept has never been used in physics previously, and it was especially unfamiliar, and, initially, even off-putting for some, because of their noncommutative nature. In fact, while matrix algebra, in both finite and infinite dimensions, was well developed in mathematics by then (Heisenberg was famously unaware of this and reinvented his matrix calculus), the *unbounded* infinite matrices were not previously studied anywhere. As became apparent later, matrices of this kind are necessary to derive the uncertainty relations for continuous variables. The concept was also used fundamentally differently physically from the way representational concepts of classical physics or relativity were used. Each such variable was a mathematical entity enabling only the probabilistic predictions concerning the relationships between *quantum phenomena*, observed in measuring instruments (initially atomic spectra), without providing a mathematically idealized representation of the behavior of *quantum objects* responsible for the appearance of these phenomena. As Heisenberg said: "What I really like in this scheme is that one can really reduce *all interactions* between atoms and the external world …to transition probabilities" (W. Heisenberg, Letter to Kronig, 5 June 1925; cited in ([19, v. 2, p. 242]; emphasis added).

It should be qualified that Heisenberg's derivation of quantum mechanics from principles cannot be considered a strictly rigorous derivation. As he noted himself in referring to his later (equivalent) derivation: "The deduction of the fundamental equation of quantum mechanics is not a deduction in the mathematical sense of the word, since the equations to be obtained form themselves the postulates of the theory. Although made highly plausible, their ultimate justification lies in the agreement of their predictions with the experiment" [4, p. 108]. While borrowing the *form* of equations from classical mechanics by the mathematical correspondence principle was decisive for establishing the mathematical architecture of quantum mechanics, Heisenberg more "guessed" than derived the variables that he needed. One may thus envision a more rigorous derivation of quantum mechanics from fundamental principles, especially by avoiding making the equations of quantum mechanics postulate of the theory. Most recent work in this direction, on some of which I shall comment below, has been in quantum information theory in dealing with discrete variables and finite-dimensional Hilbert spaces, as opposed to the infinite-dimensional Hilbert spaces, needed for continuous variables (e.g., [20–22]).[12]

[11] I have considered it on previous occasions, most recently in [8, pp. 68–83].

[12] Among the key earlier works are Wheeler's "manifesto" [23], Zeilinger's article [24], Hardy's article [25], and Fuchs's work (e.g., [26]), which "mutated" to a related but different program, that of quantum Byesianism or Qbism (e.g., [27]).

Heisenberg's approach may, however, be considered in quantum-informational terms. The quantum-mechanical situation, as he conceived of it, was defined by (a) certain *already obtained* information, concerning the energy of an electron, derived from spectral lines, observed in measuring instruments; and (b) certain possible future information, concerning the energy of this electron, *to be obtainable* from spectral lines, predictable, in probabilistic or statistical terms, again, associated with spectral lines to be observed in measuring instruments. Heisenberg, again, abandoned the task of developing a mathematical scheme representing how these data or information are connected by a spatiotemporal process, and deriving his predictions from such a scheme. Instead, he decided to try to find and succeeded in finding a mathematical scheme that would enable these predictions by probabilistically or statistically relating (a) and (b). I am not saying that Heisenberg's matrix mechanics was, or that quantum mechanics *is*, *only* a quantum information-processing scheme. Heisenberg was concerned with how quantum objects and processes work, even though these workings defied being represented, which fact is itself an effect of these workings. The information at stake was related to them and quantum mechanics was about them, rather than part of information processing by using suitable experimental technology. But then, quantum information theory, too, may serve and has served the purposes of fundamental physics, rather than only aiming at theorizing quantum information processing between devices.

In some nonrealist interpretations, such as the one the present author would favor, following W. Pauli, individual quantum events are not subject even to *probabilistic* laws of quantum mechanics, which makes these laws strictly collective, *statistical*, again, under the assumption that there are now laws describing the behavior of quantum objects themselves ([8, pp. 173–186], [14]).[13] The QP/QS principle, accordingly, becomes strictly the QS principle. According to Pauli:

> As this indeterminacy is an unavoidable element of every initial state of a system that is at all possible according to the [quantum-mechanical] laws of nature, the development of the system can never be determined as was the case in classical mechanics. The theory predicts only the statistics of the results of an experiment, when it is repeated under a given condition. Like the ultimate fact without any cause, the individual outcome of a measurement is, however, in general not comprehended by laws. This must necessarily be the case, if quantum or wave mechanics is interpreted as a rational generalization of classical physics, which take into account the finiteness of the quantum of action [h]. The probabilities occurring in the new laws have then to be considered to be primary, which means not deducible from deterministic [in effect, causal] laws. As an example of these primary probabilities I mention here the fact that the time at which an individual atom will undergo a certain reaction stays undetermined even under conditions where the rate of occurrence of this reaction for a large collection of atoms is practically certain [28, p. 32].

[13] While Bohr did not expressly state his position on this point, it appears to accord with this view [8, pp. 180–184].

Given that Pauli does not specify otherwise, this "beyond the law," in his, or in the present, view, appears to include probabilistic or, in this view, *statistical* laws of quantum mechanics, which only apply to statistical multiplicities of repeated quantum events. Elementary quantum processes and events associated with them are not only beyond representation or conception, but are also beyond even probabilistic predictions. Exact predictions, even ideal ones (but there are no other exact predictions), are excluded on experimental grounds. The outcome of an individual quantum experiment, a future individual quantum event, cannot, in general, be assigned probability. All quantum phenomena are, then, unique and unrepeatable, and are always discrete relative to each other, thus manifesting the QD principle and the ultimate form on the QP/QS principle, now as the QS principle, insofar as individual quantum experiments are not subject to laws, even to the probabilistic or statistical laws of quantum mechanics. They are random. Indeed, randomness may be defined by the impossibility of such an assignment.[14] One of the greatest mysteries, if not the greatest mystery, of quantum physics is that, in certain, but not all, circumstances, random individual events conspire to form statistically correlated and thus statistically ordered multiplicities; and these correlations are correctly predicted by quantum mechanics.

4 Reality, Phenomena, and Complementarity: From Heisenberg to Bohr

Bohr's interpretation of quantum phenomena and quantum mechanics added a new principle to those used by Heisenberg, the complementarity principle, stemming from the concept of complementarity, which, as noted from the outset, requires:

(a) a mutual exclusivity of certain phenomena, entities, or conceptions; and yet
(b) the possibility of considering each one of them separately at any given point; and
(c) the necessity of considering all of them at different moments for a comprehensive account of the totality of phenomena that one must consider in quantum physics.

Although the concept or the principle of complementarity was never quite given by Bohr a definition in such a single formulation, the definition just stated may be surmised from several of his statements. I would argue that at least by the time of his reply [2] to the argument of Einstein et al. (EPR) [29], Bohr understood the concept and principle of complementarity in this way. Parts (*b*) and (*c*) of this definition are just as important as part (*a*), and by missing them, as is often done, one would miss much of the import of Bohr's concept. That we have a free choice as concerns what kind of experiment we want to perform is in accordance with the very idea of experiment in science, including in classical physics [2, p. 699]. However, in contrast to classical physics or relativity, implementing our decision will enables us to make certain types of possible predictions and will irrevocably exclude certain other, *complementary*, types of possible predictions. We actively shape what will happen, define the course of reality. In this sense, complementarity may be seen, as it was by Bohr, as "a rational generalization of the ideal of causality" and as representing what may be called

[14] This concept of randomness is not ontological, because one cannot ascertain the *reality* of this randomness, but *epistemological*. It is ultimately a matter of assumption or belief, practically justified in a given interpretation.

"quantum causality," in the absence of classical causality or, in the first place, realism, again, of either type ([5, v. 2, p. 41], [8, pp. 203–206]).

As I stressed from the outset, complementarity is a *quantum-theoretical concept*.[15] It is, Bohr says, "an artificial word" that "does not belong to our daily concepts" and only "serves to remind us of the epistemological situation here encountered, which at least in physics is of an entirely novel character," and which "the notion of *complementarity* aims at characterizing" ([2, p. 700], [10, p. 87]). Accordingly, complementarity should be treated as such a concept, by respecting its conceptual specificity, which has not always been done, especially when it comes to the relationships between the whole and its parts, suggested by the term. Bohr's concept prevents us from ascertaining the "whole" composed from the complementary "parts," in conflict with the conventional understanding of parts as *complementing* each other within a whole. At any moment of time only one of these parts and not the other could be ascertained, as one and not the other of two complementary effects of quantum behavior, manifested in a measuring device. This ascertainable part is the only "whole" at this moment of time, and there is no overall wholeness that encompasses this local "wholes" in the history of the system. As a quantum-theoretical concept, complementarity also acquires probabilistically or statistically predictive aspects, in accordance with the QP/QS principle, as well as mathematical aspects, because one needs mathematics to make these predictions.[16]

It is worth noting that wave-particle complementarity, with which the concept of complementarity is associated most commonly, did not play a significant role in Bohr's thinking, arguably, because Bohr was always aware of the difficulties of applying the (physical) concept of waves to quantum objects. Bohr's ultimate solution to the dilemma of whether quantum objects are particles or waves—or his "escape" from the paradoxical necessity of seeing them as both—is that they and thus the ultimate nature of quantum reality are neither. Instead, either "picture" is seen by Bohr as an *effect* or set of *effects*, particle-*like* (which may be individual or collective) or wave-*like* (which are always collective), of the interactions between quantum objects and measuring

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15 It is true that the concept of complementarity, for example, as defined here, is very general and allows for applications beyond physics. Bohr (tentatively) and others, such as W. Pauli, K. G. Jung, M. Delbrück, and others proposed using it in philosophy, biology, and psychology. I shall, however, not consider these extensions here.

16 Part of the genealogy of the concept of complementarity was a conception, developed by Bohr in early psychological studies (before he began to study physics), that human cognitions must, under certain circumstances, be positioned in incompatible planes. Bohr saw this situation as analogous to the way Riemann surfaces work in the theory of functions of complex variables. A Riemann surface allow one to remove ambiguity and properly define functions of complex variables, such as $f(z) = \sqrt{z}$. It is ambiguous, and hence not properly definable, when considered on the complex plane, as $\sqrt{z}$ has two meanings, but is well defined on the corresponding Riemann surface, because it has a single meaning on each of the two mutually exclusive, "complementary," sheets of the surface. Bohr reflected on this analogy, specifically using the Riemann surface for $\sqrt{z}$ as an example, in his final interview (he died shortly thereafter) [30]. Of course, the Riemann surface for $\sqrt{z}$ rigorously defines two mutually exclusive mathematical domains without involving probability. In quantum physics, while the physical behavior of quantum objects of the same type is different and indeed incompatible in each complementary experimental setup, the quantum-mechanical formalism, while equally capably of predicting this behavior in both setups, does so only in probabilistic or statistical terms, which is, as I explained, in accord with what is observed. Nevertheless, the analogy is telling as concerns the character of Bohr's thinking and the architecture of concepts, as considered above.

instruments. Bohr used the idea of "symbolic waves" as related to probability or statistics of quantum predictions, following Born's interpretation, as a "probability wave" [31, p. 804]. This phrase must, however, be used with caution, and it is best understood in Schrodinger's terms: "probability waves" are "expectation-catalogs" concerning discrete events [3].

The concept of complementarity is arguably exemplified best by complementarities of space-time coordination and the application of momentum or energy conservation laws. There are two complementarities here: the first is that of the position and momentum measurements, and the second is that of the time and the energy measurements. These complementarities are correlative to Heisenberg's uncertainty relations and establish Bohr's interpretation of them. Technically, the uncertainty relations, $\Delta q \Delta p \cong h$ (where q is the coordinate, p is the momentum in the corresponding direction), only prohibit the simultaneous *exact* measurement of both variables, which is always possible, at least ideally and in principle, in classical physics, and allows one to maintain causality there. Even this statement needs further qualification, however, and the physical meaning of the uncertainty relations is much deeper in Bohr's interpretation and a complex subject in its own right with a long history.[17] First of all, the uncertainty relations are not a manifestation of the limited accuracy of measuring instruments, because they would be valid even if we had perfect measuring instruments. In classical and quantum physics alike, one can only measure or predict each variable *within* the capacity of our measuring instruments. In classical physics one can, in principle, measure both variables simultaneously within the capacity, and improve the accuracy of this measurement by improving this capacity, in principle indefinitely. The uncertainty relations preclude us from doing so for *both* variables regardless of this capacity in dealing with quantum objects. In Bohr's interpretation, the uncertainty relations make each type of measurement complementary to the other, in conformity with the definition of complementarity given above, as mutually exclusive yet allowing us a freedom of performing either of them at any moment in time. Furthermore, in Bohr's view, one not only cannot measure both variables simultaneously but also cannot define them simultaneously, and moreover, one cannot ever define them alternatively for the same quantum object. We always need at least two objects to define both variables. If, after determining, say, the position of the electron, emitted from a source, at time t_m after the emission, we want to determine the conjugate momentum, we need to repeat the same, identically prepared, emission from the same source of another electron, and then measure its momentum after the same time t_m after the emission. The uncertainty relations apply to the statistics of two large sets of such alternative measurements. Each measurement associated either complementary variable would be (ideally) exact. According to Bohr: "the statistical character of the uncertainty relations in no way originates from any failure of measurement to discriminate within a certain latitude between classically describable states of the objects, but rather expresses an essential limitation of applicability of classical ideas to the analysis of quantum phenomena" [34, p. 100]. Elsewhere Bohr offers a striking sentence, stating that in question in the uncertainty relations is not the accuracy

17 The uncertainty relations remain a subject of ongoing foundational discussions. Among the more illuminating contributions are [32,33].

of measurement but complementarity, defined by the irreducible difference between measuring instruments and quantum objects in quantum physics. He says: "we are of course not concerned with a restriction as to the accuracy of measurement, but with a limitation of the well-defined application of space-time concepts and dynamical conservation laws, entailed by the necessary distinction between measuring instruments and atomic objects" ([5, v. 3, p. 5], also [10, p. 86], [5, v. 2, p. 73]).

The description of effects of the interactions between quantum objects and measuring instruments is classical and can be treated as realist or objective, while no representation of quantum objects and their behavior, and hence of the efficacity of these effects, is, again, possible on this view. As Bohr notes: "It lies in the nature of physical observation … that all experience must ultimately be expressed in terms of classical concepts, neglecting the quantum of action [*h*]. It is, therefore, an inevitable consequence of the limited applicability of the classical concepts that the results attainable by any measurement of atomic quantities are subject to an inherent limitation" [5, v. 1, pp. 94–95]. Bohr's appeal to classical concepts in interpreting quantum phenomena is often misunderstood, and it is worth to briefly comment on it here. First, as this formulation makes clear, although indispensable, classical concepts are never sufficient for a proper quantum-mechanical account. Bohr, accordingly speaks of "the inability of the classical frame of concepts to comprise the peculiar feature of indivisibility, or 'individuality,' characterizing the elementary [quantum] processes" [5, v. 2, p. 34]. Secondly, the *interaction* between quantum objects and measuring instruments is quantum, and thus it is never subject to description or representation in terms of classical or any other concepts, any more than is any other quantum process, including of course the one that preceded this interaction. Thus, a realist representation is only possible and can be provided by classical physics for measuring instruments impacted by quantum objects. But it is, in Bohr's view, never possible for quantum objects and processes, including those responsible for what is observed in measuring instruments, as effects of their interactions with quantum objects, effects that could be described by classical physics but cannot be predicted by it.

This view of the quantum-mechanical situation is essentially connected to the question of the nature of quantum information. Physically, all quantum information, that is, information actually obtained in or processed through quantum experiments is physically classical: it consists of classical "bits." However, the *emergence* of this information, as individual and collective effects of the interaction between quantum objects and measuring instruments, and of the arrangement, the architecture, of these effects, such as and in particular, the correlational and complementary architecture of some of them, is *quantum*. That is, this emergence cannot be understood of in terms classical physics, nor, as a consequence, can these effects and, most importantly, this architecture be predicted by it. In nonclassical, RWR-principle-based, interpretations of quantum phenomena and quantum mechanics this emergence cannot be understood or even conceived by quantum mechanics either, or any means available to us: quantum effects emerge from the quantum unthinkable. This quantum unthinkable is the quantum "it from bit," famously invoked, as will be seen in fact following Bohr (which is often missed), by J. A. Wheeler in his quantum-information manifesto "Information, Physics, Quantum: The Search for Links" [23, p. 3]. While, however, not providing an account, in particular a representational or otherwise realist account of this emergence, of the "it," quantum mechanics can predict these effects, these "bits," and this architecture of "bits," which architecture defines quantum information most essentially.

According to Bohr, the reason for this situation is the irreducible role of measuring instruments in the constitution of quantum phenomena, including those associated with elementary quantum processes, as opposed to classical physics, where this role could in principle be neglected or compensated for [5, v. 2, p. 72]. This role is irreducible even in predictions concerning a (distant) quantum object, which are sometimes possible in quantum physics, say, in the EPR-types experiments, discussed below. These experiments with specifically prepared pairs of quantum objects, which allows one can make predictions concerning one object of each pair, without previously performing a measurement on this objects itself, but by performing a measurement on the other, spatially separated, object of the EPR pair. Bohr's argument of the irreducible role of measuring instruments in the constitution of quantum phenomena, which had defined his view of the quantum-mechanical situation all along, eventually led him to his ultimate, RWR-principle-based, interpretation and his concept of phenomenon in the late 1930s (e.g., [34]). According to Bohr:

> I advocated the application of the word phenomenon exclusively to refer to the *observations* obtained under specified circumstances, including an account of the whole experimental arrangement. In such terminology, the observational problem is free of any special intricacy since, in actual experiments, all observations are expressed by unambiguous statements referring, for instance, to the registration of the point at which an electron arrives at a photographic plate. Moreover, speaking in such a way is just suited to emphasize that the appropriate physical interpretation of the symbolic quantum-mechanical formalism amounts only to predictions, of determinate or statistical character, pertaining to individual phenomena appearing under conditions defined by classical physical concepts [describing the relevant observable parts of measuring instruments] [5, v. 2, p. 64].

Referring phenomenologically to "observations," rather than, ontologically, to the observed situations themselves, explains Bohr's choice of the term "phenomenon." Because, as explained above, the observed parts of the measuring arrangement are described classically, phenomena and, physically, measuring instruments, as objects (not quantum objects!), could be considered as identical to each other, as they are in classical physics. By contrast, as also explained above, the *emergence* of these phenomena, which is due to the interaction between measuring instruments and *quantum objects*, or quantum objects and processes themselves, are no longer available to a representation or even conception by any means, those of quantum mechanics included.

Part of Bohr's concept of phenomenon and indeed the main reason for its introduction is that this concept "*in principle* exclude[s]" any representation of quantum objects and their behavior by means of quantum mechanics or possibly any other theory, even if not a possible conception [5, v. 2, p. 62]. In other words, the concept of phenomenon is correlative to the RWR principle, reached by Bohr, in its fully-fledged form, at this stage, in conjunction with both the QD and QP/QS principles. Physical quantities obtained in quantum measurements and defining the physical behavior of certain (classically described) parts of measuring instruments are *effects* of the interactions between quantum objects and these instruments. But these properties are no longer assumed to correspond to any properties pertaining to quantum objects, even any single such property, rather than only certain joint properties, in accordance with the uncertainty relations. Bohr's earlier view up to and including his reply to EPR allowed for this type of attribution *at the time of measurement* and only then. Even this less radical view implies that the physical state of an object cannot be defined on the model of classical physics, which requires an unambiguous determination of both conjugate quantities for a given object at any moment of time and independently of measurement, which is not possible in quantum physics because of the uncertainty relations. In Bohr's ultimate interpretation, however, an attribution *even of a single property* to any quantum object as such is *never possible—before, during, or after measurement*. The conditions that experimentally obtain in quantum experiments only allow one to rigorously specify measurable quantities that can, in principle, physically pertain to measuring instruments and only to them. Even when we do not want to know the momentum or energy of a given quantum object and thus need not worry about the uncertainty relations, neither the exact *position* of this object itself nor the actual time at which this "position" is established is ever available and hence in any way verifiable. Any possible information concerning quantum objects as independent entities is lost in "the finite [quantum] and uncontrollable interaction" between quantum objects and measuring instruments [2, p. 697]. However, this interaction leaves a mark in measuring instruments, a mark, a bit of information, that can be treated as a part of a permanent, objective record, which can be discussed, communicated, and so forth. The uncertainty relations remain valid, of course. But they now apply to the corresponding (classical) variables of suitably prepared measuring instruments, impacted by quantum objects. We can either prepare our instruments so as to measure or predict a change of momentum of certain parts of those instruments or so as to locate the spot that registers an impact by a quantum object, but never do both in the same experiment. The uncertainty relations are correlative to the complementary nature of these arrangements.

This view implies and indeed arises from the assumption of the irreducible difference, again, beyond Kant, between quantum phenomena and quantum objects. According to Bohr in his reply to EPR:

> This necessity of discriminating in each experimental arrangement between those parts of the physical system considered which are to be treated as measuring instruments and those which constitute the objects under investigation may indeed be said to form a *principal distinction between classical and quantum-mechanical description of physical phenomena*. It is true that the place within each measuring procedure where this discrimination is made is in both cases largely a matter of convenience. While, however, in classical physics the distinction between object and measuring agencies does not entail any difference in the character of the description of the phenomena concerned, its fundamental importance in quantum theory, as we have seen, has its root in the indispensable use of classical concepts in the interpretation of all proper measurements, even

> though the classical theories do not suffice in accounting for the new types of regularities with which we are concerned in atomic physics. In accordance with this situation there can be no question of any unambiguous interpretation of the symbols of quantum mechanics other than that embodied in the well-known rules which allows allow to predict the results to be obtained by a given experimental arrangements described in a totally classical way, and which have found their general expression in he transformations theorems, already referred to [2, pp. 701, 697–697n].

The situation is connected to the question of the "cut" [*Schnitt*] between the quantum and the classical, the term more commonly used by Heisenberg and von Neumann than Bohr, and known as the Heisenberg or the Heisenberg–von Neumann cut. Bohr's statement may suggest that, while observable parts of measuring instruments are described by means of classical physics, the independent behavior of quantum objects is described by means of quantum-mechanical formalism. This type of view was indeed entertained, ambivalently, by Bohr in the Como lecture of 1927 [5, v. 1, pp. 52–91] and was adopted by others, such as Dirac and von Neumann. Sometimes it is even characterized (again, always a suspect characterization) as "the Copenhagen interpretation," arguably, more because of the impact of von Neumann's book [7] than Bohr's Como argument, which, however, had some influence in this respect as well, including on von Neumann. Bohr himself, however, quickly abandoned this view, following and under the impact his discussion of quantum mechanics with Einstein at the Solvay conference merely a month after the Como lecture [5, v. 2, p. 41].[18] The passage from his reply to EPR under discussion at the moment dates to 1935. Bohr does say there that observable parts of measuring instruments are described by means of classical physics, again, with a crucial qualification that this description only concerns these observable parts, because measuring instruments also have quantum strata, through which they interact with quantum objects. However, he does not say and does not mean (there is no evidence to conclude otherwise from any of his writings after the Como lecture) that the independent behavior of quantum objects is described or represented by means of the quantum-mechanical formalism, assumed to only have a probabilistically or statistically predictive role, by the QP/QS principle.

While "it is true that the place within each measuring procedure where this discrimination [between the object and the measuring instrument] is made is in both [classical and quantum] cases *largely* a matter of convenience" (emphasis added), it is true only largely, but not completely. As Bohr says: "In fact, it is an obvious consequence of [Bohr's] argumentation that in each experimental arrangement and measuring procedure we have only a free choice of this place within a region where

[18] In part correlatively, complementarity was also considered differently in the Como lecture, by using, as the main example of the concept, the complementarity of "the space-time coordination and the claim of causality" [5, v. 1, pp. 54–55]. Bohr abandoned this complementarity, along with the view in question, because he abandoned the possibility of applying the concept of causality in quantum physics in any circumstances, rather than allowing it in certain circumstances, as complementary to the space-time coordination. As noted earlier, in Bohr's later works, the concept of complementarity became primarily associated with two complementarities—that of the position and momentum measurements and that of the time and the energy measurements, correlative to the uncertainty relations.

the quantum-mechanical description of the process concerned is effectively equivalent with the classical description," in accordance with the correspondence principle [2, p. 701]. In other words, quantum objects are always on the other side of the "cut" and may even be defined accordingly. At one end, then, by virtue of their classical nature, the individual effects observed in quantum experiments can be isolated materially and phenomenally (in the usual sense)—we can perceive and analyze them as such—once an experiment is performed. They cannot be separated from the process of their physical emergence by our even conceiving of, let alone analyzing, this process. This impossibility will define what Bohr sees as the indivisible wholeness of quantum phenomena in Bohr's sense. By contrast, at the other end, quantum objects and processes can never be isolated, either materially (from the measurement process and measuring instruments) or phenomenally (in the usual sense) insofar as we cannot, even in principle, represent what actually happens at that level or how, in accordance with the RWR principle.

Defined by "the observations [already] *obtained* under specified circumstances," phenomena in Bohr sense refer only to already *registered* phenomena, phenomena that have already happened, rather than to anything that is *predicted*. For one thing, such predictions are, *in general*, probabilistic or statistical and, hence, what will happen can never be assured, unlike, ideally, in classical mechanics. In some experiments, such those of the EPR type, such predictions could be seen as ideally exact but still only ideally, and as will be seen, in the way different from classical predictions. Ultimately, all quantum predictions, by whatever means, are probabilistic or statistical, even in the EPR-type situations.

Bohr's concept of phenomenon includes a rigorous specification of each arrangement, determined by the type of measurement or prediction we want to make, which specification implies that each phenomenon corresponds to what is observed in a single experiment and only in a single experiment. This is necessary for the following reason. As explained earlier, that we always have a free choice as concerns what kind of experiment we want to perform defines the very idea of experiment in all physics. The situation is different in quantum physics because, implementing our decision concerning what we want to do will allow us to make only certain types of prediction (for example, that concerning future position measurements) and will unavoidably exclude the possibility of certain other, *complementary*, types of prediction (those concerning future momentum measurements). A rigorous specification of each experimental arrangement defines each phenomenon and the complementary nature of some of them. If seen independently of the quantum-mechanical context of its appearance, each mark on the screen in the double-slit experiment would be perceived as the same entity. It would appear as the same regardless of the difference in the physical conditions and, hence, the outcome or rather the outcome, "interference" or "no interference," of the experiment, or more accurately the *outcomes* because these two patterns are defined collectively by the two complementary setups of the experiment. The first is that with both slits open and no counters, which would allow us to know through which slit each quantum object passed, and the second is that when such a knowledge is possible in one way or another (for example, by using counters), even in principle rather than only actually. In Bohr's view, however, each mark is part of a different individual phenomenon depending on these conditions. While no single event allows one to establish, retrospectively, in which setting it had occurred, the statistical distribution of the traces on the screen will always be different in these two setups. This circumstance, again, reflects the fact defining the essence of complementarity: the behavior of quantum objects of the same type has mutually incompatible effects in complementary contexts and thus cannot be governed, as in classical physics or relativity, by the same *physical* law, especially a representational physical law, in all contexts. On the other hand, the mathematical formalism of quantum mechanics offers correct predictions, of probabilistic or statistical nature, *in all contexts*, in nonrealist, RWR-principle-based, interpretations under the assumption that quantum objects and their behavior are beyond representation or possibly even conception.

5 Complementarity, Entanglement, and Information

For half a century now, beginning with Bell's celebrated theorem of 1964, the debate concerning quantum foundations has shifted toward the subject of quantum correlations between spatially separated events (typically of the type observed in Bohm's version of the EPR experiment) and quantum entanglement. Unlike the original (idealized) thought-experiment proposed by EPR, which deals with continuous variables and cannot be performed in a laboratory, Bohm's version of the EPR experiment, which deals with discrete variables, could and has been performed, confirming the existence of quantum correlations. These correlations can be ascertained experimentally, apart from quantum mechanics. By contrast, quantum entanglement is part of the mathematical formalism of quantum mechanics, enabling one to predict these correlations. The subject was initiated by Einstein, Podolsky, and Rosen's article, which argued for the incompleteness or else the nonlocality of quantum mechanics (or possibly of the nonlocality of quantum phenomena themselves), and by Bohr's reply to it, both published under the same title, "Can Quantum-Mechanical Description of Physical Reality be Considered Complete?," in 1935 [2,29]. Neither article used the terms "correlations" and "entanglement," although EPR's argument was based on an entangled state of two EPR particles, and Bohr in effect used the idea of quantum correlations and their statistical nature as part of his argument. The concept of entanglement was introduced, in German (*Verschränkung*) and in English, by Schrödinger in three papers, including his famous cat-paradox paper, all inspired by and written in response to EPR's paper [3,35,36].

Eventually, the center stage of the debate was taken by the question of the locality of quantum phenomena or quantum mechanics, rather than of its completeness, which was primarily at stake in the Bohr-EPR exchange, although locality was important to this exchange as well. The nonlocality of quantum phenomena would imply the nonlocality of quantum mechanics, insofar as it is a correct theory of these phenomena, but not the other way around: if quantum phenomena or nature are local, then quantum mechanics, if nonlocal, would be incorrect. The question of locality became even more central to Einstein's subsequent communications on the subject, compelling him to use his famous expression "spooky action at a distance" [*spukhafte Fernwirkung*] (e.g., [37, p. 155]). However, because realism has remained a major concern, especially given the lack of realism as a possible alternative to nonlocality, the question of completeness and of suitable criteria of completeness have remained germane to this debate. For Einstein, quantum mechanics was incomplete because it did not provide a representation of individual quantum processes and, as a consequence, exact predictions concerning individual quantum events.[19] I shall call this concept of completeness "Einstein-completeness." For Bohr, by contrast, quantum mechanics, while possibly (and in his interpretation strictly) nonrealist and, as a consequence, noncausal, was as complete as nature allows our theory of quantum phenomena to be, as the experimental evidence stands now, a crucial qualification (although nothing has changed in this regard thus far). I shall call this concept of completeness "Bohr-completeness." Some see the EPR correlations as implying a violation of locality by either quantum mechanics or by quantum phenomena themselves. EPR's argument and related arguments by Einstein just mentioned contended that quantum mechanics is either incomplete, even Bohr-incomplete, or nonlocal (with some qualifications addressed below). Einstein saw this as unacceptable, because he assumed that nature is local, in part because locality was a consequence of relativity. The concept or principle locality is, however, more general and is independent of relativity. The principle states that no instantaneous transmission of physical influences between spatially separated physical systems ("action at a distance") is allowed or, which is a more current formulation, that physical systems can only be physically influenced by their immediate environment. While these two formulations are not strictly equivalent, they are equivalent in most contexts, including those considered in this article. In any event it is clear that locality in this sense is independent of relativity, which, again, implies locality, and the concepts with which locality is associated there, such as the Lorentz invariance in special relativity. Indeed, the Lorentz invariance is violated in general relativity, where it is only infinitesimally valid, while locality is strictly maintained there. Einstein's "spooky action at a distance," too, refers to an instantaneous action, which is in conflict with relativity, but is not defined by it.

Bohr contested Einstein's argumentation by offering an interpretation of quantum phenomena and quantum mechanics, defined by two of his major conceptions discussed in this article: the joint role of measuring instruments in the constitution of quantum phenomena, including those of the EPR type, and complementarity. It was Bohr's analysis of this role, which was, he argued, underappreciated by EPR, that allowed Bohr to conclude that "quantum-mechanical description of physical phenomena would seem to fulfill, within its scope, all rational demands for completeness," clearly meaning by "completeness" what I define here as Bohr-completeness [2, pp. 696, 700n]. It would, he further argued, do so without sacrificing locality, at least by virtue of the compatibility of Bohr's argument and, thus, his interpretation of quantum phenomena, including those of the EPR type, and quantum mechanics with "all exigencies of relativity theory," which implies locality [2, 700n]. Contrary to Einstein's (mis)readings of Bohr's reply, Bohr thought nonlocality to be as unacceptable as did Einstein ([38, pp. 681–682], [39, pp. 275–277]).

[19] It is conceivable that a nonrepresentational model would provide exact predictions of the outcomes of the individual processes considered. Einstein, however, always, including in EPR's article and related communications, understood completeness in terms of representational realist models (e.g., [29, p. 138]).

As noted earlier, Bohr's interpretation in his reply is different from his ultimate interpretation, developed following his exchange with EPR and Einstein's subsequent communications on the subject, which no longer allows for any assignment of elements of reality to quantum objects—before, at the time, or after measurement. In his reply this assignment is possible *at the time* of measurement and only then. However, the essential logic of his reply, especially its argument for the irreducibly role of measuring instruments and complementarity in considering quantum phenomena, could be presented in Bohr's later terms as well. Bohr argued that, because of this joint role, quantum phenomena, including of the EPR type, disallow EPR's (realist) conception of physical reality and the corresponding criterion of physical reality they introduce, or at least, the unqualified *way* in which the criterion is used by EPR. Hence, rather than demonstrating a deficiency of quantum mechanics with respect to either completeness or locality, EPR's *argument* itself appears as insufficient in view of a more rigorous analysis (than that given by EPR) of the nature of quantum phenomena, including those considered in the EPR *experiment*. It is true that Bohr only argued for (along with locality), the Bohr-completeness of quantum mechanics, rather than for its Einstein-completeness, a requirement that quantum mechanics expressly does not satisfy in Bohr's interpretation by virtue of its nonrealist, the RWR-principle-based, character. However, this is all that Bohr needed to do. This is because EPR did not argue that quantum mechanics was Einstein-incomplete, but rather that, to return to Bohr's locution, "its predictions [did] not exhaust the possibilities of observation," which is to say, that it was not even Bohr-complete [5, v. 2, p. 57]. In his later communications, Einstein acknowledged that the statistical predictions of quantum mechanics might be seen as exhausting the possibilities of observation, thus making quantum mechanics Bohr-complete (in present terms), as well as local. Einstein still found this to be unsatisfactory, because, unlike Bohr, Einstein saw the Einstein-completeness as an essential requirement for a fundamental theory.

Ironically, any classical-like Einstein-complete theory that would predict the EPR-type correlations appears to be nonlocal in view of Bell's and the Kochen–Specker theorems, and related findings, which thus far deal with discrete variables. Among the most famous of them are those of D. M. Greenberger, M. Horne, and A. Zeilinger, L. Hardy, and, from the experimental side, A. Aspect's experiments and related experimental work, such as that by Zeilinger and his group [40–43].[20] The advent of quantum information theory during recent decades, gave this problematic new prominence and significance. The meaning of these findings, or, in the first place, the EPR experiment and Einstein's and Bohr's arguments concerning it, and their implications continue to be intensely debated. I shall bypass these debates here, given that my main concern is the role of the concept of complementarity, rarely invoked and underappreciated in these debates.[21] I would argue, however, that at stake in these theorems are still situations that "the notion of *complementarity* is aimed at characterizing" [2, p. 700].

[20] I only cite some of the key earlier experiments. There have been numerous experiments performed since, some in order to find loopholes in these and other experiments, seen as confirming Bell's theorem.

[21] I might add that most key issues at stake in this debate have been extensively discussed and *debated* during the two decades of Växjö conferences on quantum foundation and the proceedings of these conferences offer an invaluable resource, such as, to cite only most recent ones [44–46].

The EPR-type correlations reflect or are given a physical meaning by a form of complementarity, which I call "the EPR complementarity," which and, thus, or rather, to begin with, "entanglement," are ultimately found in any complementarity of quantum phenomena. I would extend the present article well beyond its scope to give justice to EPR's argument and Bohr's reply, which I have considered in detail elsewhere (e.g., [8, pp. 136–154], [39, pp. 237–312]). I would like, however, to offer a brief sketch of what is at stake in order to define the EPR complementarity, which may be the instance of the concept that reflects its deepest aspects, as Bohr appears to have thought as well [2, p. 700].

The crux of the EPR argument is that the EPR (idealized) thought-experiment allows for predictions with certainty concerning quantum objects without physically interfering with them by means of measurement, or, in EPR's language, "without in any way disturbing the system" [29, p. 138]). This possibility defines their "criterion of reality," on which they base their argument—"*If, without in any way disturbing a system, we can predict with certainty (i.e., with the probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity*" [29, p. 138]). This possibility would seem to circumvent the irreducible role of measuring instruments in the constitution of all quantum phenomena, specifically in making predictions concerning them. EPR clearly realized that, if a prediction concerning the future behavior of a given quantum system required a measurement performed on this system, quantum mechanics would then predict all that is possible to predict in view of the uncertainty relations. But they did not appreciate the fact that the uncertainty relations, which, they thought, their argument allows them de facto (although not in practice) to circumvent, would still affect the situation at of the EPR experiment in the way that impairs their argument, even though the type of predictions they consider are indeed possible.

An EPR prediction concerning a given quantum object, S_2, of the EPR pair (S_1, S_2), is enabled by performing a measurement on another quantum object, S_1, with which, S_2, has previously been in interaction, but from which it is spatially separated at the time of the measurement on S_1. Specifically, once S_1 and S_2, forming an EPR pair (S_1, S_2), are separated, quantum mechanics allows one to simultaneously assign both the *distance between* the two objects and *the sum of their momenta*, because the corresponding Hilbert-space operators *commute*. With these quantities in hand, by *measuring* either the position or, conversely, the momentum of S_1, one can (ideally) *predict exactly* either the position or the momentum for S_2 without physically interfering with, "disturbing," S_2, which would, EPR assume, imply that one can simultaneously assign to S_2 both quantities as elements of reality pertaining to S_2. "The authors," Bohr summarizes in his reply, "therefore want to ascribe an element of reality to each of the quantities represented by such variables. Since, moreover, it is a well-known feature of the present formalism of quantum mechanics that it is never possible, in the description of the state of a mechanical system [object], to attach definite values to both of two canonically conjugate variables, [EPR] consequently deem this formalism to be incomplete, and express the belief that a more satisfactory theory can be developed" [2, p. 696]. It follows, then, that the formalism would not even be Bohr-complete, insofar as quantum mechanics does not predict all that it is possible to ascertain. The only alternative, as EPR saw it, would be the nonlocal nature of quantum phenomena or of quantum mechanics [29, p. 141]. They disallow this possibility, as does Bohr, although this alternative has been entertained, especially in the wake of Bohmian mechanics (which is nonlocal) and then the Bell and the Kochen–Specker theorems.

Bohr argued that the situation does not allow one to dispense with the role of measuring instruments, because this role entails limitations on the *types* of measuring arrangements used in determining the quantities in question, even if one does so in terms of predictions, including in the EPR case, rather than measurement. These limitations result from "an influence *on the very conditions which define the possible types of predictions, regarding the future behavior of the system* [S_2]" [2, p. 700]. It is disregarding this influence, as EPR do, that gives EPR's criterion of reality its "essential ambiguity" when it applied to quantum phenomena, which ambiguity disables their argument. By contrast, by exposing the irreducible nature of this influence and by taking it into account, Bohr was able to argue that neither EPR's argument for the incompleteness of quantum mechanics nor their alternative reasoning that entails nonlocality could be sustained. Bohr's thinking concerning the situation eventually led him to his ultimate interpretation, in which *only what has already occurred* determines any physical quantity considered, and *not what has been predicted* (even with certainty) and is yet to be confirmed by a measurement. As I noted, however, Bohr's reply assesses EPR's argument on their own terms, whereby it is possible to assign certain properties to quantum objects themselves under the constraints of the uncertainty relations, rather than in terms of Bohr's ultimate epistemology, where such an assignment is no longer possible even for a single physical quantity. Bohr also assumes, as do EPR, that this assignment to be possible on the basis of *predictions* rather than only *measurements*, albeit only predictions that are *in principle verifiable*. This is a crucial qualification, necessary in a rigorous analysis of all quantum phenomena, those of the EPR-type included, but it is not considered by EPR either.

Both EPR and Bohr assume that the EPR experiment for (S_1, S_2) can be set in two alternative ways so as to predict (using quantum mechanics), with a probability equal to unity, either one or the other of the measurable quantities associated with these variables for S_2 on the basis of measuring the corresponding quantities for S_1. Let us call this assumption "*assumption A*."

On the basis of this assumption, EPR infer that *both* of these quantities can be assigned to the quantum object under investigation, $S_{2,}$ even though it is impossible to do so simultaneously (in view of the uncertainty relations for measurements on S_1). This makes quantum mechanics incomplete because it has no mechanism for this assignment, unless one allows for nonlocality, which is unacceptable [29, p. 141]. Let us call this inference "*inference E*" (for Einstein).

Bohr argued that, while *assumption A* is legitimate, *inference E* is unsustainable because a realization of the two situations necessary for the respective assignment of these quantities would involve two incompatible, in fact complementary, experimental arrangements and, thus, in effect *two different quantum objects* of the same type (e.g., electrons or photons). There is no physical situation in which this joint assignment is ever possible for *the same object*, either simultaneously (in view of the uncertainty relations) or separately. If one makes the EPR prediction, with a probability equal to unity, for the second object, $S_{12,}$ of a given EPR pair, (S_{11}, S_{12}),one would always need a different EPR pair $(S_{21}, S_{22})$to get to "the last critical stage of the measuring procedure," performed on S_{21}, in order to make an alternative EPR prediction concerning S_{22} [2, p. 700]. I designate this inference as "*inference B*" (for Bohr).

Nor is an identical assignment of the single quantity ever possible, or in any event, ever guaranteed, for two "identically" prepared *objects* in the way it can be in classical physics. This is because, as explained earlier, quantum experiments cannot be controlled so as to identically prepare quantum objects but only so as to identically prepare the measuring instruments involved, because this behavior is classical. The quantum strata of measuring instruments throughout which they interact with quantum objects do not affect these preparations but only the outcome of an actual measurement. On the other hand, this interaction itself is uncontrollable. This fact is central to Bohr's argument, which invokes this "finite and incontrollable interaction" at two key junctures of his argument [2, pp. 697, 700]. It follows that the outcomes of repeated, identically prepared, experiments, including those of the EPR type, cannot be controlled, even ideally (as in classical physics), and these outcomes will, in general, be different. This circumstance makes statistical considerations unavoidable in the EPR experiment, even though each actual predictions involved can be made with a probability equal to unity. This aspect of the situation does not appear to have been realized by EPR, whose *inference E* and their argumentation based in it implicitly depends on the possibility of the identical repetition of the EPR experiment, precluded by *inference B*.

One can diagrammatically represent the situation as follows. Let X and Y be two complementary variables in the Hilbert-space formalism ($XY – YX \neq 0$) and x and $y$the corresponding physical measurable quantities ($\Delta x \Delta y \approx h$); (S_1, S_2) is the EPR pair of quantum objects; and p is the probability of prediction, via the wave function, Ψ. Then:

The EPR experiment (EPR/Einstein's view):

$$\begin{array}{ccc} S_1 & & S_2 \\ X_1 & \Psi_1(\text{with } p = 1) \rightarrow & X_2 \\ Y_1 & \Psi_2(\text{with } p = 1) \rightarrow & Y_2 \end{array}$$

The EPR experiment (Bohr's view):

$$\begin{array}{ccc} S_{11} & & S_{12} \\ X_{11} & \Psi_1(\text{with } p = 1) \rightarrow & X_{12} \\ S_{21} & & S_{22} \\ Y_{21} & \Psi_2(\text{with } p = 1) \rightarrow & Y_{22} \end{array}$$

This diagram is that of a complementarity, which I would like to call "the EPR complementarity." It can be described as a follows. Once one type of measurement (say, that of variable X) is performed on S_{11}, enabling the corresponding prediction on S_{12}, we irrevocably cut ourselves off from any future possibility of making the alternative, complementary, measurement (that of Y) on S_{11} and, thus, equally irrevocably, from the possibility of ever predicting the second variable for S_{12} [2, p. 700]. There is simply no way to define that variable for S_{12}, except by a measurement, which, however, defeats the very purpose of EPR's argument. This could only be done for S_{22}, which is to say by preparing another EPR pair and performing a complementary measurement of Y on S_{21}, which will irrevocable prevent us from establishing X for S_{22}. As discussed earlier, stemming from "our freedom of handling the measuring instruments, characteristic of the very idea of experiment" in all physics, classical or quantum (or relativity), our "free choice" concerning what kind of experiment we want to perform is essential to complementarity (part (*b*) of the main definition given earlier) [2, p. 699]. However, as against classical physics or relativity, implementing our decision concerning what we want to do will allow us to make only certain types of predictions and exclude the possibility of certain other, *complementary*, types of predictions. In the EPR case, it is only possible to establish both complementary quantities for two EPR pairs, (S_{11}, S_{12}) and (S_{21}, S_{22}), and never for one, and if we had predicted the second quantity, instead of the first one, for S_{12}, it would not, in general, be the same, even ideally, as it is for S_{22}. It follow that the situation representing EPR/Einstein's view of the EPR experiment, defined by *inference E* is physically unrealizable.

Bohr does not explain the EPR experiment in terms of two different objects and EPR pairs necessary in order to make the second EPR prediction, which might have helped to make his argument clearer. This, however, is at least an implication of his argument, given his insistence in his reply and elsewhere that "in the problem in question we are not dealing with a *single* specified experimental arrangement, but are *referring* to two different, mutually exclusive arrangements" ([2, p. 699], [5, v. 2, pp. 57, 60]). In view of this mutual exclusivity, which is, again, due to the irreducible role of the measuring instruments, the second quantity in question cannot in principle be assigned to the *same quantum object*, *once one such quantity is assigned*. More accurately, one should speak of complementarity, because we always have a freedom of choice in making another arrangement and thus measuring or predicting the other conjugate variable in question, which is a defining aspect of complementarity. This assignment is not possible even if one accepts EPR's criterion of reality, whereby such an assignment is made on the basis of a prediction. It is not possible once an experiment enabling one to make the first prediction is performed, because the first object S_1 (using the notation corresponding to EPR's view of the experiment) or S_{11}(using the notation corresponding to Bohr's view of the experiment) is no longer available. The simultaneous assignment of both quantities is precluded by the uncertainty relations, which fact is not only recognized by EPR but is also germane to their argument. They aim to show that, while valid, the uncertainty relations could be circumvented by arguing that both variables could in fact *be assigned* to a given quantum object at any moment of time, although only one of them could be actually measured or predicted at any moment of time, which is the physical content of the uncertainty relations. This leads them to reason that quantum mechanics is incomplete, or else nonlocal. Bohr, by contrast, argues that the uncertainty relations disallow one *ever* to simultaneously assign both quantities to, or simultaneously define them for, for any quantum object, even in the EPR case.

The considerations just offered could be transferred, with a few easy adjustments, to Bohm's version of the EPR experiment and spin variables, which can be actually performed in a laboratory.[22] In this case, too, there is the EPR complementarity (which corresponds to generalized uncertainty relations for spin-measurements) insofar as any assignment of the alternative spin-related quantity to the same quantum objects becomes impossible, once one such quantity is assigned. An assignment of the other would require an alternative type of measurement, mutually exclusive with the first, on the first object of a given pair, and hence, at least, another fully identically behaving EPR-Bohm pair, which is, again, not possible or, in any event, cannot be guaranteed. Nothing else than statistical correlations between such assignments is possible, which is consistent with the Bell-EPR correlations (as they are often called in this context), which are statistical (e.g., [47, pp. 107–108]).[23] The argument concerning locality, to which I now turn, could be similarly transferred to the case of discrete variables as well.

EPR acknowledged a possible loophole in their argument by admitting that they did not demonstrate that one could ever *simultaneously* ascertain both quantities in question for the same quantum object, such as S_2in the EPR experiment, in the same location, either that of S_1 or S_2.[24] They, however, see this requirement as implying nonlocality in the EPR situation and hence as unreasonable. According to EPR:

> One could object to this conclusion [that the quantum-mechanical description of physical reality given by wave functions is incomplete] on the grounds that our criterion of reality is not sufficiently restrictive. Indeed, one would not arrive at our conclusion if one insisted that two or more physical quantities can be regarded as simultaneous elements of reality *only when they can be simultaneously measured or predicted*. On this point of view, since either one or the other, but not both simultaneously, of the quantities P and Q can be predicted, they are not simultaneously real. This makes the reality of P or Q depend upon the process of measurement carried out on the first system [the first object of the EPR pair considered, S_1], which does not disturb the first system in any way. No reasonable definition of reality could be expected to permit this [29, p. 141].

Nonlocality indeed follows if one assumes, as EPR do, that the measurement, say, of P, on S_1 *fixes the physical state itself* of S_2 by "a spooky *action* at a distance," rather than allows for what one might call "a spooky *prediction* at a distance," by fixing *the possible conditions* of such a prediction, as explained above. It follows, under the first assumption, that an alternative measurement of Q on S_2 would discontinuously

[22] The thought experiment (dealing with continuous variables), proposed by EPR cannot be physically realized because the EPR-entangled quantum state is not normalizable, which does not affect the fundamentals of the case. There are experiments (e.g., those involving photon pairs produced in parametric down conversion) that statistically approximate the idealized entangled state constructed by EPR for continuous variables. I cannot consider these experiments here, but they can be shown to be consistent with the present argument.

[23] See also the remainder of Mermin's excellent discussion, which contains an elegant proof of Bell's theorem [47, pp. 110–176].

[24] As Schrödinger was the first to note, one could simultaneously make alternative (complementary) measurements on S_1, say, the position measurement, which determines its position, and S_2, the momentum measurement, which determines its momentum, and thus simultaneously predict (ideally exactly) the alternative second variable for each system, the momentum for S_1 and the position for S_2. This joint determination, however, is *not simultaneous* in the same location, and, thus, is in accord within EPR's initial criterion of reality, without the restriction in question.

change this fixed state, although EPR do not examine this last eventuality. Or, as Einstein argued on later occasions, one is left with a paradoxical situation insofar as (assuming that quantum mechanics is complete) two mutually incompatible states could be assigned to the same quantum object or system, S_2, by a different "spooky action at a distance," defined by a different measurement performed on S_1 (e.g., [37, p. 205]).[25] This is why EPR contend that, if quantum mechanics *is* complete by their criterion, then the physical state of a system (object), here S_2, could be *determined* by a measurement on a spatially separate system (object), S_1, in violation of locality, while their criterion of *reality* no longer applies in its original form. If it is local, their main argument, based on their criterion of reality, showed (they believed) that it is incomplete. Einstein, as I noted, thought that Bohr accepted the alternative of locality versus completeness, and retained completeness by allowing for nonlocality [38, pp. 681–682]. Einstein, however, misread Bohr's argument, which only allows for a spooky *prediction*, and *not action*, at a distance. As Bohr said, "the singular position of measuring instruments in the account of quantum phenomena … together with the relativistic invariance of the uncertainty relations … ensures the compatibility between [his] argument and all exigencies of relativity theory" [2, p. 701n]. This compatibility implies locality and enables Bohr's critique of EPR's argument without sacrificing locality.

There is a difference between determining (fixing) the state of a physical object *by* a prediction and *possibly* (only possibly!) doing so *on the basis* of a prediction. In Bohr's view, physical states of, at least, quantum objects cannot be seen as finally determined (even when we have predicted them exactly) unless either the actual measurement is made or unless the possibility of *verifying* the prediction is assured insofar as such a measurement could, in principle, be performed so as to yield the predicted value. This last requirement in turn becomes a necessary qualification of EPR's criterion of reality in the case of quantum phenomena. This is because, if one assumes the validity of EPR's criterion in its original (unrestricted) form, the measurement of the alternative quantity, Q, on S_2 would automatically disable any possible verification of the original prediction. It is crucial that it is always possible to perform this alternative measurement. This is one of the reasons why, the assumption of the *independent existence or reality* of quantum objects or something in nature to which one can relate by using this term becomes especially important for Bohr's analysis of the EPR experiment and of the question of locality in it. This independent existence opens the possibility of this measurement. Once this alternative measurement is performed, the original prediction becomes meaningless as in principle unverifiable, and, as I explained, we cannot identically repeat the experiment so as to confirm this prediction. This, again, implies that both quantities in question could never be experimentally ascertained, either simultaneously or separately, for the same quantum object and hence that quantum mechanics could not be shown to be nonlocal by EPR's logic here, anymore than it can be shown to be incomplete by their logic.

[25] Einstein does note on the same occasion (and elsewhere) that the paradox is eliminated if quantum mechanics is only a statistical theory of ensembles and not of individual events, because, in this case, no single measurement of a given variable on S_1 or, more accurately, S_{1n} determines the value of the corresponding variable on S_{2n} [37, p. 205].

Thus, according to Bohr, EPR's logic is disabled by the nature of quantum phenomena, as defined by the irreducible role of measuring instruments in the constitution of these phenomena, and thus by the impossibility of considering the behavior of quantum objects independently of their interaction with these instruments. The application of EPR's criterion of reality becomes ambiguous by virtue of the lack of qualifications of this criterion required by these conditions, which is to say, by complementarity. Hence, Bohr concludes in the elaboration, which in effect defines the EPR complementarity:

> From our point of view we now see that the wording of the above mentioned criterion of physical reality proposed by A. Einstein, B. Podolsky, and N. Rosen contains an ambiguity as regards the meaning of the expression "without in any way disturbing a system." Of course there is in a case like that just considered no question of a mechanical disturbance of the system under investigation during the last critical stage of the measuring procedure. But even at this stage there is essentially the question of *an influence on the very conditions which define the possible types of predictions regarding the future behavior of the system*. Since these conditions constitute an inherent element of the description of any phenomenon to which the term "physical reality" can be properly attached, we see that the argumentation of the mentioned authors does not justify their conclusion that quantum-mechanical description is essentially incomplete. On the contrary this description, as appears from the preceding discussion, may be characterized as a rational utilization of all possibilities, of unambiguous interpretation of measurements, compatible with the finite [quantum] and uncontrollable interaction between the object and the measuring instruments in the field of quantum theory. In fact, it is only the mutual exclusion of any two experimental procedures, permitting the unambiguous definition of complementary physical quantities, which provides room for new physical laws the coexistence of which might at first sight appear irreconcilable with the basic principles of science. It is just this entirely new situation as regards the description of physical phenomena that the notion of *complementarity* aims at characterizing ([2, p. 700]; Bohr's emphasis).

Bohr's claims concerning "the essential ambiguity" of EPR's use of their criterion and specifically that this ambiguity concerns EPR's expression "without in any way disturbing a system," and this elaboration as a whole have posed difficulties for Bohr's readers and frequently caused confusion on their part. However, this elaboration and Bohr's meaning in this particular clause should pose no special difficulties given the preceding analysis. Once one conjugate quantity in question is established (even on the basis of a prediction, in accordance with EPR's criterion of reality) for S_{12}, we cannot ever establish the second quantity involved without measuring and hence disturbing S_{12}. Only one of these quantities could be established for S_{12} without disturbing it, but once it is established, never the other quantity without disturbing it. We can establish such a quantity only for a different quantum object, S_{22}, via a different EPR pair (S_{21}, S_{22}), by a measurement of a complementary type on S_{21}. These two determinations cannot be coordinated so as to assume that both quantities could be associated with the same object of the same EPR pair. The coordination of such events can only be statistical. We cannot establish *both quantities* for the same system *without in any way disturbing it*. The only way to establish the second quantity for this system would be to perform a measurement on and thus disturb it, which, however, would erase the determination of the first quantity, if one assumes, as EPR do, that it could be made on the basis of a prediction on the first object of the corresponding EPR pair. This point, as we have seen, is also crucial for maintaining locality. Thus, the ambiguity in question indeed relates to the clause "without in any way disturbing the system," which, if one wants to apply this clause rigorously, requires qualifications explained in the present analysis but not provided by EPR. As Bohr noted earlier in his reply:

> In the phenomena concerned we are not dealing with an incomplete description characterized by the arbitrary picking out of different elements of physical reality at the cost of [sacrificing] other such elements [for a given quantum object], but with a rational discrimination between essentially different experimental arrangements and procedures which are suited either for an *unambiguous* use of the idea of space location, or for a legitimate application of the conservation theorem of momentum. Any remaining appearance of arbitrariness concerns merely our freedom of handling the measuring instruments, characteristic of the very idea of experiment ([2, p. 699]; emphasis added).

Any attempt to apply both is ambiguous, and complementarity provides a necessary disambiguation, in correspondence with the uncertainty relations, which, as explained earlier, are interpreted by Bohr in accordance with this view, in the first place.

Eventually Bohr came to realize that any quantum measurement can be seen in terms of the EPR experiment by considering the quantum object under investigation and the quantum part of the measuring instrument, interacting with this object, as entangled, with the quantum part of the instrument as the second object of the EPR pair thus formed ([5, v. 2, pp. 59–60], [34, pp. 101–104]). Before addressing this argument and in order to bring the concepts of complementarity and entanglement together, I would like to consider in Schrödinger's discussion of entanglement in his response to EPR's paper. Without defining the concept, Bohr invokes what is in effect "entanglement" at the outset of his reply (Bohr 1935, pp. 697–698, n.). Schrödinger, however, makes the case more effectively, via *the concept of entanglement*, which, as I said, he introduced in both German and English. According to Schrödinger:

> When two systems, of which we know the states by their respective representatives, enter into temporary physical interaction due to known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before, viz. by endowing each of them with a representative of its own. I would not call that *one* but rather *the* characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought. By the interaction the two representatives [quantum states] have become entangled. Another way of expressing the peculiar situation is: the best possible knowledge of a *whole* does not necessarily include the best possible knowledge of all its *parts,* even though they may be entirely separate and therefore virtually capable of being 'best possibly known,' i.e., of possessing, each of them, a representative of its own. The lack of knowledge is by no means due to the interaction being insufficiently known—at least

> not in the way that it could possibly be known more completely—it is due to the interaction itself [35, p. 555].

That "the lack of knowledge is by no means due to the interaction being insufficiently known—at least not in the way that it could possibly be known more completely—it is due to the interaction itself" is strictly in accord with Bohr's view. Bohr gives this point a nonrealist interpretation (which Schrödinger resists, along with quantum mechanics itself) as concerns the ultimate character of this interaction, which is "finite [quantum] and uncontrollable" [2, pp. 697, 700].[26] One must keep in mind that, as related to "questions about the future," this "knowledge" is probabilistic or statistical [3, p. 160]. This is the only knowledge the wave function provides, by way of an expectation-catalog, and, when it concerns predictions, any complementarity is a complementarity of expectation-catalogs. Complementary *measurements* give us definitive knowledge, but never both measurements concerning the same quantum object. In any event, the maximal expectation-catalog for the combined system does not contain the maximal expectation-catalog for each part. Speaking of the situation in this way is preferable to speaking of a whole and parts, because, as explained earlier, it may not be rigorously possible to speak of the combined system as the "whole" of its two "parts." When we have a complete expectation-catalog (the maximal probabilistic or statistical knowledge concerning the outcomes of possible future experiments) for each of the two completely separate systems, then we also have such a catalog for both systems together. But converse—if we have a complete catalog for a combined system, then we have a complete catalogue for each separate system—is not necessarily true:

> This is the point. Whenever one has a complete expectation-catalog—a maximal total knowledge—of a ψ-function for two completely separate bodies, or, in better terms, for each of them singly, then one obviously has it also for the two bodies together, i.e., if one imagines that neither of them singly but rather the two bodies together make up the object of interest, of our questions about the future.
>
> But the converse is not true. *Maximal knowledge of a total system does not necessarily include total knowledge of all its parts, not even when these are fully separated from each other and at the moment are not influencing each other at all.* Thus it may be that some part of what one knows may pertain to relations or stipulations between the two subsystems (we shall limit ourselves to two), as follows: if a particular measurement on the first system yields *this* result, then a particular measurement on the second the valid expectation statistics are such and such, but if the measurement in question on the first system should have *that*

[26] There are realist views of quantum entanglement, either in realist interpretations of quantum mechanics (for example, the many worlds interpretation) or in alternative theories, such as Bohmian mechanics, or those in which the level of reality handled by quantum mechanics is underlain by a deeper reality (even within the proper, low-energy, scope of quantum mechanics), such as that of classical random fields, recently proposed by Khrennikov [48]. The so-called superdeterminism is another realist view of entanglement and quantum phenomena that might be mentioned here, as offering a particularly striking contrast to the present view, because it explains away the complexities discussed here by denying a free choice of performing one or the other EPR measurement, a free choice central to Bohr's position and defining complementarity (e.g., [49]).

> result, then some other expectation holds for that on the second; should a third result occur for the first, then still another expectation applies to the second; and so on, in the manner of a complete disjunction of all possible measurement results which the one specifically contemplated measurement on the first system can yield. In this way, any measurement process at all or, what amounts to the same, any variable at all of the second system can be tied to the not yet known value of any variable at all on the first, and of course *vice versa* also. If that is the case, if such conditional statements occur in the combined catalog, *then it can not possibly be maximal in regards to the individual systems*. For the content of two maximal individual catalogs would by itself suffice for a maximal combined catalog; the conditional statement could not be added on [3, p. 160].

The situation is clearly due to the complementary nature of the measurements and predictions involved, which prevents us from speaking of the whole of two complementary parts of these measurements or of expectation-catalogs they provide, or, again, of a "whole" and "parts." Schrödinger continues:

> These conditional predictions, moreover, are not something that has suddenly fallen in here from the blue. They are in every expectation-catalog. If one knows the ψ-function and makes a particular measurement and this makes a particular result, then one again knows the ψ-function, *voilà tout*. It's just that for the case under discussion, because the combined system is supposed to consist of two fully separated parts, the matter stands out as a bit strange. For thus, it becomes meaningful to distinguish between measurements on the one and measurements of the other subsystem. This provides to each full title to a private maximal catalog; on the other hand, it remains possible that a portion of the attainable combined knowledge is, so to say, squandered on conditional statements, that operate between the subsystems, so that the private expectancies are left unfulfilled—even though the combined catalog is maximal, that is even though the ψ-function of the combined system is known.
>
> Let us pause for a moment. This result in its abstractness actually says it all: Best possible knowledge of a whole does not necessarily include the same for its parts. … The whole is in a definite state [ψ-function], the parts taken individually are not [3, p. 161].

This is a profound insight, enabled by the concepts of entanglement and, I would add, expectation-catalog, in to the nature of quantum phenomena and quantum mechanics. This insight was importantly developed in quantum information theory in the recent work, cited earlier, by G. M. D'Ariano and coworkers, enabling them to derive the finite-dimensional quantum theory from quantum-informational principles. They convert Schrödinger's assertion that "the best possible knowledge of a *whole* does not necessarily include the best possible knowledge of all its *parts*" into what they call "the purification principle," which, I would argue, also shed a new light on complementarity. The purification principle plays a particularly, indeed uniquely, important role in their program, as an essentially quantum principle, which, in correspondence with Schrödinger's view and giving the situation yet new dimensions, distinguished quantum information theory from classical information theory. They say: "In a sense, our work can be viewed as the concrete realization of Schrödinger's claim: the fact that every physical state can be viewed as the marginal of some pure state of a compound system is indeed the key to single out quantum theory within a standard set of possible theories. It is worth stressing, however, that the purification principle assumed in this paper includes a requirement that was not explicitly mentioned in Schrödinger's discussion: if two pure states of a composite system AB have the same marginal on system A, then they are connected by some reversible transformation on system B. In other words, we assume that all purifications of a given mixed state are equivalent under local reversible operations" [20, p. 2]. D'Ariano et al. do not discuss complementarity as such. Given, however, that complementarity, specifically the EPR complementarity, is unavoidable once entanglement is at stake as it is, again, in a defining way in their approach, their work connects complementarity, entanglement, and quantum information, thus taking Bohr's signature concept in a new direction, crucial for the development of quantum theory.

What then would be "*the* characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought," complementarity or entanglement? Perhaps both, as they are essentially linked in quantum measurement, which is a defining aspect of complementarity as a concept, in its ultimate, definitive form. Any quantum measurement can be seen in terms of the EPR experiment by considering the quantum object under investigation and the quantum part of the measuring instrument, interacting with this object, as entangled, with the quantum objects and the quantum part of the instrument as forming the EPR pair ([5, v. 2, pp. 59–60], [34, pp. 101–104]). One can reproduce the EPR situation for this EPR pair by the set of additional measurements for $q_1 - q_2$ and $p_1 + p_2$, both of which can be exact, and by using these measurement to make the EPR-type (ideally) exact predictions.

Thus, the conditions of the EPR type apply to *all quantum predictions*, although there are some (epistemologically inessential) differences between the standard and the EPR case. We always predict the value of either the position or the momentum associated with a quantum object in the corresponding measuring arrangement *after* the object has already left the region of its interaction with the measurement apparatus, which interaction defines the data on the basis of which we make such a prediction. Thus, we make this (spooky) prediction "at a distance" without any further interference with this object, whether we assume that this quantity is already defined by this prediction, in accordance with EPR's criterion, or that it is yet to be defined by a subsequent measurement, in accordance with Bohr's ultimate view [5, v. 2, p. 57]. The object and that (quantum) part of the measuring instrument that originally interacted with it, which interaction enables our prediction, are entangled, and it is in fact this entanglement that makes our predictions is possible. Or, more accurately, analogously to our knowledge concerning the EPR objects, our actual knowledge of the state of the apparatus (analogous to S_1 of an EPR pair) obtained by measurements becomes entangled with our possible knowledge (now provided by the wave function or density matrix, which define the mathematical "quantum state") concerning the outcome of a specified future measurement performed on this object (analogous to S_2). To cite Roger Penrose: "A measurement, after all, merely consists of the quantum state under consideration becoming entangled with a more extended part of the physical universe, e.g., with a measuring apparatus" [50, p. 290]. In the present view (although not that of Penrose), "quantum state" would only refer to the mathematical object—the state vector, defined by the wave function or density matrix—enabling our expectation concerning a given experiment.[27]

In the EPR situation, which involves two quantum objects rather than a single object and a measuring apparatus, the case is a bit more complicated (given that we need to involve additional measurements), but it is not fundamentally different, and, as indicated above, it can, at least in principle, be reenacted by making additional measurements in the standard measurement [5, v. 2, pp. 59–60]. A certain quantum part of the apparatus involved appears in the same role as the first object, S_1, of an EPR pair (S_1, S_2), which has previously interacted with the second object, S_2. Conversely, in the case of the EPR experiment, the first object, S_1, may be seen as analogous to—or may be treated as—a quantum part of a measuring instrument, although one would still need to add a proper classical part to it (which is essentially done by performing a measurement on it, since this measurement involves a measuring apparatus). In this case, as against that of the standard measurement, we are no longer concerned with predictions concerning S_1 but only with those concerning S_2, which is what makes the analogy in question possible. Bohr came to realize this analogy at least by the time of the called Warsaw lecture of 1938, and he used it from that point on (e.g., [5, v. 2, p. 60], [34, pp. 101–103]).

Of course, in the standard case, as opposed to that of the EPR experiment (possibly enacted via the standard quantum measurement), a prediction, say, concerning the location of a collision between the object and the silver bromide screen, will not, in general, have a probability equal to unity, but some other probability. In order to get a probability equal to unity, one would, again, need to perform some intermediate measurements, just as one would in any EPR case. These nuances, however, do not affect my main point at the moment. All our predictions concerning quantum objects, that is, concerning the outcomes of their future interactions with measuring instruments, are defined by their previous entangling interactions with other quantum objects, whether a part of the quantum strata of measuring instruments or one of the objects of the EPR pair. By the same token, these predictions are defined by a measurement performed on this other object, which is, at the time of the measurement enabling the prediction, spatially separated from the object under investigation. That interaction, as it were, endows the object with which we interfere in a measurement with certain possible information necessary for such a prediction, information that we acquire through this measurement. We can always (in a delayed-choice manner) make our plans and set up one arrangement or another after this interaction has already taken place ([2, pp. 698–699], [5, v. 2, p. 57]). Thus, although ingenious, EPR's contrivance in designing the EPR experiments of making such predictions on the basis of performing a measurement on a different quantum object so as not to interfere with the object in question merely reproduces the essential aspects of the quantum-mechanical situation. The EPR experiment itself, however, the understanding of which was helped by Bohr's analysis of it in terms of complementarity and by Schrödinger's analysis and the con-

[27] See [51,52] for elegant experimental illustrations of this point by S. Haroche and coworkers. S. Haroche was awarded a Nobel Prize for this work.

cept of entanglement, revealed the deeper relationships among complementarity and entanglement, and between both and quantum information.

The question of quantum information is part of this situation because in the EPR-type experiments, this prediction also enable a transmission of quantum information, once the measurement corresponding to this prediction is made, but, in nonrealist, the RWR-principle-based, interpretation not otherwise. But then, as both R. Feynman and J. A. Wheeler (Feynman's dissertation advisor), two towering pioneering figures of quantum information theory, realized, the question of quantum information has always been at the core of quantum mechanics. Indeed, as I have argued here, this is apparent, in retrospect, in Heisenberg's derivation the theory. It might be a bit too bold to link the relationships among complementarity, entanglement, and quantum information, to "the age-old question: 'How come Existence?,"' as Wheeler (who is not afraid of boldness) does in his quantum-information, "it-from-bit," manifesto, "Information, Physics, Quantum: The Search for Links," already cited here [23, p. 3]. Wheeler, it is true, asks more broadly what "quantum physics and information theory have to tell us" about this question [23, p. 3]. This broader terms hardly change what is at stake, because, as least as things stand now, the conjunction of quantum physics and quantum information cannot be thought apart from the relationships among complementarity, entanglement, and quantum information. This is quite apparent in Wheeler's manifesto, beginning with the fact that he immediately invokes Bohr's concept of phenomenon, which embodies these relationships and is central for Wheeler's argument there and all of Wheeler's thinking concerning quantum physics, thinking in the spirit of Copenhagen. "No element in the description of physics," Wheeler says, "shows itself … closer to primordial that the elementary quantum phenomenon, that is, the elementary device-intermediated set of posing a yes-no physical question and eliciting an answer or, in brief, the elementary act of observer-participancy" [23, p. 3]. This clearly refers Bohr's concept of phenomenon, defined by the interaction between quantum objects and measuring instruments, a concept fundamental to complementarity, entanglement, and, as Wheeler confirms next, information. For Wheeler adds, bringing in his "*it from bit*," famous ever since: "Otherwise stated, every physical quantity, every it, derives its ultimate significance from bits, binary yes-or-no indications, a conclusion which we epitomize in the phrase, *it from bit*" [23, p. 3]. It is, again, difficult to assess now how close this view may be to answering or, in the first place, meaningfully asking in terms of fundamental physics "the age-old question: 'How come Existence?"' However, as Wheeler's manifesto and the subsequent developments in quantum information theory make clear, we are as yet far from having explored the deep truth of the quantum. This deep truth may be imageless or perhaps, as all deep truth, must be imageless, as Shelley and Bohr thought, or unthinkable altogether. But this does not mean, quite the contrary, that we cannot explore it, or, given that we have been doing so for over a century, with the help of complementarity, entanglement, and information, and their relationships, for over a century, that we cannot continue to explore it, often groping in the dark, as Bohr saw his own thinking. The spirit of Copenhagen has always been defined by this exploration of the deep truth of the quantum. This is what has kept and continues to keep it alive.

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