The Sports Product Market - Part 3

ECONOMICS OF SPORTS (ECON 325)

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Competition in sports, 2 different notions

- On the field. Defined as:
  - Close games.
  - Absence of extremely dominant and wretched teams, “parity.”
  - Churn in the standings, from year-to-year.
- Free entry into the league.
  - Expansion.
  - The league’s quality-quantity tradeoff.

Okay, it's time

- We've danced around competitive balance long enough.
  - Or uncertainty of outcome? *I'd* still pay even if I was certain my team would win. Well maybe not as often.
- So many league policies implicitly or explicitly aim to increase it.
  - Is more balance (always) good?
  - How to model it?
  - How to measure it?

Modeling uncertainty of outcome

- Following Vrooman (2009) and applying a familiar model of club revenue.
  - 2 (representative) team league;
  - Win probability is my team's share of league talent, $w_1 = \frac{t_1}{t_1+t_2}$;
  - The markets in the league are asymmetric, giving #1 (bigger) an advantage in the form of $\sigma = \frac{m_1}{m_2} > 1$;
  - Uncertainty of outcome gets part of the "weight" $(1-\phi), \phi \in [0,1]$, in the revenue function.
    - Equals the product of the 2 opponents' win probabilities, $w_1 w_2$.

$$\text{The revenue function}, R_1 = \sigma[\phi w_1 + (1-\phi)w_1(1-w_1)],$$

- Gets really simple if you make the (strong) assumption that winning and balance have the same weight to customers:

$$\phi = 0.5 \rightarrow R_1 = \sigma(w_1 - 0.5w_1^2); R_2 = (w_2 - 0.5w_2^2).$$

Equilibrium

- As before the equilibrium is where both teams' marginal revenues of wins are equal:

$$\sigma(1-w_1) = (1-w_2); \frac{w_1}{w_2} = \sigma.$$

- So if the big market is 1.5 times as large, its team wins 0.6 fraction of the games. Imbalance!
- You've already seen that revenue sharing, in theory, does nothing to this imbalance.
  - Unless it's done in a way conditional on winning.
  - Called the invariance proposition in the literature.

"Sportsman" owners

- A different perspective on revenue sharing.
- Objective differs: maximize wins subject to a break even constraint.
- Equilibrium determined by average revenue:

$$AR_1 = AR_2.$$

- Competitive balance is worse, because no one cares about it anymore

$$\frac{w_1}{w_2} = \frac{2\sigma - 1}{1 + \sigma} \div \frac{2 - \sigma}{1 + \sigma} = \frac{2\sigma - 1}{2 - \sigma},$$

which would imply, for $\sigma = 1.5, w_1 = 0.8.$

- But . . .

From: twitter.com/mcuban

Sportsmen respond to revenue sharing

- With pure revenue sharing, it's obvious that the average revenues are equal:

$$\frac{TR}{w_1} = \frac{TR}{w_2} \rightarrow \frac{w_1}{w_2} = 1,$$

And you get the competitive balance you were looking for.

  - "Everyone just wants to win, so if you give them equal money, they will all try equally hard."

- Other features of this equilibrium.
  - When you sub in $w_1 = w_2 = 0.5$, the TR per team is $\frac{3}{8}(1+\sigma)$.
  - Since the sportsmen make zero profit, TR is equal to the payroll per team, e.g., for my $\sigma = 1.5$ example it would be $\frac{15}{16} = 0.9375$ ("\$million").
  - Without context this means nothing, but it is actually <u>not the maximum</u> that can be attained, i.e., even though fans would like this competitive balance, *players* would want more revenue/salary.
  - The league could balance this tension by <u>partial revenue sharing</u>, whereby the teams keep a fraction of their revenue and share the rest.

The big 4

- After a brief history lesson, Vrooman asks where MLB, the NFL, the NBA and the NHL reside vis-à-vis the sportsman league and the profit maximization league.
  - Evidenced by player payroll shares of the overall revenue,
  - And the degree of revenue sharing in each league.
- That the players' share of revenues is roughly 2/3 in all 4 leagues, the league monopsony power from days of yore has essentially vanished.
  - The big 4 are essentially sportsmen leagues.
- Since this evolution has so much to do with their labor markets, I postpone analysis of it until a later chapter.

From Vrooman (2009), p. 24

Table 2 Player Costs in NFL and MLB

	National Football League					Major League Baseball				
Year	League Revenue	Player Costs	Player Percent	Mean Salary	Percent Change	League Revenue	Player Costs	Player Percent	Mean Salary	Percent Change
1990	1,314†	539	41.0	395	15.1	1,348†	450	33.4	598	20.2
1991	1,469	693	47.2	463	17.0	1,504	681	45.3	851	42.5
1992	1,491	893	59.9	484	4.6	1,585	916	57.8	1,029	20.8
1993	1,753	1,129	64.4	666	37.7	1,775**	1,005	56.6	1,076	4.6
1994	1,730†	1,110	64.2	628	−5.7	1,132†	717	63.4	1,168	8.6
1995	2,059**	1,399	68.0	717	14.1	1,385	927	66.9	1,111	−4.9
1996	2,235	1,372	61.4	788	9.9	1,775†	939	52.9	1,120	0.8
1997	2,382	1,402	58.9	737	−6.5	2,067	1,117	54.0	1,337	19.3
1998	3,183†	1,770	56.4	993	34.7	2,479**	1,272	51.3	1,399	4.7
1999	3,423*	2,040	59.6	1,056	6.4	2,787	1,490	53.5	1,611	15.2
2000	3,938	2,414	61.3	1,116	5.7	3,178	1,847	58.1	1,896	17.7
2001	4,284	2,383	55.6	1,101	−1.4	3,548†	2,141	60.3	2,139	12.8
2002	4,944*	2,497	50.5	1,316	19.6	3,652	2,455	67.2	2,296	7.3
2003	5,330	2,931	55.0	1,259	−4.3	3,878	2,540	65.5	2,372	3.3
2004	6,029	3,169	52.6	1,331	5.7	4,269	2,514	58.9	2,314	−2.5
2005	6,160	3,269	53.1	1,396	4.9	4,733	2,702	57.1	2,477	7.0
2006	6, 359†	4,015	61.4	1,630	16.8	5,111	2,799	54.8	2,699	9.0

Revenues and player costs $ millions, average player salaries $ thousands. Player costs include benefits and bonuses
Source: *Forbes*, NFLPA and MLBPA and MLB Blue Ribbon Commission
† New TV rights fees
* Expansion teams

From Vrooman (2009), p. 26

Table 3 Player Costs in NBA and NHL

Season	National Basketball Association					National Hockey League				
	League Revenue	Player Costs	Player Percent	Mean Salary	Percent Change	League Revenue	Player Costs	Player Percent	Mean Salary	Percent Change
1989–90	606**	246	40.6	717	24.7	465	139	29.9	221	17.3
1990–91	843†	331	39.2	927	29.3	518	169	32.6	271	22.8
1991–92	1,000	436	43.6	1,100	18.7	575*	224	38.9	368	35.8
1992–93	1,050	509	48.5	1,300	18.2	694†**	284	41.0	467	26.9
1993–94	1,259	521	41.4	1,500	15.4	817**	337	41.2	572	22.5
1994–95	1,403†	647	46.1	1,800	20.0	728†	277	38.1	733	28.1
1995–96	1,664**	781	46.9	2,000	11.1	1,099	563	51.2	892	21.7
1996–97	1,874	827	44.1	2,300	15.0	1,336	623	54.8	984	10.3
1997–98	1,836	995	54.2	2,640	14.8	1,356	697	58.4	1,168	18.7
1998–99	1,220†	720	59.0	3,000	13.6	1,427†*	802	56.2	1,289	10.4
1999–00	2,316	1,381	59.6	3,620	20.7	1,697*	968	57.0	1,356	5.2
2000–01	2,496	1,551	62.1	4,200	16.0	1,906**	1,093	57.3	1,435	5.8
2001–02	2,664	1,524	57.2	4,500	7.1	2,077	1,270	61.0	1,643	14.5
2002–03	2,721†	1,697	62.4	4,546	1.0	2,094	1,389	66.3	1,790	9.0
2003–04	2,932	1,728	58.9	4,917	8.2	2,238	1,477	66.0	1,830	2.2
2004–05	3,185*	1,842	57.8	4,900	−0.3	Lockout season				
2005–06	3,367	1,985	59.0	5,000	2.0	2,267†	1,211	53.4	1,460	−20.2
2006–07	3,573	2,061	57.7	5,215	4.3	2,436	1,325	54.4	1,770	21.2

Revenues and player costs $ millions, average player salaries $ thousands. Player costs include benefits and bonuses
† New TV rights contract
* Expansion teams
Source: *Forbes*

Back to competitive balance

- One way to measure (the absence of) balance would be to look at the variation across team revenues.
  - Note: his data originate from Forbes. You could calculate this yourself for a different year.
- Comparisons using the coefficient of variation (CV).

$$CV \equiv \frac{s}{\bar{x}};\bar{x}(sample\ mean) \equiv \frac{1}{n}\sum_{i=1}^{n} x_i\,;s(standard\ deviation) \equiv \left[\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2\right]^{\frac{1}{2}},$$

i.e., it’s the variation across team revenues, controlling for the units (by dividing by the mean).

Revenue imbalance

- Vrooman's estimates based on 2007, in parentheses (my estimates from Vrooman's 2014 data):
  - NFL : $CV = 0.126\ (0.212)$,
  - NBA: $CV = 0.240\ (0.259)$,
  - MLB: $CV = 0.218\ (0.261)$,
  - NHL: $CV = 0.229\ (0.281)$.
- List is from most to least equal and revenue includes the shared league revenue.
- Every league got less balanced in the last 10 years, especially the NFL.
  - NBA held the line pretty well.

Does it translate into competitive imbalance?

- The concept employed by Vrooman considers imbalance in the same terms used in labor economics to analyze the intergenerational transmission of income inequality.
  - It uses regression analysis.

- You regress the team's ("i") present winning percentage in season "t" (w_{it}) on its winning percentage from the previous season ($w_{i,t-1}$).
  - In labor you regress children's (permanent) incomes on their parents' (permanent) incomes.
  - Fun fact: this was roughly the *original* application of regression analysis by Sir Francis Galton in 1886. He was trying to see the relationship between the height of sons and fathers.

$$E(w_{it}|w_{i,t-1}) = \beta_0 + \beta_1 w_{i,t-1}$$

- This is not the only way to conceive of imbalance, but it does get at how "persistent" success in each league is from year to year.
  - $\beta_0 = 0.5$ and $\beta_1 = 0$ → winning is independent from year to year, ultra balanced.
  - $\beta_0 = 0$ and $\beta_1 = 1$ → winning is deterministic, whoever won last year will win again, unbalanced.

Persistence of winning, year to year

- This is another finding we could replicate using current data, but for now I report Vrooman's estimates.
  - Broken down by league and period.
- The NFL and NHL have trended toward more churn and less persistence over time.
- Baseball had a period of declining persistence, then it increased again in recent years.
- The NBA is the opposite of MLB: a steep rise during the 1980s and falling off in recent years.

	NFL	**MLB**	**NBA**	**NHL**
1971-1982	0.530	0.575	0.580	0.749
1983-1995	0.447	0.313	0.716	0.597
1996-2007	0.286	0.513	0.626	0.556
2008-2017 (Ben)	0.335	0.447	0.634	0.465

Revenue imbalance and competitive imbalance

- Vrooman doesn't make strong conclusions about a connection between the two.
- But Ross Booth (in HSE pp. 552-564) does a nice job analyzing the long history of the Australian Football League, as it went through various institutional changes. The figure summarizes the findings.

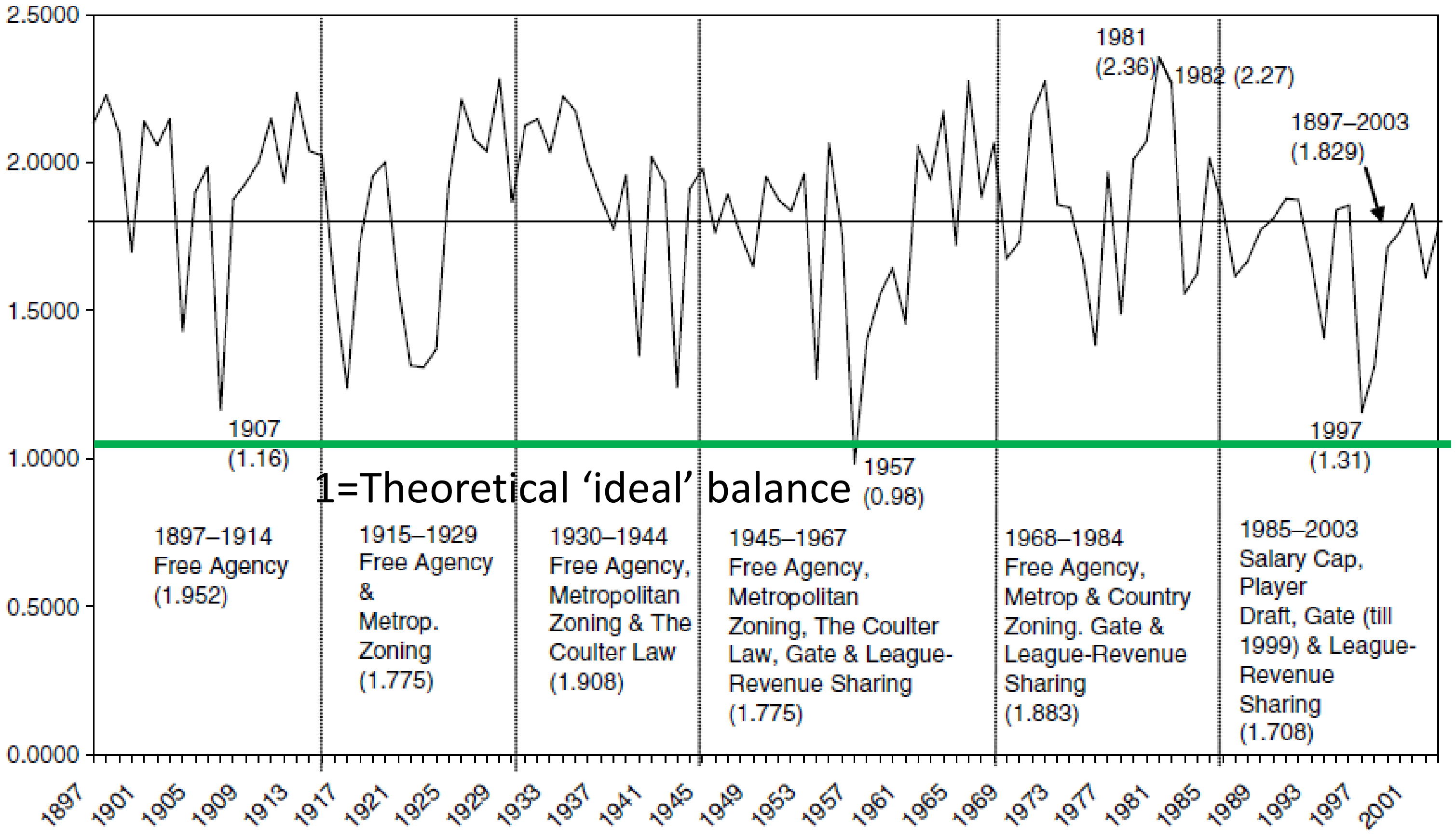


Figure 58.1 Competitive balance ratios in the VFL/AFL, 1897–2003

"Alternative measures of competitive balance . . ."

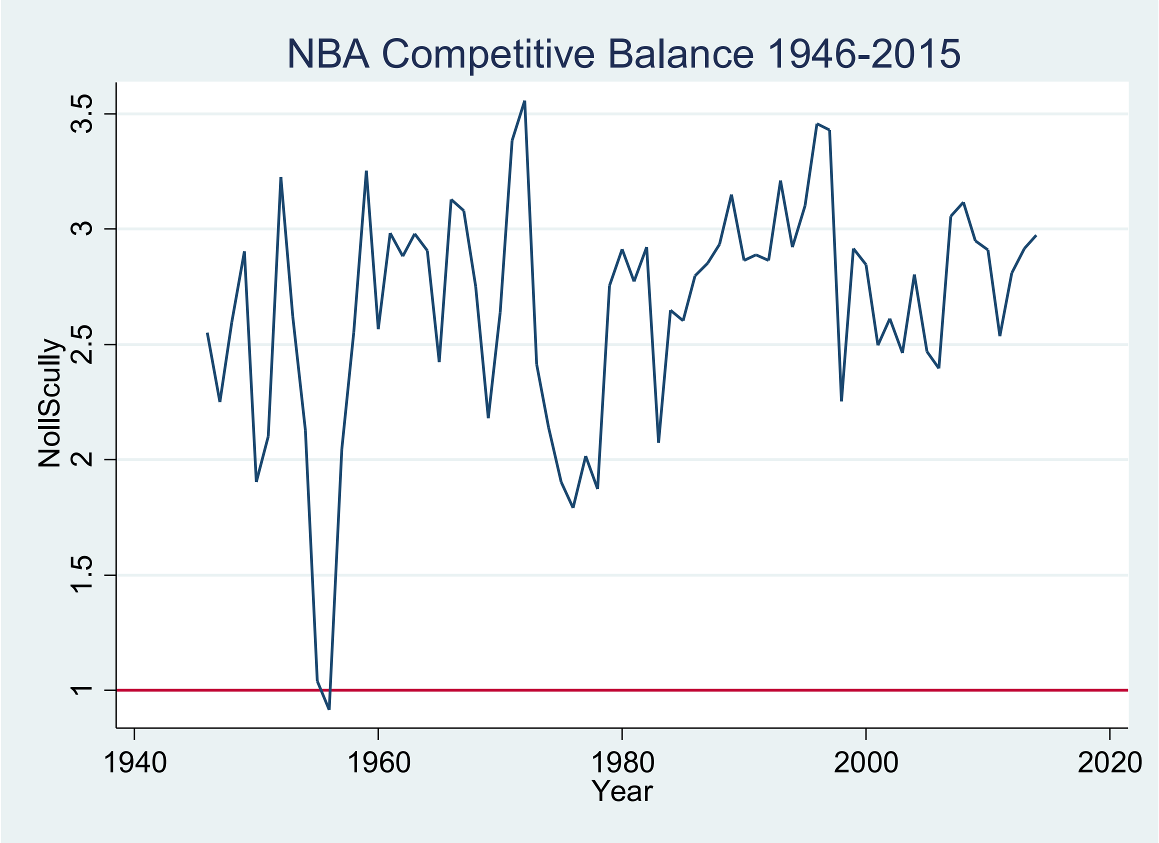
- From the paper by the same name (Humphrey 2002).
- The Noll-Scully statistic: like CV but uses standard deviation of win %s.
  - Named for economists, Gerald Scully and Roger Noll.
  - As a fraction of the s.d. in an "idealized" league, in which all teams have equal strength and variations about 0.500 are random. What Booth's graph is showing.
- Numerator is:

$$s_o = \left[\frac{1}{n}\sum_{i=1}^{n}(w_i - 0.5)^2\right]^{\frac{1}{2}}.$$

- Denominator is: $s^* = G^{-\frac{1}{2}} * 0.5$. This is the standard error of the sample proportion, with $p = 0.5$. G is the number of games played in the league per team.
- Then just divide: $NS \equiv \frac{s_o}{s^*}$, to see how much larger than 1 it is.

Noll-Scully calculations

- As an example, here are textbook author Rodney Fort's Noll-Scully calculations for the NBA.
- A time series graph at right. Except for 2 anomalous seasons in the late 1950s, you see roughly the same pattern as in the Vrooman measure.
  - Dip in the 1970s,
  - Steep rise from the mid 1980s through 1990s,
  - Leveling off after that.



Pros and cons

- Pro: you only need 1 year of data.
- Con: Noll-Scully doesn't capture the churn in the standings that Vrooman's autoregression coefficients do.
- If you have more than 1 year, you can calculate and compare the following to see whether:
  - The imbalance is cross-sectional, same teams are good (bad) every year, or
  - The imbalance is time series, different team on top every year.
  - Calculate:

$$s_i^2 = \frac{1}{n}\sum_{i=1}^{n}(\bar{w}_i - 0.5)^2 \text{ and } s_t^2 = \frac{1}{nT}\sum_{i=1}^{n}\sum_{t=1}^{T}(w_{it} - w_i)^2 ,$$

to see where the most variance originates.

- The ratio of these is eerily similar to what Humphreys calls the "CBR" or Competitive Balance Ratio.

Exercise

- Calculate the Noll-Scully for both of these hypothetical leagues, pooling together all teams and years.
  - $nT = 25.$
- Then try doing the variance decomposition on the previous slide.

Humphreys / COMPETITIVE BALANCE IN SPORTS LEAGUES 135

TABLE 1: Won-Loss Records in Two Hypothetical Leagues

League 1						*League 2*					
Team	*1*	*2*	*3*	*4*	*5*	*Team*	*1*	*2*	*3*	*4*	*5*
A	4-0	4-0	4-0	4-0	4-0	F	4-0	3-1	2-2	1-3	0-4
B	3-1	3-1	3-1	3-1	3-1	G	3-1	2-2	1-3	0-4	4-0
C	2-2	2-2	2-2	2-2	2-2	H	2-2	1-3	0-4	4-0	3-1
D	1-3	1-3	1-3	1-3	1-3	I	1-3	0-4	4-0	3-1	2-2
E	0-4	0-4	0-4	0-4	0-4	J	0-4	4-0	3-1	2-2	1-3

Measures from *industrial organization*

- The Herfindahl-Hirschman Index (HHI) is similar to variance and was devised to measure monopoly power in a given industry.
  - After economists Orris (!) Herfindahl and Albert Hirschman.
  - See Owen et al. (2007) as it applies to sports.
- The idea is to measure each firm's (club's) share of the market (wins), square them and sum them up:

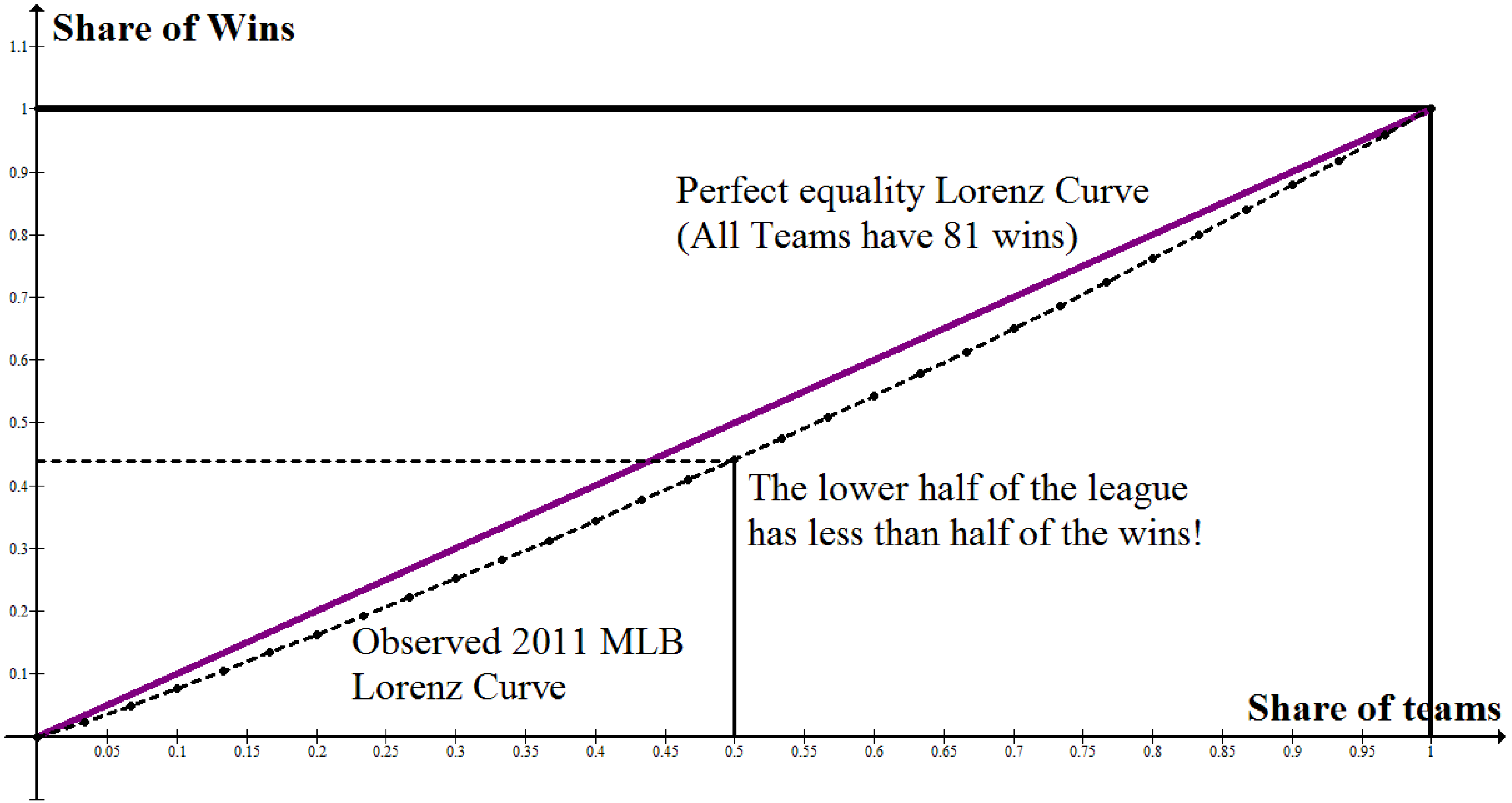
$$HHI \equiv \sum_{i=1}^{n} share_i^2 \,.$$

- The lower bound is 1/n, if all have equal shares, and the upper is 1, if a single monopolist controls the whole market.
  - Anything over, say, 0.2 is getting pretty concentrated.

Measures from *labor economics*

- There are 2 related methods used to measure income inequality.
  - Graphical: the Lorenz curve, and
  - Numerical: the Gini coefficient.
- Both represent the degree of departure from an egalitarian "ideal."
- The Lorenz curve plots the cumulative share of wins, starting from the bottom of the distribution, against the cumulative share of teams.
  - The further is lies from a 45° line, the more inequality there is.
  - The graph is based on Prof. Fort's MLB data for the 2011 season.

2011 MLB Lorenz curve



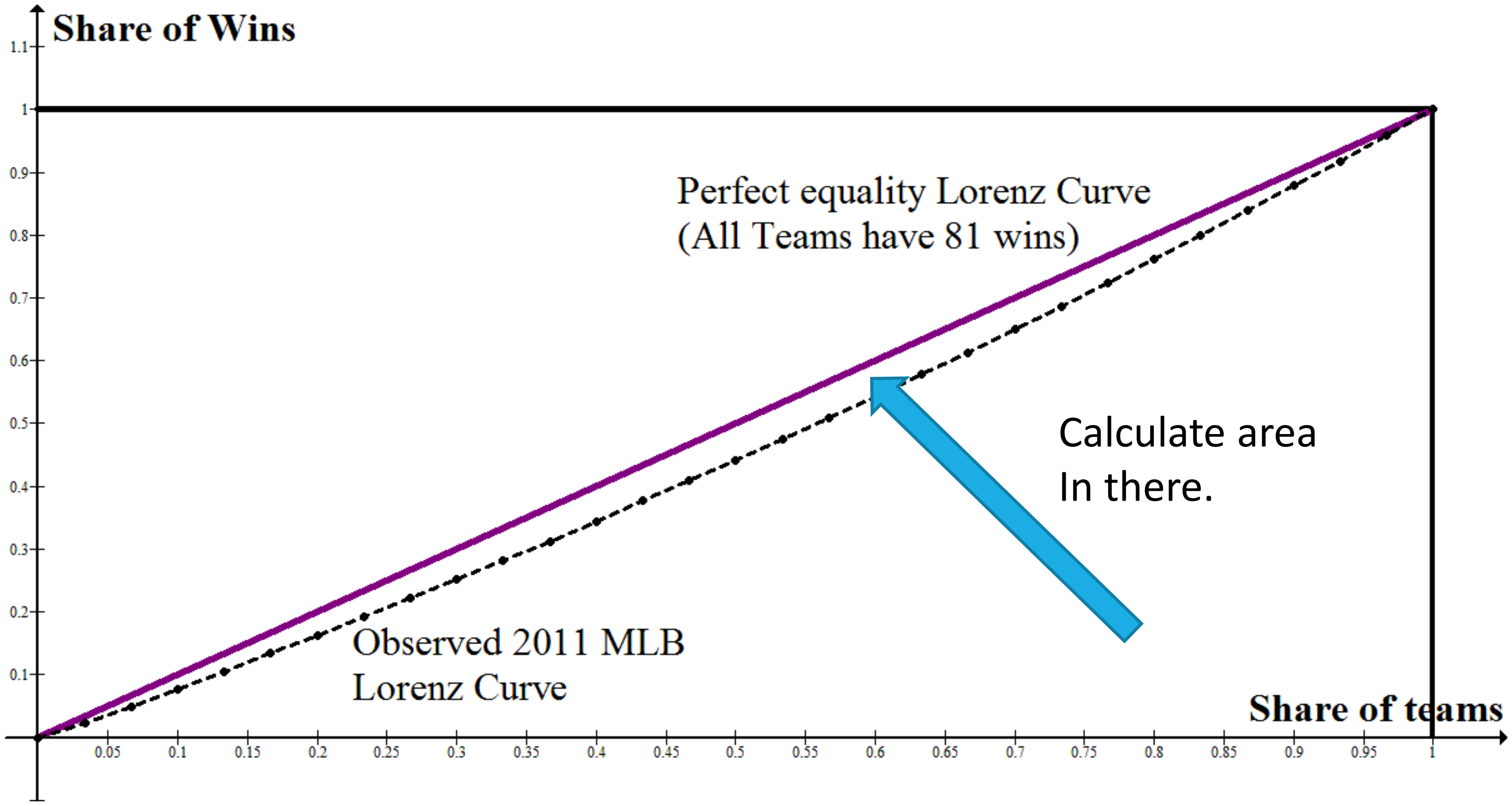
Quantifying the Lorenz curve

- If you looked at 2 of them overlapping, you might be able to compare the inequality shown on 2 Lorenz curves.
  - Sometimes they cross twice, though, which is both interesting and complicated.
- The Gini coefficient quantifies inequality by measuring the area between the observed Lorenz and the (purple) perfect equality Lorenz.
  - Careful inspection reveals it's just geometry (triangles and rectangles).

$$Gini = \left[0.5 - \frac{1}{n} * \sum_{i=1}^{n} \left(shareofwins_{equal\ or\ lower\ than\ i} + shareofwins_{lower\ than\ i}\right)\right],$$

- Lots of them. The above is my best attempt to collapse them into a formula.
  - It's still a lot of number crunching, but there *are* a lot of shapes for 30 teams!

Gini coefficient, illustrated



Gini coefficient, calculated

- Performing the calculations is <u>much</u> easier using software like Excel. I'll show you my calculations used to arrive at the figure of **0.039**, for the 2011 MLB season.

- Compared to the U.S. income distribution (and most others in the world), this is quite low.
  - But baseball is a sport where they say,
  - "You know when a season starts that the best team is going to get beaten a third of the time; the worst team is going to win a third of the time. The argument, over 162 games, is that middle third." – George Will, in <u>Baseball</u> by Ken Burns.

Competitive balance and attendance

- Humphreys concludes the paper by regressing MLB attendance on the various measures of competitive imbalance.
  - He concludes based on the fit of the regression and the precision of the coefficient estimates that the CBR is the best measure of competitive balance,
  - Having the predicted positive effect on attendance.
- Just the tip of the iceberg in this area.
  - Schmidt and Berri (2001) is one prominent example.

Is it the same thing as "uncertainty of outcome" of a given game?

- Kringstad & Gerard (2004) do a good job of summarizing the distinction in the literature.
  - "Competitive balance is the distribution of sporting quality between the teams in a league/tournament." And,
  - "The uncertainty of outcome is the probability distribution of outcomes in individual contests, individual tournaments and repeated tournaments."
- Conclusion, vis-à-vis the sports demand function (regression if we're looking at data), is to structure it like this:

$$Demand = Economic\ factors + Geographic, Demographic\ factors + sporting\ (quality)\ factors + unobservables$$

Uncertainty of outcome, 3 dimensions

- For a single match/game, sporting factors include:
  - Talent of each team (home, away),
  - The historical rivalry between them,
  - The significance to each team, say, with respect to playoff eligibility,
  - 3 dimensions of outcome uncertainty: match-specific, aggregate season uncertainty and long-run league uncertainty.
- The latter is (the opposite of) the Celtics winning all those championships under Red Auerbach.
  - There wasn't much uncertainty in the NBA in early 1960s (they won it 9/10 years!).
- Season uncertainty is the thing I've been getting at the most with the competitive balance measures

Match uncertainty

- Tainsky & Winfree (2010) use an attractive method to estimate the uncertainty about a given MLB game.
  - Use a probit estimation (similar to regression, but used to estimate the *probability that something happens*, in this case the home team wins game "g"), based on the 2 teams' win-loss records.
  - The predicted probability "$p(win_{ig})$" is turned into an uncertainty measure by multiplying by its complement and taking the square root: $GAMEU_{ig} = \left[p(win_{ig}) * \left(1 - p(win_{ig})\right)\right]^{\frac{1}{2}}$.
- I haven't found this in the literature, but I'd like to see a similar measure that incorporates more game idiosyncratic information, e.g.,
  - How good are the 2 starting pitchers?
  - How healthy are the 2 teams' lineups *that day*?
- I'm not very satisfied with how the literature measures variations in team quality within a season, from a theoretical or empirical point of view.

Literature, tip of the iceberg

- Forrest & Simmons (2002): (representative of papers that) use betting odds to measure uncertainty about Premier league matches. FS & Buraimo (2005), too.
  - Knowles, Sherony, Haupert (1992): same with MLB
  - Garcia & Rodriguez (2002): no evidence Spanish soccer fans care
  - Pawlowski & Anders (2012): German soccer fans apparently *don't* care
- Schreyer & Torgler (2016): Formula 1 fans like balance
- Paul, et al (2011) NFL fans *say* they care
- There are dozens of papers on this and no unanimous consensus.

Competition, “free entry” sense

- The premise is that sports are a natural monopolies or club goods. And that the league is the firm.
  - If I start up a basketball team, the NBA doesn’t have to let me play against their teams.
  - Unless I get their “permission” to join the league as an expansion team.
  - Sounds like a major barrier to entry.
- What follows is based on Vrooman (1997) and Kahn (2007).
- The league tries to maximize the value of the incumbent clubs.

Optimal league size

- Clubs are added beginning with the (geographically) most valuable market.
- As long as the value of the marginal club exceeds the average of the incumbents' values, the marginal club should be added.
  - i.e., average is rising.
- Once marginal falls below average, average is declining.
  - No more clubs should be added.
- The league just has to find V*.

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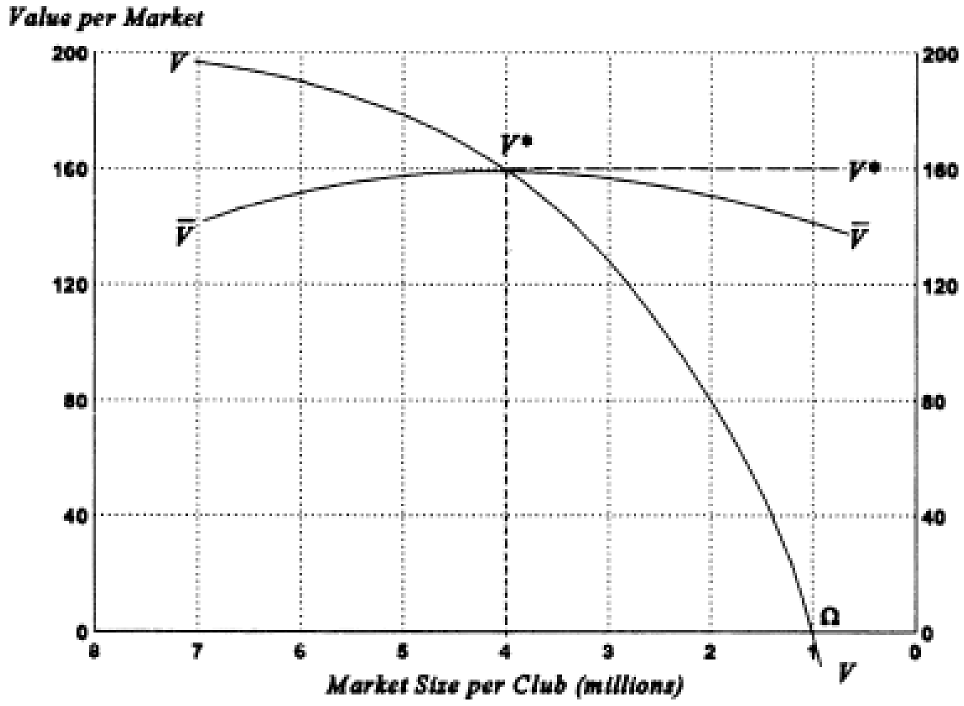

Figure 1. Optimum League Expansion (after Buchanan)

Extensions

- If there is no revenue sharing, the incumbents don’t care about the average anymore, so they would let anyone in with a positive marginal value.
- Or it they could charge the entrant an expansion fee to compensate themselves for lost value, they will expand larger than V*.
- Parallel reasoning that gets you to the same conclusion:
  - the quality of games is a positive (but diminishing) function of the number of games (which increases with more teams),
  - but quality decreases in talent per team, and talent is diluted with more teams,
  - so there is a maximum quality that dictates optimal expansion.

Formal model

- The willingness to pay of fans to watch an individual team in a league with *T* total teams is:

$$P(t) = At^{-a}T^{-q}; 0 \leq a \leq 1.$$

  - Note the cleverness of using *t* as an argument to order the markets by size: 1 to a negative exponent is higher than 2 to a negative exponent! They'll enter in this order (highest value to lowest).

- The relevant costs of operating a franchise are:

$$C(T) = T^c; c > 1, \text{i.e., increasing marginal costs.}$$

- The marginal team has $P(T) = AT^{-(a+q)}$, and free entry would have teams entering up to the point at which this equals the marginal cost of expansion:

$$AT^{-(a+q)} = cT^{c-1} \Leftrightarrow \log(T_{comp}) = \frac{\log(A) - \log(c)}{c - 1 + a + q}.$$

Formal model, continued

- Of course that's not what a monopoly league does, but free entry is a benchmark comparison.
- The monopoly league tries to maximize the sum of incumbents' profits:

$$\Pi = \frac{A}{1-a} T^{1-a-q} - T^{c}$$

  - Note: Total revenue arrived at using integral calculus on the demand function, definite integral [0,T].

- The maximum is where:

$$\frac{A}{1-a}(1-a-q)T^{-(a+q)} = cT^{c-1}$$

$$\Leftrightarrow \log(T_{mono}) = \frac{\log(A) - \log(c) - \log(1-a) + \log(1-a-q)}{c-1+a+q} < \log(T_{comp}).$$

Formal model, continued

- The unusual thing about Kahn's result on the previous slide is that the monopoly outcome is actually the one that maximizes the sum of consumer and producer surplus!
- This comes from the fact that entrants have a negative externality on the quality of the other teams' games that goes un-internalized in the competitive model.
  - But the league would internalize them as monopolist!
- Kahn's model can predict inefficient barriers to entry, too, though. If prices are constrained to be uniform across markets, the league is unable to earn more than $T * P(T)$, the WTP of the marginal market.
  - This would be the case if a large share of the revenue came from fans paying a uniform fee for television broadcasts, for example.
  - The optimum is:

$$\log\left(T_{mono,uniform}\right) = \frac{\log(A) - \log(c) + \log(1 - a - q)}{c - 1 + a + q} < \log(T_{mono}) \, .$$

Formal model, concluded

- Lastly if the league can price discriminate across markets, but still shares the revenue among the teams:
  - Free entry occurs until marginal cost equals the average revenue of the league:

$$\frac{A}{1-a}T^{-(a+q)} = cT^{c-1} \rightarrow \log\left(T_{comp,discr}\right) = \frac{\log(A) - \log(c) - \log(1-a)}{c-1+a+q} < \log\left(T_{comp}\right).$$

  - Actually an improvement compared to the competitive league size under local revenue.

- Not surprisingly when the monopoly league can extract all surplus through price discrimination, it chooses the optimal league size.

Conclusions

- Kahn's "congestion" model of sports league size paints a rosier-than-usual picture of monopoly.
- When it internalizes an externality, a monopoly can produce the socially optimal "output."
  - In this case output is number of teams.
- We have not considered true "entry" in the sense of another firm (league).
  - There are not many historically successful examples (that have not merged with incumbent leagues).
  - Further proof that sports is a natural monopoly?
- Important to make sure "$t \leq T \; \forall \; t$," i.e., you have the best teams in the league.
  - A case for relegation/promotion? Noll (2002)
- Team relocation: a topic for another day when we revisit league policies.
  - First: labor markets.

The invariance proposition

- To see this, pretend that there was full revenue sharing, such that:

$$R_1 = R_2 = \frac{1}{2}(R_1 + R_2).$$

- The MRs would be:

$$\frac{1}{2}[\sigma(1 - w_1) - w_1] \text{ and } \frac{1}{2}[w_1 - \sigma(1 - w_1)].$$

- Setting them equal and solving for the win ratio yields the same result as without revenue sharing!

$$\frac{w_1}{1 - w_1} = \sigma$$

The invariance proposition, continued

- The ratio of marginal products are the same, i.e., vis-à-vis one another neither team is trying harder/less hard than before.
- What is different is the absolute amount they're trying. That's lower for both teams, but proportionally for both. This can be seen by comparing the revenue sharing MR to the non-sharing MR:

$$\frac{1}{2}[\sigma(1 - w_1) - w_1] < \sigma(1 - w_1).$$

- What does this mean in practical terms?

The invariance proposition, concluded

- Equilibrium moves from point A to point C.
- Competitive balance is unchanged.
- Payroll is lower because teams aren't bidding as aggressively for talent.
- Back.

Theory of the Perfect Game 9

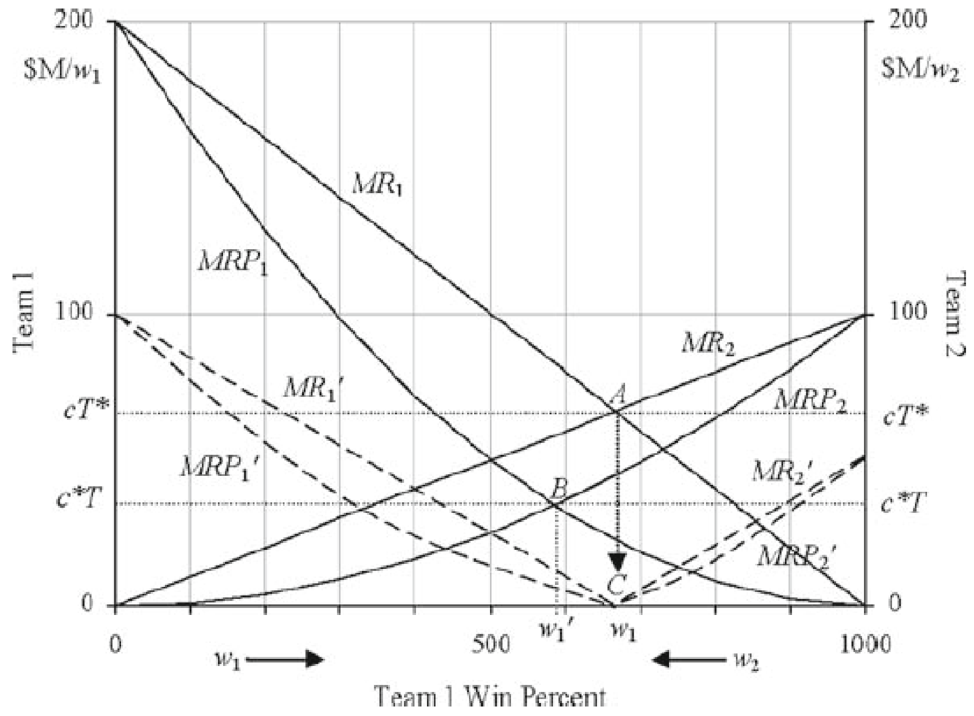

Fig. 1 Invariance Proposition

From Vrooman "Theory of the Perfect Game."