Analogies and metaphors as a mediational strategy for conveying the proportional scale cognition at a middle school level

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Introduction

Notions of scale have been identified as one of the four powerful common themes that transcend disciplinary boundaries and levels. (American Association for the Advancement of Science [AAAS] 2006).

On the other hand, and as explained by Marsh, Parkes, and Boulter (2001), there has been very little research on children's understanding of scale. There is also a tendency of teachers of underestimating the scale aspect of representation of an object, and do not spend enough time teaching scaling concepts.

By analyzing summative evaluation reports and surveys conducted in informal educational settings, it was found that the general public have limited knowledge and consistent misconceptions about atoms, molecules, DNA, cells, and other things the interviewees cannot see (Edu.Inc, n.d.); misconceptions such as the smallest thing that they can think of is something they can actually see (early elementary students) or objects at the microscopic scale (Holladay, 2005). Another common naive conception is the fact that most of the interviewees had no working concept of one billion and did not understand 10^{-9} (Edu.Inc). Today's middle school students do not demonstrate an adequate understanding of concepts of scale and size on the micro and the nano level (Tretter, Jones, Andre, Negishi, & Minogue, 2006). Students are unable to identify the relative sizes between micrometer-sized and nanometer-sized objects (Edu.Inc, n.d., 2004, 2005; Holladay 2005; Jones et al. 2004; Waldron, 2006).

These results can further be corroborated with research done by Nakhleh, Samarapungavan, and Saglam (2005). They have found that although students knew that matter was composed of atoms and molecules, the middle school students could not be classified as having consistent knowledge frameworks because their ideas were very fragmented. They found that some students had the conception that they could see atoms or molecules under an optical microscope in the same way they could see microbes.

Since scale is an abstract concept and human are not able to perceive it with the naked eye, our identified problem is well described by Sabelli et al. (2005) as: "The problem is conceptual and practical" (p.3), objects and concepts at the atomic, nano, and micro scale are hard to visualize, difficult to describe, abstract, and their relationships to the observable world can be counterintuitive. According to this problem definition, we suggest then that middle school students may require abstract thought in order to understand the concept of scale (Eylon & Linn, 1988; Inhelder & Piaget, 1958).

In the following sections of this literature review we will focus first on characterizing scale cognition, second on identifying the main learning theories that address the cognitive processes required to attain scale cognition, and finally, on exploring the role of analogies and metaphors as learning strategies that will scaffold the cognitive processes in order to attain the proportional schema of scale cognition.

Characterizing size and scale cognition

For the purpose of characterizing size and scale cognition, it is required to determine a taxonomy for these conceptions. Informed by Gagne's taxonomy of learning outcomes, it is identified that conceptions of size and scale may be related to intellectual skills in the form of students' ability to apply knowledge; namely conceptions of size and scale, across a variety of previously unencountered instances. (Smith & Ragan, 2005). We suggest then, that initial cognitive processes for attaining scale cognition are generalization and discrimination among objects of different sizes. The generalization part will depend on the Piagetian logic of classes, and the discrimination one will depend on the Piagetian logic of relations for serial ordering. Since these two processes suggest a qualitative process, both are considered as 'size'; and will be referred as class inclusion conception of size and serial ordering conception of size.

To exemplify these two conceptions the following set of objects is considered: bacteria, virus, red blood cell, atom, human egg cell, ant, diameter of a DNA double strand, and human. The process of generalization of sizes will consist on classifying the atom as part of the atomic scale, the virus and the diameter of a DNA double strand as corresponding to the nanoscale, the bacteria, red blood cell and human egg cell as part of the microscale and the ant and human as part of the macroscale. In contrast, the process of discrimination will consist on ordering the objects according to its sizes: atom < virus < bacteria < red blood cell < human egg cell < ant < human.

A particular concept of our interest is the concept of proportion and the cognitive process behind it; proportional reasoning. We believe that it is this concept of proportion the bridge between a qualitative conception of size, to a quantitative conception of scale. Inhelder and Piaget describe the proportional schema as composed of two aspects; the logical and the mathematical. "In its general logical form, a proportion is the equivalence of the relations connecting two terms α and β to the relations connecting two other terms γ and σ." (p. 314). The authors also argue that the logical aspect leads to the mathematical one; once the learner has acquired the schemata of the logical one, later on, at any point the numerical values could be inserted.

To exemplify the proportional conceptions of size and scale let's consider some differences in relative sizes among some of the objects of the previous example: DNA double strand, bacteria, ant, and human. For the logical proportional conception of size an example would be: the difference in sizes between the height of a human and the length of an ant is approximately the same–proportional as the difference in sizes between a bacteria and the diameter of a DNA double strand. In contrast, an example illustrating the case of the mathematical proportional conception of scale would be: the difference in length of an ant compared to the height of a human is that the length of an ant is about thousand times smaller than the height of a human as the difference in size between a bacteria and the diameter of a DNA double strand is that the diameter of a DNA double strand is about thousand times smaller than the size of a bacteria. It is important to point out that the mathematical proportional conception of scale involves a process of measurement that according to Campbell, "... is the process of assigning numbers to represent qualities" (p.223 as cited in Rozeboom 1966) and therefore the numerical proportional conception is considered as 'scale'.

Finally the last conception of scale will be referred as the numerical absolute size of an object, and the general cognitive process behind the numerical absolute conception of scale is mathematical cognition. According to Campbell (2005), mathematical cognition deals with questions such as how does the mind represent numbers and make mathematical calculations. Therefore, this conception of scale consists on the assignment of an absolute number or measurement to an object. For example, the size of a bacteria is one micrometer, also expressed as 1 μm or in scientific notation as $1x10^{-6}$ m.

For a summary of our taxonomy of size and scale, and the cognitive processes behind each conception, see Table 1.

Table 1: Taxonomy for characterizing size and scale cognition and the cognitive processes behind them.

Cognitive process	**Taxonomy**
Generalization	qualitative categorical conception - size
Discrimination	qualitative relational conception - size
Logical proportional reasoning	qualitative proportional conception - size
Numerical proportional reasoning	quantitative proportional conception - scale
Mathematical reasoning	quantitative absolute conception - scale

In contrast with the characterization of size and scale proposed by Delgado et al. (2007), our characterization differs slightly in the following way. Delgado et al. identify four conceptions of size: a) qualitative relative concerning on Piaget's seriations by relative size, b) Piaget's categorical, related to classification of objects according to their attributes, c) quantitative relative, if unit is another object, and d) absolute, when exact unit is assigned. The justification that make us suggest the that the quantitative relative is actually composed by the logical and mathematical proportional is that although Delgado's quantitative relative conception of size is not referred as the proportional schema, it does involve a proportional relationship.

The logical and mathematical conceptions of size and scale are the focus of this literature review.

Requirements for eliciting abstract reasoning

From our problem definition it has been identified that abstract reasoning is required to attain size and scale cognition. Abstract thought is the focus of the developmental genetic epistemology of Piaget. According to Piaget's cognitive development stage view (Inhelder & Piaget, 1958), young children are different kinds of learners and in general thinkers than adults. Until they reach the age of 12 or so they are not capable of reasoning as an adult and show abstract thinking. This theory of cognitive development proposed four major stages of development: the sensomotor stage, the preoperational thought stage, the concrete operations, and the formal operations; this same stage continues to the adulthood.

It is important to point out that even though Piagetian processes are used as a way to characterize size and scale cognition, we are not suggesting that size and scale cognition is attained under the developmental perspective, but rather through principles of the cognitive information processing perspective. Therefore, although Inhelder and Piaget conclude that abstract reasoning cannot be shown until formal operations stage, the alternative theories of cognitive development suggest that students appear quite able to think abstractly as long as they have appropriate science knowledge of the subject matter even if they are young and if their ideas are incomplete (Eylon & Linn, 1988). These alternative theories incorporate elements of the cognitive information processing perspective.

Case (1993) suggested that the mental space, a concept similar to working memory, increases during development. He suggested that this increase occurs because of three processes: brain maturation and its resulting myelinization that increase processing speed, cognitive strategies become automatic, and prior knowledge becomes more extensive and better organized (as cited in Smith and Ragan, 2005). This view proposed that it is the process of encoding that distinguishes cognitive development.

Another important aspect to consider is the role of experience in specific domains. This aspect was deeply studied by Siegler (1983, as cited by Driscoll, 1994). He was focused on local description as well as on specific task requirements. He suggested that when emphasized the role of encoding, it resulted in children's construction of more advanced knowledge. Siegler (1994, as cited by Driscoll) developed a performance model for problem solving. He incorporated four steps consisting on children actively encoding the features of a problem by trial and error, monitoring these features and selecting specific rules ad hoc to the problem. Then, combining the dimensions into the rule, and executing it correctly.

A study conducted Linn, Clement, Pulos, and Sullivan (1989) corroborates the role of experience in scientific domain. These researchers assessed the role of science topic instruction combined with logical reasoning strategy instruction in teaching adolescent students. They found that topic-related training enhanced knowledge about relevant variables and resulted in reasoning changes. This suggests that reasoning strategies and knowledge of the subject matter influence abstract thinking. This improvement results from student's restructuring of their naive frameworks modified by exposure to formal instruction (Nakhleh et al., 2005), and as individuals learn a new domain of science (Carey, 1986).

Eylon and Linn (1988) considered the important role that plays working memory capacity. They suggested that adequate instruction, together with learners consolidation of information into procedures, would result in learners becoming able to handle more complex problems using the same amount of processing capacity. Ben-Zvi, Eylon, and Silberstein (1986a) also proposed that abstract reasoning may vary as a function of working memory demand rather than development, and therefore, if a given problem overloads working memory, students cannot reason abstractly and revert to a more concrete approach. One way to reduce this overload of working memory is by using instructional strategies that will scaffold abstract concepts.

Therefore for conveying size and scale cognition, we will rely on the alternative theories of cognitive development arguing that deep coverage of the topic may elicit abstract reasoning (Eylon & Linn, 1988). Thus, in order to overcome the limits of working memory capacity it is necessary to help students in their process of encoding and automatization (Case, 1993), by providing adequate topic-related formal instruction, knowledge of the subject matter, reasoning and reflective strategies, feedback for self-monitoring, and integration of the new information with prior knowledge.

Proportional reasoning: the cognitive process behind the proportional conceptions of size and scale

According to Lesh, Post, and Behr, (1988), proportional reasoning is a form of mathematical reasoning involving multiple comparisons, inference and prediction, as well as both qualitative and quantitative methods of thought. In their work, Lesh et al. examine it from the perspective of proportional reasoning as a capstone of elementary arithmetic, number, and measurement concepts.

Proportional reasoning is the cognitive process behind the ability to reason about the relationship between two rational expressions. Therefore, our first inference is that proportional reasoning is the required cognitive process in order to attain the proportional size and scale cognition. We have identified that scale cognition is composed by the logical proportional and numerical proportional conceptions of size and scale; these conceptions and the cognitive processes behind them are explained below.

For describing the logical proportional conception of size we referred to the work conducted by Tretter et al. (2006). Tretter et al. describe unitizing as the first required cognitive process in order to attain scale cognition. Unitizing refers to the process of creating new meaningful units from the existing objects. Tretter et al. suggest that students must create scale conceptions focusing on relative sizes and not much on exact size information. These conceptions could be based on everyday uses of size with relations based on experience, organized into categories and containing prototypes as exemplars of categories; such as well known landmarks and reference points. For example, a well known unit for a novice learner can be the relationship in sizes between two objects from the macroscale, that will serve as a landmark or reference size for objects in the microscale, atomicscale and nanoscale. (e.g. the difference in sizes between the height of a human and the length of an ant is approximately the same–proportional as the difference in sizes between a bacteria and the diameter of a DNA double strand).

The numerical proportional conception of scale, on the other hand, point us to consider the mathematical relationships with pairs of rational expressions; such as ratios, proportions, rates, quotients, and fractions. For the case of ratios and proportions, Hart, (1988) and Behr, Lesh, Post, and Silver, (1983) agree that a ratio is a comparative index between tow entities that conveys the notion of relative magnitude. On the other hand, a proportion refers to the equivalence of two ratios, "When two ratios are equal they are said to be in proportion to one another. A proportion is simply a statement equating two ratios." (Behr et al., p.95). For the case of rates, and as explained by Lesh et al., (1988), it refers to a single quantity (e.g. 30 miles/hour), while ratios involve two quantities. Fractions include percentages, decimal expressions, and operations or points on the number line (Person, Berenson, and Greenspon, 2004)and finally, a quotient is simply an operation of indicated division (Karplus, Pulos, and Stage, 1983).

The proportional mathematical conception of scale would be considered then as the comparison of two equivalent ratios; namely a proportion. (e.g. the difference in length of an ant compared to the height of a human is that the length of an ant is about thousand times smaller than the height of a human; therefore, the difference in size between a bacteria and the diameter of a DNA double strand is that the diameter of a DNA double strand is about thousand times smaller than the size of a bacteria).

To better understand proportional reasoning and at the same time infer instructional strategies that will address this cognitive process, research related to this area was analyzed. For this purpose three authors and their research were considered: perspectives proposed by Piaget, Karplus, and Lesh. Piaget's perspective was selected because he and his colleagues conducted the first attempts to measure proportional reasoning in their experiments for clarification of young people's development of the logico-mathematical concept (Lesh et al., 1988). But because the designs of their tasks were not designed specifically to illustrate proportional reasoning, more specialized studies were consulted. Robert Karplus, in contrast, focused on proportional reasoning by trying to minimize the need for knowledge of physical principles Lesh et al.. Finally, the research conducted by Lesh was also considered because of his attempts to contrast and compare prior research in this area.

Piaget (1968, as cited in Lesh et al., 1988) describes the development of adolescent's proportional reasoning in three stages. The first one is the global compensatory strategy focused on additive strategies, the second one is based on a multiplicative strategy, and the third one is based on a formulation of a law of proportions. In contrast, Karplus focused on categorizing the responses of children as a demonstrative of a level of understanding of proportion (Hart, 1988). Karplus et al., (1983) argue that Piagetian tasks were not selected exclusively for characterizing subjects' proportional reasoning, leaving unanswered many questions related to how proportional reasoning is applied in problem solving in different contexts and numerical relationships. Instead Karplus et al. and the research group at the Lawrence Hall of Science at Berkeley, focused on assessing children's proportional reasoning using tasks in which knowledge of the physical principles was minimized. They found that "academically upper-track or upper middle-class students used proportional reasoning increasing after about age 12 years, only a small fraction of urban low-income and academically lower track- students used proportions at age 14 or even 17 years." (Karplus, 1981 as cited in Karplus et al., 1983, p. 47).

Compared to the results obtained by Karplus et al., as well as Inhelder and Piaget, Lesh et al. (1988) concludes that in mathematics education research, proportional reasoning is characterized by a gradual increase in local competence; and not by a global ability related to a cognitive structure. These results point us to consider that environmental interactions are a very influential factor for the development of proportional reasoning.

Singer-Freeman and Goswami (2001) also investigated children's intuitive approach to proportions, arguing that their intuitions of proportional reasoning were based on their ability to recognize relational similarity. This idea is consistent with Lesh et al. (1988) remark that "proportional reasoning deals with one of the most common forms of structural similarity "(p.95). This recognition of structural similarity is what points us to consider analogies and metaphors as a way to scaffold proportional reasoning in young learners for the following reasons. First, although similarity and analogy are not the same, Gentner and Markman (1997) suggest that the process of carrying out a comparison is the same in both cases, summarizing the idea as similarity is like analogy, involving a process of "structural alignment and mapping between mental representations" (Gentner and Markman, 1997, p.45).

Second, classical or conventional analogies take the form of A:B::C:D (L. English, 2004), where the A and B can be termed as the base or source, and C and D can be termed as the target (Gentner, Holyoak, & Kokinov, 2001). These analogies are basically proportional or relational problems.(L. English; Gentner and Markman).

Finally, we rely on the fact that it has been identified that one reasonable way to convey proportions is through means of existing conceptual knowledge as a basis for teaching (Singer-Freeman & Goswami, 2001), as well as on the fact that children's ability to recognize relational similarity may be present as early as infancy; namely the capacity for analogical thinking (Goswami, 2001; Holyoak and Thagard, 1997).

Analogies and metaphors for attaining proportional scale cognition

Analogies are a fundamental cognitive mechanism that people use to map processes by identifying relevant information from a more familiar domain to a less familiar one (Mason, 2004). Since early times it has been well recognized the powerful role of analogies in enabling people to communicate, explore, and infer about novel phenomena, as well as to transfer learning across subject domains (L. English, 2004; Gentner and Markman, 1997; Goswami, 2001; Richland, Morrison, and Holyoak, 2006).

While analogy is a sophisticated process used in creative discovery, similarity is, as described by Gentner and Markman, (1997), brute perceptual process. Gentner (1983) define an analogy as "a device for conveying that two situations or domains share relational structure despite arbitrary degrees of difference in the objects that make up the domains This promoting of relations over objects makes analogy a useful cognitive device, for physical objects are normally highly salient in human processing" (as cited in Gentner and Markman (1997), p.46). They emphasize the importance of common relations to analogy and not common objects.

In contrast, L. D. English, (1997) describes metaphors as characterized by cross domain mappings. She explains that reasoning by metaphor implies conceptualizing the phenomena in terms of another mental domain by finding a mapping between the target domain and the source domain. "Like analogical reasoning, metaphorical reasoning can generate new inferences and lead to the construction of mental models based on the relational structure shared by the source and target".(L. D. English, p.7).

Although Piaget concludes that reason by analogy must depend on the development of categorization skills and therefore developed in the formal operations stage; recent research proves that the ability to reason by analogy was shown by young children (Vosniadou, 1995). Vosniadou argues that in the Piagetian perspective the focus was on "how the development of analogical reasoning could be explained in the context of his [Piaget's] theory of intellectual development, rather than how analogical reasoning might contribute to intellectual development".(p.300). Supporting this idea Goswami (1992) and her colleagues found no support for the claims made by Piaget in his theory of analogical development, including the idea that reasoning about relational similarity requires the ability to reason about proportional equivalence (as cited in Vosniadou, 1995). Goswami also concludes that "domain knowledge is the primary constraint in children's analogical reasoning" (p.250, as cited in Richland et al., 2006).

We conclude that due to the facts that a) analogical reasoning does not depend on the ability to reason about proportional equivalence, b) young children have the ability to reason by analogy, and c) analogies and metaphors are powerful tools in enabling people to communicate, explore, infer about novel phenomena, and to transfer learning across subject domains; we therefore propose that proportional analogies may be a sense-making way to unitize and at the same time serve as a scaffold that will emphasize relative sizes of objects in different scales. These analogs, as explained by Tretter et al. (2006) could be based on everyday uses and conceptions of size with categorical relations that conceptions of scaling must be based on experience, and containing prototypes as exemplars of categories; such as landmarks and reference points.

Conclusion

In the final analysis, we conclude that scaling related concepts have been identified as one of the important unifying topics in science, engineering, and technology education. We have also identified that today's middle school students do not demonstrate the adequate understanding of concepts of scale and size on the micro and the nano level.

We characterized scale cognition in four components; class-inclusion, serial-ordering, logical proportional, and mathematical proportional. We also identified the abstract nature of scale cognition and have proposed the alternative theories of cognitive development as the learning theories that suggest strategies for eliciting abstract thinking. We have also suggested that proportional reasoning is the cognitive process behind the proportional conceptions of scale. We identified the process of unitizing as the first required cognitive process in order to attain the logical proportional scale cognition and mathematical relationships of ratios as a comparative process for attaining the mathematical proportional conception of scale.

To sum up, it is our expectation that classical or conventional analogies would facilitate the logical aspect of the proportional schema of scale cognition. These same analogies, together with a scale metaphor, would also facilitate the mathematical aspect of the proportional schema of scale cognition. Analogies and metaphors will serve as a scaffold for the learners in which two pairs of objects that students are familiar with and visible to the naked eye will serve as the source, and two other objects that are in the micro, nano, and/or atomic scale will serve as the target. Students then may map the structural similarity of the objects; namely their difference in sizes.

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