INTERACTION BETWEEN A PAIR OF DROPS ASCENDING IN A LINEARLY STRATIFIED FLUID

M. Bayareh  
Department of Aerospace and Mechanical Engineering  
University of Notre Dame  
Notre Dame, Indiana 46556  
Email: bayareh.1@nd.edu

S. Dabiri  
Department of Aerospace and Mechanical Engineering  
University of Notre Dame  
Notre Dame, Indiana 46556  
Email: sadegh.dabiri.1@nd.edu

A. M. Ardekani  
Department of Aerospace and Mechanical Engineering  
University of Notre Dame  
Notre Dame, Indiana 46556  
Email: arezoo.ardekani.1@nd.edu

ABSTRACT

In this paper, we provide fully resolved three-dimensional direct numerical simulations of ascending motion of a pair of drops in a linearly stratified fluid using a finite-volume/front-tracking method. We study the density stratification effects on the rise velocity of drops and their stable position for different initial alignments. Due to the formation of a jet in the lee of a single drop rising in a stratified fluid, a decrease in rise velocity (or an increase in drag) is observed compared to the homogeneous case. The hydrodynamic interaction between two drops in a linearly stratified fluid depends on the properties of both fluids and initial orientation of the two drops. For the case of drops rising side by side, the lateral separation of drops is suppressed due to stratification effects. In contrast to homogeneous case, two nearly spherical drops collide with each other at low Froude numbers and oscillate around their neutrally buoyant density level. Two spherical drops rising in tandem in a linearly stratified fluid at finite Reynolds number regime undergo drafting, kissing, and tumbling unlike their counterpart in a homogeneous fluid.

Key words: Stratification, hydrodynamic interaction, drop motion

INTRODUCTION

Since many flows relevant to aquatic and atmospheric environments include solid objects, there have been several studies on the settling of rigid particles in stratified fluids, for example, the aggregation of marine snow particles and micro-organisms in oceans [1] and settling of dust particles in the atmosphere [2]. Greenslade [3] showed that the stratification leads to an increase in the ratio of the wake drag to the wave drag of a sphere moving horizontally in a vertical stratified fluid. Srdić-Mitrović et al. [4] experimentally showed that the settling velocity of a rigid particle falling in a stably density-stratified fluid decreases upon crossing the density interface before it increases again to the terminal velocity of the particle in the higher density fluid layer. Abaid et al. [5] found that a rigid particle may exhibit a levitation phenomenon after crossing a sharp density stratified layer. The motion of a rigid sphere settling in a linearly stratified fluid has been considered in several studies [6, 8, 9]. Torres et al. [6] showed that there is a critical Froude number in which the rear vortex completely collapses and forms a buoyant jet in the lee of the particle at moderate Reynolds numbers (25 ≤ Re ≤ 100). Their study revealed that the particle velocity decreases significantly due to the formation of the jet. The buoyant jet is not generated in the lee of a rigid sphere at small Reynolds numbers (0.05 ≤ Re ≤ 2.1) [7]. Hanazaki et al. [8] demonstrated that the jet velocity increases with decreasing the molecular diffusivity and increasing the stratification strength. More recently, Hanazaki et al. [9] investigated the wake structure behind a rigid sphere settling in a linearly stratified fluid at Prandtl number Pr = 700 corresponding to salt stratified water and discussed different types of columnar structures behind the settling particle. These different jet structures were observed for a wide range of Froude numbers and Reynolds numbers (0.2 ≤ Fr ≤ 70 and 30 ≤ Fr ≤ 4000).
In the Stokes regime, Ardekani and Stocker [10] used the solution of point-force singularities in Stokes flow of a stratified fluid and showed that density stratification affects the motion of particles and organisms as small as O(100μm-1mm). Recently, Doostmohammadi et al. [1] demonstrated that the swimming of small organisms living at pycnoclines (i.e. region of sharp variation in fluid density) can be largely influenced by vertical density gradient when the full Navier-Stokes equations are solved.

Despite widespread applications for the motion of drops in stratified fluids, such as oil spills in the ocean and aeration systems in water reservoirs, limited work has been done on the motion of drops in the presence of stratification. Zhang et al. [11] analyzed the motion of a drop in a vertical temperature gradient when the gravity and thermocapillary effects are included. Recently, Blanchette and Shapiro [12] numerically investigated the case of a spherical drop settling in a sharp stratified fluid and demonstrated that a drop crossing the density interface entrains smaller volume of lighter fluids compared to a rigid particle.

Hydrodynamic interaction between two drops/bubbles rising in a homogeneous fluid has received significant attention. For example, Vélez-Cordero et al. [13] compared the dynamical behavior of two bubbles moving vertically in Newtonian and shear-thinning fluids. They revealed that two nearly spherical bubbles (Eo = 0.9) rising in line in a Newtonian fluid exhibit drafting-kissing-tumbling (DKT) phenomenon at Re = 1.3, but they do not tumble for the shear-thinning case. Hallez and Legendre [14] and Legendre et al. [15] showed that the interaction between the vorticity fields around two close side-by-side bubbles leads to a repulsive force acting on the bubbles which increases their separation distance. Bunner and Tryggvason [16] studied the role of the bubble deformability on the dynamics of a pair of bubbles at Re = 23 and found that in contrast to spherical bubbles at this Reynolds number, two deformable bubbles rising in tandem follow DKT process.

Here, in this study we consider the interaction between two drops in a linearly stratified fluid. Different initial configurations of drops are considered and the role of stratification on the dynamical behavior of drops are studied.

**BASIC EQUATIONS AND NUMERICAL METHODS**

The governing equations for the problem of the interaction between two drops rising in a linearly stratified fluid can be described by the conservation of mass:

\[ \nabla \cdot \mathbf{u} = 0, \quad (1) \]

conservation of momentum:

\[ \rho \frac{D\mathbf{u}}{Dt} = -\nabla P + (\rho - \bar{\rho})\mathbf{g} + \nabla \cdot \mu \left( \nabla \mathbf{u} + \nabla \mathbf{u}^T \right) + \int \sigma \kappa' \hat{n'} \delta^{\beta'}(\mathbf{x} - \mathbf{x'})dA', \quad (2) \]

and conservation of energy:

\[ \frac{DT}{Dt} = \kappa \nabla^2 T, \quad (3) \]

where \( \mathbf{u} \) is the velocity vector, \( P \) the pressure, \( \mathbf{g} \) the acceleration of gravity, and \( \kappa \) is the diffusion coefficient. \( \rho, \rho_0, \) and \( \mu \) are the local density, reference density, and viscosity. They are equal to drop properties inside the drop (dispersed phase) and properties of the background fluid in the continuous phase. \( \sigma \) is the surface tension between two fluids (the dispersed and continuous phases). \( \kappa' \) is twice the mean curvature of the interface of the drop, \( \hat{n'} \) the unit vector normal to the interface, \( dA' \) the three-dimensional delta function. \( x \) and \( x' \) are the position in Eulerian and Lagrangian coordinates, respectively. \( \bar{\rho} = \frac{1}{V} \int_V \rho dV \) is the mean density over the entire computational domain \( V \). The acceleration term, \( \bar{\rho} / \rho g \), is acting on the entire domain to set the net momentum flux of the computational domain across outer boundaries \( \partial \Omega \) to zero (\( \int_{\partial \Omega} \rho \mathbf{u} dA = 0 \)). In its absence, the entire domain accelerates due to gravity. The density varies linearly with temperature, \( T \), as follows

\[ \rho = \rho_0 (1 - \beta T), \quad (4) \]

where \( \beta \) is the thermal expansion coefficient. The above governing equations are solved by using a front-tracking technique [17]. A computational code is developed based on the finite volume method and projection scheme to solve incompressible Navier-Stokes equations. The details of the numerical method have been presented and fully verified in ref. [18]. The method is extended to account for stratification of the background fluid. Convective terms of the Navier-Stokes equations are discretized explic-
itly using third-order QUICK scheme and diffusion terms are described by central difference scheme.

RESULTS

In this study we consider the interaction between two nearly spherical drops rising in a linearly stratified fluid with different initial angles, focusing on two drops rising in line and side by side. Figure 1 shows the schematic of the problem for two drops interacting in a linearly stratified fluid. For this problem, the governing dimensionless parameters are the Reynolds number, $Re = \rho_f w_d / \mu_f$, the Froude number, $Fr = w / (N d)$ where $N = \sqrt{-dp/dz(g/\rho_f)}$ denotes Brant-Väisälä frequency, the Prandtl number, $Pr = \nu_f / \kappa_f$, the Eötvös number, $Eo = (\rho_f - \rho_d)gd^2 / \sigma$, and the ratio of material properties $\eta = \rho_d / \rho_f$ and $\lambda = \mu_d / \mu_f$. Here, $z$ represents the vertical position which is positive upward. $\rho_{f0}$ is the fluid reference density (density of the background fluid at the initial location of the drop), and subscripts $d$ and $f$ correspond to drop and background fluid properties, respectively. The Froude number represents the strength of stratification and the Eötvös number represents the ratio of the drop gravitational force to the surface tension force. We use $w = \sqrt{gd}$, particle diameter, $d$, and $t = \sqrt{d/g}$ as velocity, length, and time scales, respectively. In this paper, we use crude oil properties and dimensionless parameters are set as: $Re = 790$, $Eo = 0.6388$, $Pr = 7$, $\lambda = 5$, and $\eta = 0.89$. Also, the initial dimensionless separation distance is set to $d^* = 1.25$ for all cases. The boundary condition imposed on the top and bottom of the domain is wall and periodic boundary condition is used on the slide planes. A large domain is used in all directions to avoid boundary effects (the size of the computational domain is $15d \times 15d \times 60d$).

Single drop

We performed the domain independency test for the case of single drop ascending in a linearly stratified fluid and found that domain size of $30d \times 30d \times 120d$ leads to 0.5% difference in the rise velocity.

The snapshots of the motion of a single drop are presented in Fig. 2. This figure shows contours of constant density (isopycnals) along with contours of the vertical component of velocity at $Fr = 5$. The drop is initially at rest and as it accelerates, it perturbs isopycnals and entrain a shell of lighter fluid behind it. As Froude number decreases, the deflection of isopycnals increases. The isopycnals return to their neutrally buoyant levels far from the drop. As the drop reaches its neutrally buoyant density level,
In-line configuration ($\theta = 90$)

Two spherical bubbles rising in line in a homogeneous fluid at finite Reynolds number regime do not collide with each other due to the local velocity gradients induced by the leading bubble [16]. The trailing bubble tends to move away from the wake of the leading one. In other words, the second stage of DKT phenomenon does not occur during the interaction between two spherical bubbles. Our results for two drops rising in a homogeneous fluid show that the drops do not collide but they tumble (Fig. 3q). To understand pair-drop interaction in the presence of stratification, we have performed a simulation at Froude number $Fr = 10$. Figure 3 shows consecutive snapshots of the motion of two drops ascending in a linearly stratified fluid compared to those rising in a homogeneous one. At stronger stratification, the background density gradient is larger and thus the leading drop reaches its neutrally buoyant level earlier. The deflection of isopycnals hinders the rise velocity of drops. When the leading drop reaches its neutrally buoyant level, it decelerates and collides with the trailing drop. Therefore, two drops oscillate due to the simultaneous effects of buoyancy and inertia. While the levitation of drops continues, the trailing drop rotates around the leading one, oscillates, and eventually stops in a configuration where the drops line of center is perpendicular to the direction of gravity. The trailing drop has non-zero lateral velocity (along $x$ and $y$ directions) and undergoes a spiral motion. ($x$, $y$, $z$) are the cartesian position vector where gravitation acceleration is along positive $z$ axis. Even though we observe the drafting-kissing-tumbling process at low Froude numbers, it is different from those occurring for deformable drops rising in a homogeneous fluid. In the presence of stratification, the rise velocity of the leading drop is suppressed. Thus, the trailing drop collides with the leading one and they tumble.

Side-by-side configuration ($\theta = 0$)

In this section, we consider the interaction between two side-by-side drops ($\theta = 0$). Two close side-by-side drops ascending in a homogeneous fluid repel each other due to the generation of vorticity around them. At $Fr = 10$, the drops repel each other but at smaller rate compared to a homogeneous fluid. As the stratification strength increases ($Fr = 5$), drops are attracted to each other. At this Froude number, smaller value of vorticity is generated near the drops which allows the drops to collide and remain together as a single body. In the absence of stratification, drops deform more and the lateral velocity of drops changes sign during their motion as they repel each other. Initially, the deformation of drops induces a lift force which leads to their attraction. After the drops come to close contact, the lift force changes its sign and drops repel each other. The higher deformability of drops rising in a homogeneous fluid induces larger values of vorticity at the surface of each drop, and thus the repulsive force is stronger compared to the stratified fluid. At low Froude numbers, the sign of transverse force does not change, resulting in attraction of drops. The drops in a stratified fluid remain spherical and do not repel each other. At moderate Froude number, $Fr = 10$, we observe an attraction between drops, then a repulsion, followed by another attraction phase (see Fig. 5). The third stage of this dynamics is different from the homogeneous case. The reason for the last attraction stage is related to the reduced lateral motion of drops in stratified fluids. The strength of drop attraction depends on the value of the Froude number. At high Froude numbers, the attraction-repulsion dynamics occurs simi-
lar to the homogeneous case, but stratification decreases the rate of separation between drops.

**Different initial orientation**

In this section, we consider the binary interaction between two drops with different initial alignments. The variation of normalized separation distance between two drops rising in a linearly stratified fluid is shown as a function of dimensionless time in Fig. 6. For the initial angle $\theta = 15$, homogeneous results are plotted as well as those of stratified fluids. We can see that the rate of change of separation distance between drops decreases in the presence of stratification. In Fig. 6 we can see how the separation distance between two drops varies with the initial orientation. Drops repel each other for all cases and the relative velocity of the center of mass of drops along x direction is zero.

**CONCLUSION**

In this paper, we have investigated the interaction between two drops rising in a linearly stratified fluid with different initial configurations. We have shown that linear stratification significantly alters the motion of a pair of drops. The rate of repulsion between the drops decreases as the Froude number decreases due to the reduced lateral motion of drops in a stratified fluid. At low Froude numbers, two side-by-side drops tend to collide and remain together. Unlike the homogeneous case, two drops as-
ascending in tandem in a linearly stratified fluid collide with each other when the drops reach their neutrally buoyant density level. At this position, the trailing drop rotates around the leading one. The drops oscillate together and eventually stop at a horizontal alignment.

ACKNOWLEDGMENTS
This work is supported by National Science Foundation Grant CBET-1066545.

REFERENCES