Data Structures And Algorithms

Office : LWSN 3151-1
Office Hours : Tuesdays 10-12pm
Topics

- Maximum Bipartite Matching
  - Augmented Paths
  - Using Reduction
Maximum Bipartite Matching

- Suppose we have a set of people \( L \) and set of jobs \( R \).
- Each person can do only some of the jobs.
- We can model this as a bipartite graph.
Maximum Bipartite Matching

- A matching gives an assignment of people to tasks.
- Want to get as many tasks done as possible.
- So, want a maximum matching: one that contains as many edges as possible.
Maximum Bipartite Matching

- A matching gives an assignment of people to tasks.
- Want to get as many tasks done as possible.
- So, want a maximum matching: one that contains as many edges as possible.
- **perfect matching** if every vertex is matched.
- **Maximum** is not the same as **maximal**: greedy will get to maximal
Maximum Bipartite Matching

- **Maximal matching**
  - matching is maximal if we cannot add any edge to the existing set.

- **Maximum matching**
  - matching $M$ is said to be maximum if for any other matching $M'$, $|M| \geq |M'|$

- A maximum matching $M$ implies $M$ is also maximal
Maximum Bipartite Matching

- A maximum matching $M$ implies $M$ is also maximal
  - The converse of the above is not true
Topics

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Augmented Paths

Definitions

- **Matched Vertex**: Given a matching \( M \), a vertex, \( v \) is said to be matched if there is an edge \( e \) in \( M \) which is incident on \( v \).

- **Augmenting Path**: Given a graph, \( G = (V,E) \) and a matching \( M \), a path \( P \) is called an augmenting path for \( M \) if:
  - The two end points of \( P \) are unmatched by \( M \).
  - The edges of \( P \) alternate between edges of \( M \) and edges of \( M' \).
Augmented Paths

Berge's Theorem

- A matching $M$ is maximum if and only if it has no augmenting path
Augmented Paths

Algorithm:

\[ \text{M} = \text{empty set} \]
\[ \text{While (there is an augmenting path P)} \]
\[ \quad \text{M} = \text{M} + p \]
\[ \text{return M} \]
Augmented Paths

Algorithm:
- \( M = \text{empty set} \)
- \( \text{While (there is an augmenting path } P) \)
  - \( M = M + p \)
- \( \text{return } M \)

But how to find augmenting paths?
Topics

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  - Using Reduction
Reduction

- Given an instance of bipartite matching
- Create an instance of network flow
- Where the solution to the network flow problem can easily be used to find the solution to the bipartite matching.
Reduction
Reduction

- Given bipartite graph $G = (A, B; E)$, direct the edges from $A$ to $B$.
- Add new vertices $s$ and $t$.
- Add an edge from $s$ to every vertex in $A$.
- Add an edge from every vertex in $B$ to $t$.
- Make all the capacities 1.
- Solve maximum network flow problem on this new graph $G'$.
Reduction

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Reduction

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- Add an edge from every vertex in \( B \) to \( t \).
- Make all the capacities 1.
- Solve maximum network flow problem on this new graph \( G' \).

The edges used in the maximum network flow will correspond to the largest possible matching!
Reduction
The maximum flow from source to sink is five units. Therefore, maximum five people can get jobs.
Questions