Modeling and Analysis of Simple Market Strategies

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This project aims to observe what happens when simple market strategies are used in a particular market place.

Modeling based on real-valued functions.

Simple software developed in SageMath \([S^{+17}]\) (arbitrary precision computing) to track statistics and plot data for analysis.
We analyze the following model.

- There is a market place for one product, with \( n \) sellers and a market price \( mp \).
- Each seller has a pricing strategy which depends on at most \((n - 1)\) other sellers and/or the current or starting market price.
- Each seller has an initial price for the market.
- The market price also has a strategy.

This model can be thought of as a *seller’s only market*, where the pricing strategies of the sellers is the sole determinant of the market as a whole.
Experiments in this model follow the following outline.

- The market spans over $t$ rounds or time instances.
- During a round $t_i$, we determine which seller(s) apply their pricing strategy(-ies) and update their price(s).
- At the end of a round $t_i$, the market price is updated.
Model

- Model is broad in order to allow a wider range of possible results and observations, but at the same time is limited.
  - No classical supply and demand for example.

- Many parameters are fixed to allow comparable results to be generated.

- General assumptions
  - Sellers cannot price below $0.
  - Sellers may not update their price more than once per round.
  - Each seller strategy is round independent.

- Fixed Market Price Strategy
  - the average of all seller prices at the end of round $t_i$. 
Seller Strategies

- Guideline for seller strategies
  - Deterministic vs. Nondeterministic
  - Unbounded vs. Bounded

- Experiments are performed with each of the above combinations, in addition to a mix of bounded and unbounded constraints.
  - For deterministic strategy experiments, all sellers have deterministic strategies.
  - For nondeterministic, at least one seller must have a nondeterministic strategy.
What is Market Skew?

- When the strategies of the seller pull the market upwards or downwards (in terms of price).

Three possible: Low-Skew, No-Skew, and High-Skew

- Low-Skew inherently floored at $0$ by our assumptions, but is otherwise symmetric to High-Skew.
- No-Skew means *roughly on average*, the market doesn’t skew up or down “too much”.
- High-Skew not considered, as it is symmetric to Low-Skew (especially when a market cap is provided).

Broad categorization, does not distinguish between the rate at which a market skews in a particular direction.
How to choose a seller to update their price in a round?

- One Random Seller: one seller chosen uniformly at random out of $n$.
- Two Random Sellers: two distinct sellers chosen uniformly at random.
- All Sellers: each seller updates their price

More parameter fixing

- Number of rounds: $t = 500$.
- Number of seller: $n = 5$ and $n = 10$.
- Starting market price: $smp = $2500.

Still a great degree of freedom left in the model because of seller strategies.

We run each experiment for 20 trials; i.e., at the end of 500 rounds, statistics are collected, the market is reset, and the experiment is run again.
### Results from a Low-Skew Market

<table>
<thead>
<tr>
<th>Seller</th>
<th>Nondeterministic Unbounded</th>
<th>Nondeterministic Bounded</th>
<th>Nondeterministic Mix</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seller 1</td>
<td>( f_1(x_{[n]}) = \text{ave}_i { x_i } - \text{rr} (5, 16) )</td>
<td>( g_1(x_{[n]}) = \max { 100, f_1(x_{[n]}) } )</td>
<td>( h_1(x_{[n]}) = g_1(x_{[n]}) )</td>
</tr>
<tr>
<td>Seller 2</td>
<td>( f_2(x_{[n]}) = \frac{\text{rr}(145,161)}{100} \text{ave}_i { x_i } )</td>
<td>( g_2(x_{[n]}) = \min { 5(mp), f_2(x_{[n]}) } )</td>
<td>( h_2(x_{[n]}) = f_2(x_{[n]}) )</td>
</tr>
<tr>
<td>Seller 3</td>
<td>( f_3(x_{[n]}) = \frac{\text{rr}(67,74)}{100} \text{ave}_i { x_i } )</td>
<td>( g_3(x_{[n]}) = \max { 0.15(mp), f_3(x_{[n]}) } )</td>
<td>( h_3(x_{[n]}) = f_3(x_{[n]}) )</td>
</tr>
<tr>
<td>Seller 4</td>
<td>( f_4(x_{[n]}) = \frac{\text{rr}(75,86)}{100} \text{min}_i { x_i } )</td>
<td>( g_4(x_{[n]}) = \max { 75, f_4(x_{[n]}) } )</td>
<td>( h_4(x_{[n]}) = g_4(x_{[n]}) )</td>
</tr>
<tr>
<td>Seller 5</td>
<td>( f_5(x_{[n]}) = \text{ave}_{x_i &lt; mp} { x_i } - \text{rr} (20, 31) )</td>
<td>( g_5(x_{[n]}) = \max { 0.1(mp), f_5(x_{[n]}) } )</td>
<td>( h_5(x_{[n]}) = f_5(x_{[n]}) )</td>
</tr>
<tr>
<td>Seller 6</td>
<td>( f_6(x_{[n]}) = \frac{\text{rr}(107,116)}{100} \text{max}_i { x_i } )</td>
<td>( g_6(x_{[n]}) = \min { 3(mp), f_6(x_{[n]}) } )</td>
<td>( h_6(x_{[n]}) = f_6(x_{[n]}) )</td>
</tr>
<tr>
<td>Seller 7</td>
<td>( f_7(x_{[n]}) = \text{min}_{x_i \geq 0.75(mp)} { x_i } - \text{rr} (9, 21) )</td>
<td>( g_7(x_{[n]}) = \max { 50, f_7(x_{[n]}) } )</td>
<td>( h_7(x_{[n]}) = g_7(x_{[n]}) )</td>
</tr>
<tr>
<td>Seller 8</td>
<td>( f_8(x_{[n]}) = \frac{75}{100} \text{ave}_{i \text{ even}} { x_i } )</td>
<td>( g_8(x_{[n]}) = \max { 100, f_8(x_{[n]}) } )</td>
<td>( h_8(x_{[n]}) = g_8(x_{[n]}) )</td>
</tr>
<tr>
<td>Seller 9</td>
<td>( f_9(x_{[n]}) = \frac{\text{rr}(120,136)}{100} \text{ave}_{i \text{ odd}} { x_i } )</td>
<td>( g_9(x_{[n]}) = \min { 500 + (mp), f_9(x_{[n]}) } )</td>
<td>( h_9(x_{[n]}) = f_9(x_{[n]}) )</td>
</tr>
<tr>
<td>Seller 10</td>
<td>( f_{10}(x_{[n]}) = \text{ave}_i { x_i } )</td>
<td>( g_{10}(x_{[n]}) = \max { 60, f_{10}(x_{[n]}) } )</td>
<td>( h_{10}(x_{[n]}) = g_{10}(x_{[n]}) )</td>
</tr>
</tbody>
</table>
Results from a Low-Skew Market

Figure 1: Data plots for Low-Skew, 5 Sellers, Deterministic Unbounded. Clockwise from the top left: One Update, Two Updates, All Update. The $x$-axis is number of rounds $t$; the $y$-axis is price in dollars. Market Price in black.
Results from a Low-Skew Market

Figure 2: Data plots for Low-Skew, 5 Sellers, Deterministic Bounded. Clockwise from the top left: One Update, Two Updates, All Update. The $x$-axis is number of rounds $t$; the $y$-axis is price in dollars. Market Price in black.
Results from a Low-Skew Market

Figure 3: Data plots for Low-Skew, 5 Sellers, Nondeterministic Bounded. Clockwise from the top left: One Update, Two Updates, All Update. The $x$-axis is number of rounds $t$; the $y$-axis is price in dollars. Market Price in black.
Results from a Low-Skew Market

Figure 4: Data plots for Low-Skew, 10 Sellers, Deterministic Unbounded. Clockwise from the top left: One Update, Two Updates, All Update. The $x$-axis is number of rounds $t$; the $y$-axis is price in dollars. Market Price in black.
Results from a Low-Skew Market

Figure 5: Data plots for Low-Skew, 10 Sellers, Nondeterministic Unbounded. Clockwise from the top left: One Update, Two Updates, All Update. The $x$-axis is number of rounds $t$; the $y$-axis is price in dollars. Market Price in black.
## Results from a No-Skew Market

<table>
<thead>
<tr>
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<th>Deterministic Bounded</th>
<th>Deterministic Mix</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seller 1</td>
<td>$f_1(x_{[n]}) = \text{ave}_i {x_i}$</td>
<td>$g_1(x_{[n]}) = \max {0.3(smp), f_1(x_{[n]})}$</td>
<td>$h_1(x_{[n]}) = g_1(x_{[n]})$</td>
</tr>
<tr>
<td>Seller 2</td>
<td>$f_2(x_{[n]}) = 1.613 \left(\text{ave}_i {x_i}\right)$</td>
<td>$g_2(x_{[n]}) = \min {4000, f_2(x_{[n]})}$</td>
<td>$h_2(x_{[n]}) = f_2(x_{[n]})$</td>
</tr>
<tr>
<td>Seller 3</td>
<td>$f_3(x_{[n]}) = 0.55 \left(\text{ave}_i {x_i}\right)$</td>
<td>$g_3(x_{[n]}) = \max {500, f_3(x_{[n]})}$</td>
<td>$h_3(x_{[n]}) = f_3(x_{[n]})$</td>
</tr>
<tr>
<td>Seller 4</td>
<td>$f_4(x_{[n]}) = 1.1 \left(\min_i {x_i}\right)$</td>
<td>$g_4(x_{[n]}) = \max {0.5(mp), f_4(x_{[n]})}$</td>
<td>$h_4(x_{[n]}) = f_4(x_{[n]})$</td>
</tr>
<tr>
<td>Seller 5</td>
<td>$f_5(x_{[n]}) = 0.9 \left(\max_i {x_i}\right)$</td>
<td>$g_5(x_{[n]}) = \min {mp + 1500, f_5(x_{[n]})}$</td>
<td>$h_5(x_{[n]}) = f_5(x_{[n]})$</td>
</tr>
</tbody>
</table>

![Graph showing results from different sellers](image-url)
Figure 6: Data plots for No-Skew, 10 Sellers, Deterministic Unbounded. Clockwise from the top left: One Update, Two Updates, All Update. The $x$-axis is number of rounds $t$; the $y$-axis is price in dollars. Market Price in black.
Results from a No-Skew Market

Figure 7: Data plots for No-Skew, 10 Sellers, Deterministic Bounded. Clockwise from the top left: One Update, Two Updates, All Update. The $x$-axis is number of rounds $t$; the $y$-axis is price in dollars. Market Price in black.
Results from a No-Skew Market

Figure 8: Data plots for No-Skew, 10 Sellers, Deterministic Mix. Clockwise from the top left: One Update, Two Updates, All Update. The $x$-axis is number of rounds $t$; the $y$-axis is price in dollars.
These results weren’t formally part of the overall experiments, but are interesting to mention and put in the report.

More or less achieved what I thought of as a “realistic” looking market.

Has 5 sellers, mix of bounded and unbounded functions. 2 of 5 sellers have nondeterministic strategies.
Results from Program Testing

- The functions from this testing
  - $f_1(x_n) = 0.9(\text{ave } \{x_i\})$
  - $f_2(x_n) = \min\{1.05(mp), 1.1(\text{ave } \{x_i\})\}$
  - $f_3(x_n) = \begin{cases} 
  b = \text{rr}(0, 4) \\
  \text{if } b == 0: \text{ return } 1.25(mp) \\
  \text{else: return } 0.95(mp) 
\end{cases}$
  - $f_4(x_n) = \begin{cases} 
  \text{do not exceed } 1.2(mp) \\
  \text{do not go below } 0.85(mp) \\
  \text{otherwise, return } \sum_i \frac{i}{n}x_i 
\end{cases}$
  - $f_5(x_n) = \max\{(\text{rr} (65, 76) /100)(mp), 80\}$
Results from Program Testing
Results from Program Testing
Conclusions

- Pricing purely based on other sellers and market price can quickly fall prey to groups who undercut/overcut the market price.
- Even randomness in determining which sellers update their prices doesn’t help much (think expectation over a large number of trials).
- Randomness in seller strategies contributes more than the randomness tested in determining seller updates (in these experiments).
- Difficult with this model to get a relatively “stable” no-skew market when trying more intuitive functions (i.e., having two seller strategies designed to cancel each other).
- Much room for other testing, different functions, different market price update strategies, different seller choice strategies, assumptions, ...
Thank You!

Any questions?
[S+17] W. A. Stein et al.

*Sage Mathematics Software (Version 7.4.*).